3D applications of trigonometry
HKCEE Paper 1 past paper sorted by topic
(1980 - 2009)

	Vertical pole 鉛垂柱	Solid figures 立體	Shadow 影子	Paper folding 摺紙	Inclined plane 斜面
1980	X				
1981	NA				
1982		X			
1983	X				
1984	X				
1985	X				
1986	X				
1987		X			
1988			X		
1989			X		
1990	X				
1991	NA				
1992				X	
1993	X				
1994	X				
1995			X		
1996					X
1997	NA				
1998			X		
1999				X	
2000	Laser projection on vertical wall 激光投射				
2001				X	
2002	X				
2003			X		
2004					X
2005		X			
2006				X	
2007		X			
2008	X				
2009		X			

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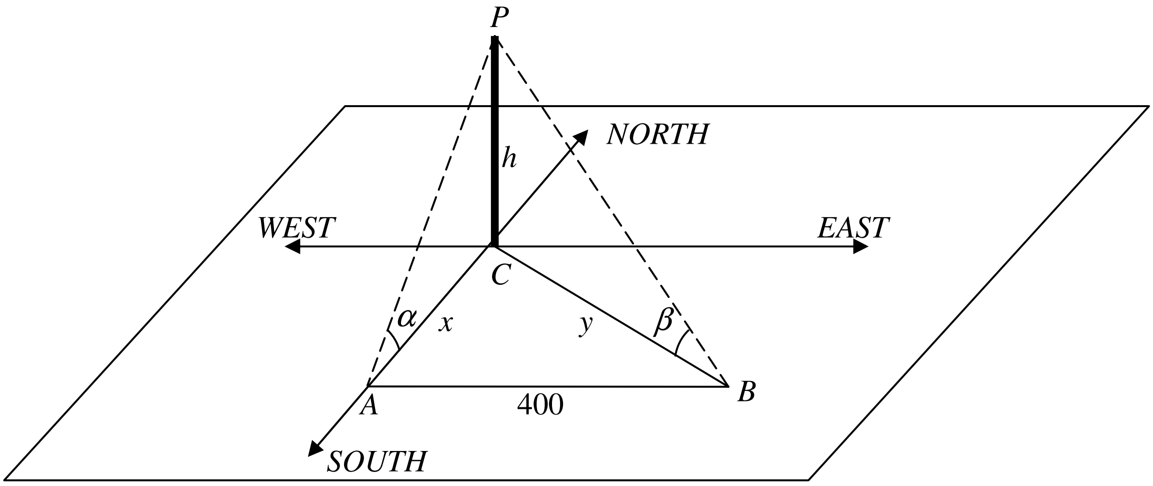
Vertical pole on horizontal triangular plane

1980B9



In the figure, *PC* represents a vertical object of height *h* metres. From a point *A*, south of *C*, the angle of elevation of *P* is α. From a point *B*, 400 metres east of *A*, the angle of elevation of *P* is β. *AC* and *BC* are *x* metres and *y* metres respectively.

(a) (i) Express *x* in terms of *h* and α.

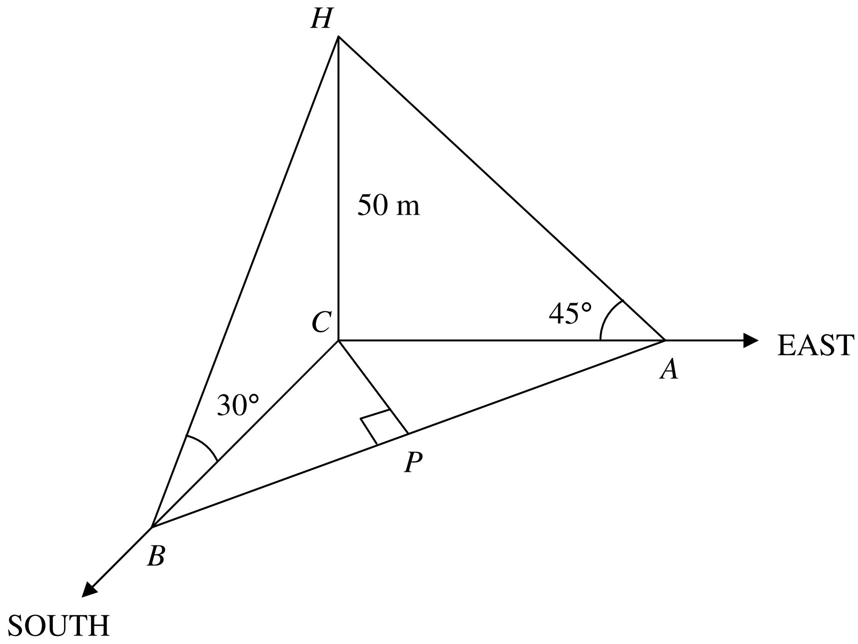
(ii) Express *y* in terms of *h* and β.

(4 marks)

(b) If $\alpha = 60°$ and $\beta = 30°$, find the value of *h* correct to 3 significant figures.

(6 marks)

1983B13



In the figure, A, B and C are three points on the same horizontal ground. HC is a vertical tower 50 m high. A and B are respectively due east and due south of the tower. The angles of elevation of H observed from A and B are respectively $45°$ and $30°$.

(a) Find the distance between A and B.

(6 marks)

(b) P is a point on AB such that $CP \perp AB$.

(i) Find the distance between C and P to the nearest metre.

(ii) Find the angle of elevation of H observed from P to the nearest degree.

(6 marks)

1984B13

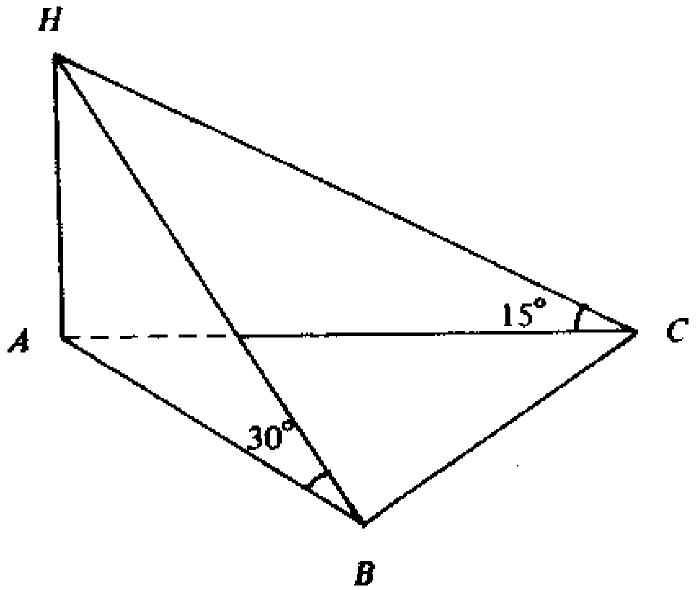


Figure 4

In Figure 4, A, B and C lie in a horizontal plane. $AC = 20$ m. HA is a vertical pole. The angles of elevation of H from B and C are 30° and 15° respectively.

(In this question, give your answers correct to 2 decimal places.)

(a) (i) Find, in m, the length of the pole HA.

(ii) Find, in m, the length of AB.

(6 marks)

(b) If A, B and C lie on a circle with AC as diameter,

(i) find, in m, the distance between B and C;

(ii) find, in m², the area of ΔABC.

(6 marks)

1985B8

8.

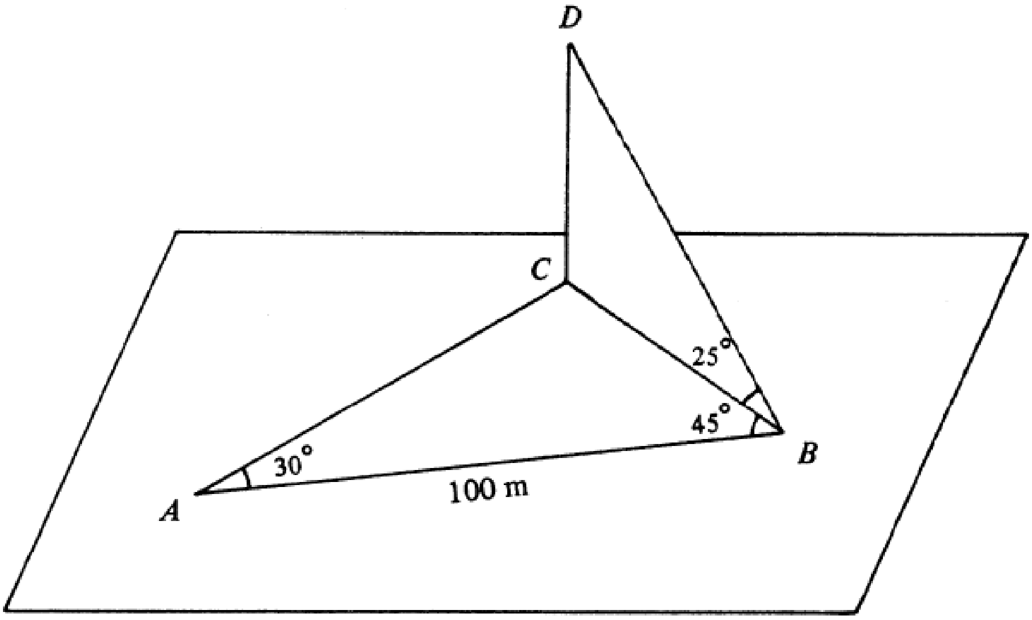


Figure 3

In Figure 3, A, B and C are three points in a horizontal plane. $AB = 100$ m. $\angle CAB = 30°$, $\angle ABC = 45°$.

(a) Find BC and AC, in metres, correct to 1 decimal place.

(5 marks)

(b) D is a point vertically above C. From B, the angle of elevation of D is $25°$.

(i) Find CD, in metres, correct to 1 decimal place.

(ii) X is a point on AB such that $CX \perp AB$.

(1) Find CX, in metres, correct to 1 decimal place.

(2) Find the angle of elevation of D from X, correct to the nearest degree.

(7 marks)

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1986B10

10.

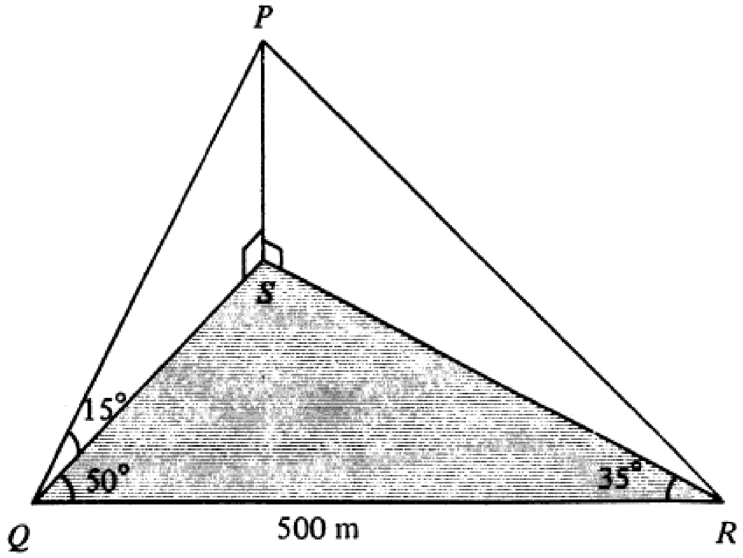

Figure 4

In Figure 4, Q, R and S are three points on the same horizontal plane. QR = 500 m, $\angle SQR = 50°$ and $\angle QRS = 35°$. P is a point vertically above S. The angle of elevation of P from Q is 15°.

(a) Find the distance, in metres, from P to the plane, correct to 3 significant figures.

(6 marks)

(b) Find the angle of elevation of P from R, correct to the nearest degree.

(6 marks)

1990B10

10.

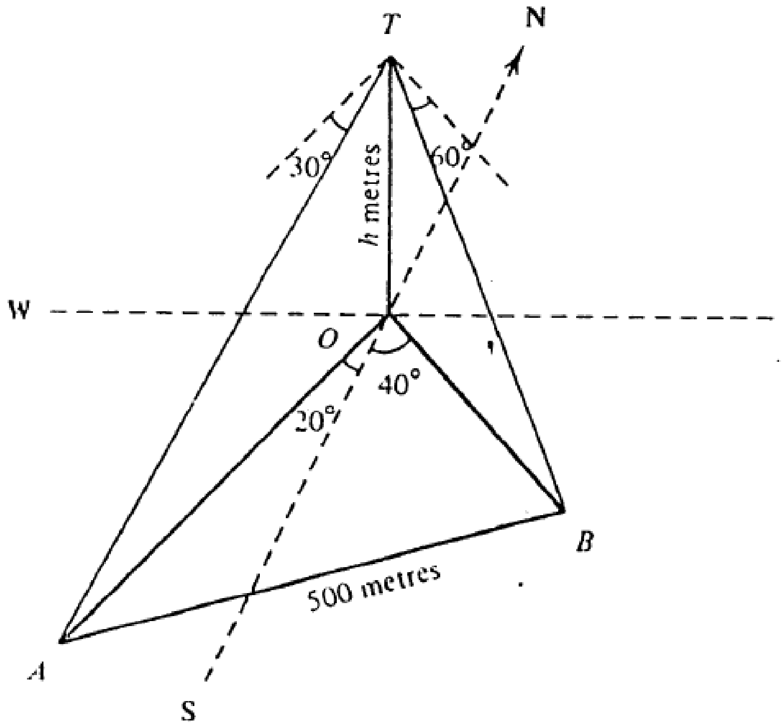

Figure 4

In Figure 4, OT represents a vertical tower of height h metres. From the top T of the tower, two landmarks A and B, 500 metres apart on the same horizontal ground, are observed to have angles of depression 30° and 60° respectively. The bearings of A and B from the tower OT are S20°W and S40°E respectively.

(a) Find the lengths of OA and OB in terms of h.

(3 marks)

(b) Express the length of AB in terms of h. Hence, or otherwise, find the value of h.

(5 marks)

(c) Find $\angle OAB$, correct to the nearest degree.

Hence write down

(i) the bearing of B from A,

(ii) the bearing of A from B.

(4 marks)

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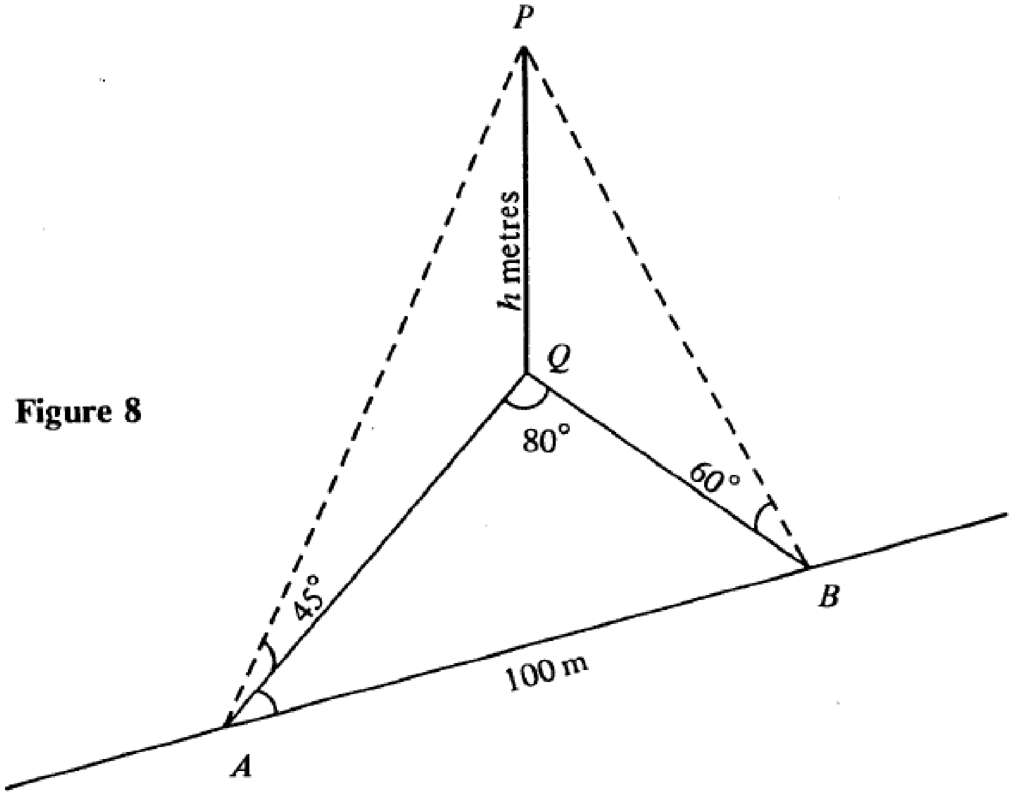
1993B12

12.

Figure 8



In Figure 8, PQ is a vertical television tower h metres high. A and B are two points 100m apart on a straight road in front of the tower with A, B and Q on the same horizontal ground and $\angle AQB = 80°$. The angles of elevation of P from A and B are 45° and 60° respectively.

(a) (i) Express the lengths of AQ and BQ in terms of h.

(ii) Find h and $\angle QAB$.

(8 marks)

(b) A person walks from A along the road towards B. At a certain point R between A and B, the person finds that the angle of elevation of P is 50°. How far away is R from A ?

(4 marks)

1994B14

14. In Figure 8, OT is a vertical tower of height h metres and O, P and Q are points on the same horizontal plane. When a man is at P, he finds that the tower is due north and that the angle of elevation of the top T of the tower is 30°. When he walks a distance of 500 metres in the direction N50°E to Q, he finds that the bearing of the tower is N70°W.

(a) Find OQ and OP. (3 marks)

(b) Find h. (2 marks)

(c) Find the angle of elevation of T from Q, giving your answer correct to the nearest degree. (2 marks)

(d) (i) If he walks a further distance of 400 metres from Q in a direction Nθ°E to a point R (not shown in Figure 8) on the same horizontal plane, he finds that the angle of elevation of T is 20°. Find $\angle OQR$ and hence write down the value of θ to the nearest integer.

(ii) If he starts from Q again and walks the same distance of 400 metres in another direction to a point S on the same horizontal plane, he finds that the angle of elevation of T is again 20°. Find the bearing of S from Q, giving your answer correct to the nearest degree.

(5 marks)

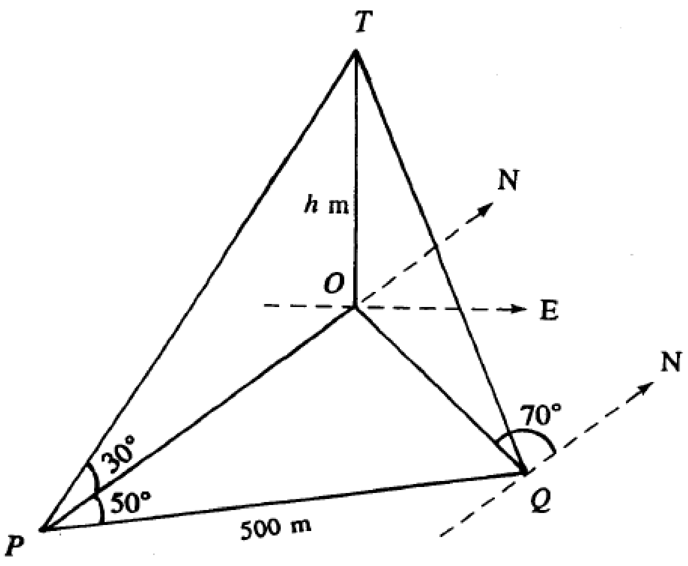


Figure 8

2002B14

14. In Figure 8, AB is a straight track 900 m long on the horizontal ground. E is a small object moving along AB. ST is a vertical tower of height h m standing on the horizontal ground. The angles of elevation of S from A and B are $20°$ and $15°$ respectively. $\angle TAB = 30°$.

(a) Express AT and BT in terms of h. Hence find h. (5 marks)

(b) (i) Find the shortest distance between E and S.

(ii) Let θ be the angle of elevation of S from E. Find the range of values of θ as E moves along AB.

(6 marks)

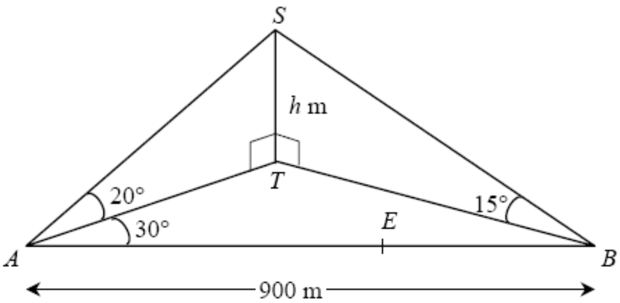


Figure 8

2008B15

15. In Figure 5, H is the top of a tower and A is vertically below H. AB, BC and CA are straight paths on the horizontal ground and D is a point on AB. Christine walks from A to D along AD and finds that the angle of elevation of H from D is $50°$. She then walks 50 m to B along DB and finds that the angle of elevation of H from B is $35°$.

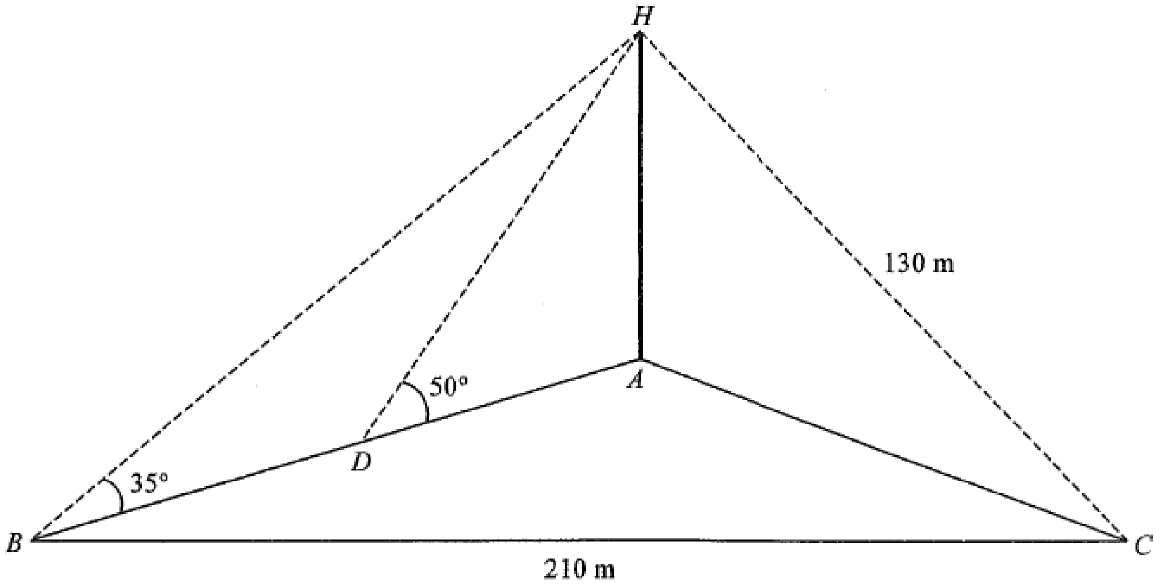


Figure 5

(a) Find the distance between B and H. (2 marks)

(b) Christine walks 210 m from B to C along BC. It is given that the distance between C and H is 130 m.

(i) Find $\angle CBH$.

(ii) Find the angle between the plane BCH and the horizontal ground.

(iii) When Christine walks from B to C along BC, is it possible for her to find a point K on BC such that the angle of elevation of H from K is $75°$? Explain your answer.

(9 marks)

Solid figures

1982B8

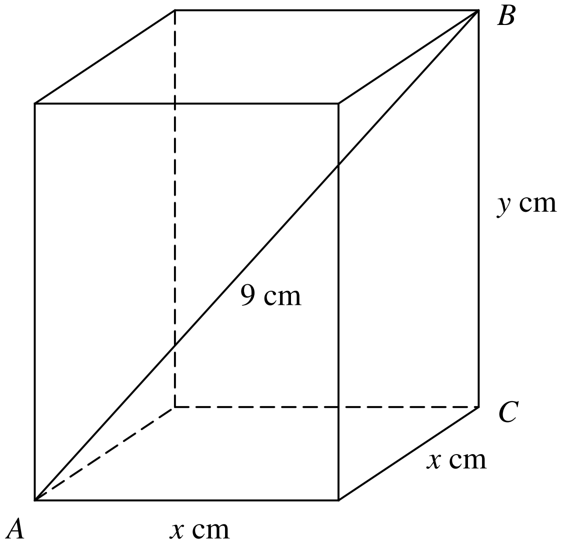


Figure not drawn to scale

The figure represents the framework of a cuboid made of iron wire. It has a square base of side x cm and a height of y cm. The length of the diagonal AB is 9 cm. The total length of wire used for the framework (including the diagonal AB) is 69 cm.

(a) Find all the values of x and y.

(10 marks)

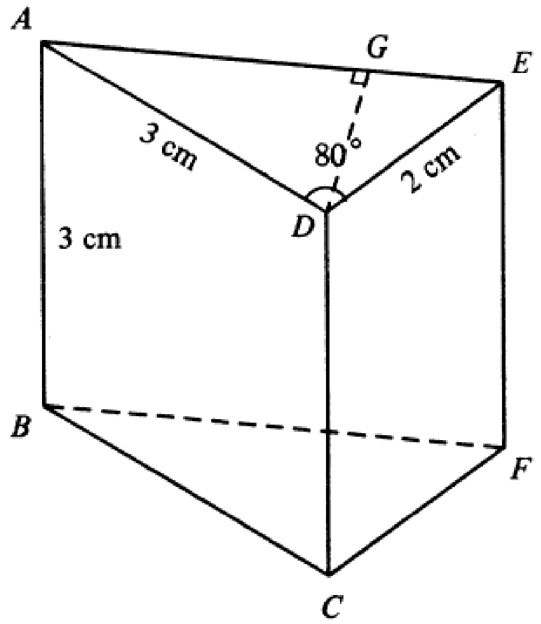
(b) Hence calculate $\angle ABC$ to the nearest degree for the case in which $y > x$.

(2 marks)

1987B11

11.

Figure 6



In this question, you should give your answers in cm or degrees, correct to 3 decimal places.

Figure 6 shows a solid in which $ABCD$, $DCFE$ and $ABFE$ are rectangles. DG is the perpendicular from D to AE. AB = 3 cm, AD = 3 cm and DE = 2 cm. $\angle ADE = 80°$.

(a) Find AE.

(3 marks)

(b) Find $\angle DAE$.

(3 marks)

(c) Find DG.

(2 marks)

(d) Find BD.

(2 marks)

(e) Find the angle between the line BD and the face $ABFE$.

(2 marks)

2005B14

14.

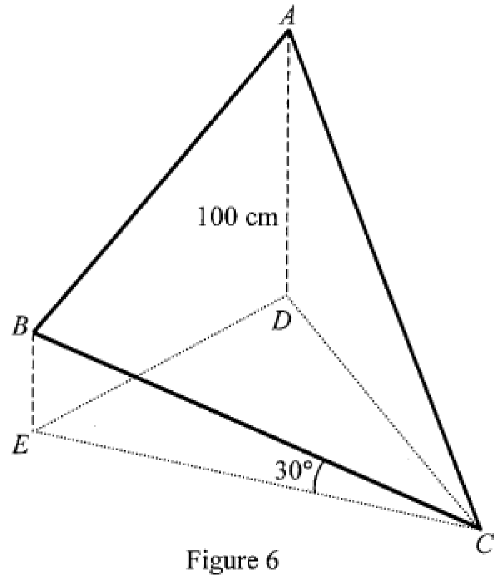


Figure 6

In Figure 6 , a thin triangular board ABC is held with the vertex C on the horizontal ground. D and E are points on the ground vertically below A and B respectively. BC is inclined at an angle of $30°$ with the horizontal. It is known that $AD = 100$ cm , $BC = 120$ cm , $\angle CAB = 60°$ and $\angle ABC = 80°$.

(a) Find BE and CE . (2 marks)

(b) Find AB and AC . (3 marks)

(c) Find $\angle CDE$ and the shortest distance from C to DE . (6 marks)

2007B16

16. Figure 6 shows a solid wooden souvenir $ABCDEF$ with the triangular base ABC lying on the horizontal ground. A, B and C are vertically below E, F and D respectively. DEF is an inclined triangular plane. It is given that $AB = 9$ cm, $BC = 5$ cm, $AC = 6$ cm, $AE = BF = 20$ cm and $CD = 23$ cm .

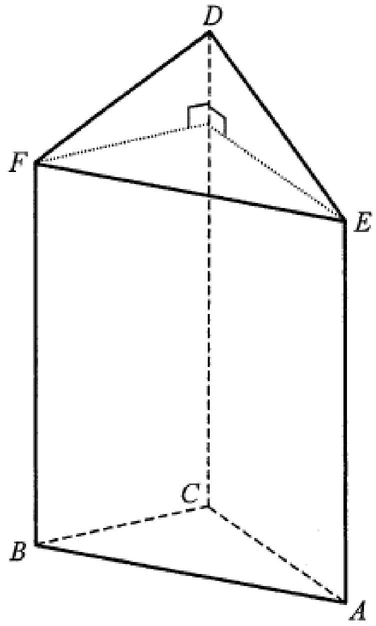


Figure 6

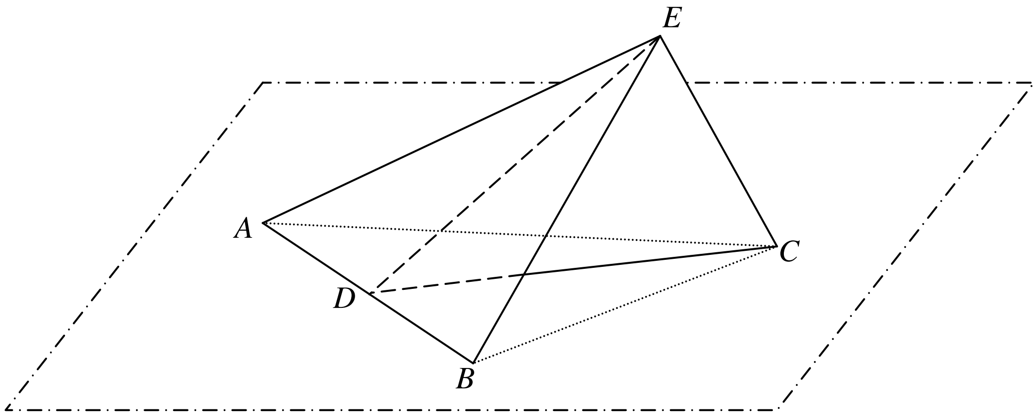
(a) Find the area of the triangular base ABC and the volume of the souvenir $ABCDEF$. (4 marks)

(b) Find $\angle DFE$ and the shortest distance from D to EF. (5 marks)

(c) Can a piece of thin rectangular metal plate of dimensions $5\text{ cm} \times 4\text{ cm}$ be fixed onto the triangular surface DEF so that the thin metal plate completely lies in the triangle DEF ? Explain your answer. (2 marks)

2009B17

The figure shows a geometric model fixed on the horizontal ground. The model consists of two thin triangular metal plates *ABE* and *CDE*, where *D* lies on *AB* and *CE* is perpendicular to the thin metal plate *ABE*. It is given that *A*, *B*, *C* and *D* lie on the horizontal ground. It is found that $AC = 28$ cm, $BC = 25$ cm, $BD = 6$ cm, $BE = 24$ cm and $\angle ABC = 57°$.



(a) Find

- (i) the length of *CD*,
- (ii) $\angle BAC$,
- (iii) the area of ΔABC,
- (iv) the shortest distance from *E* to the horizontal ground.

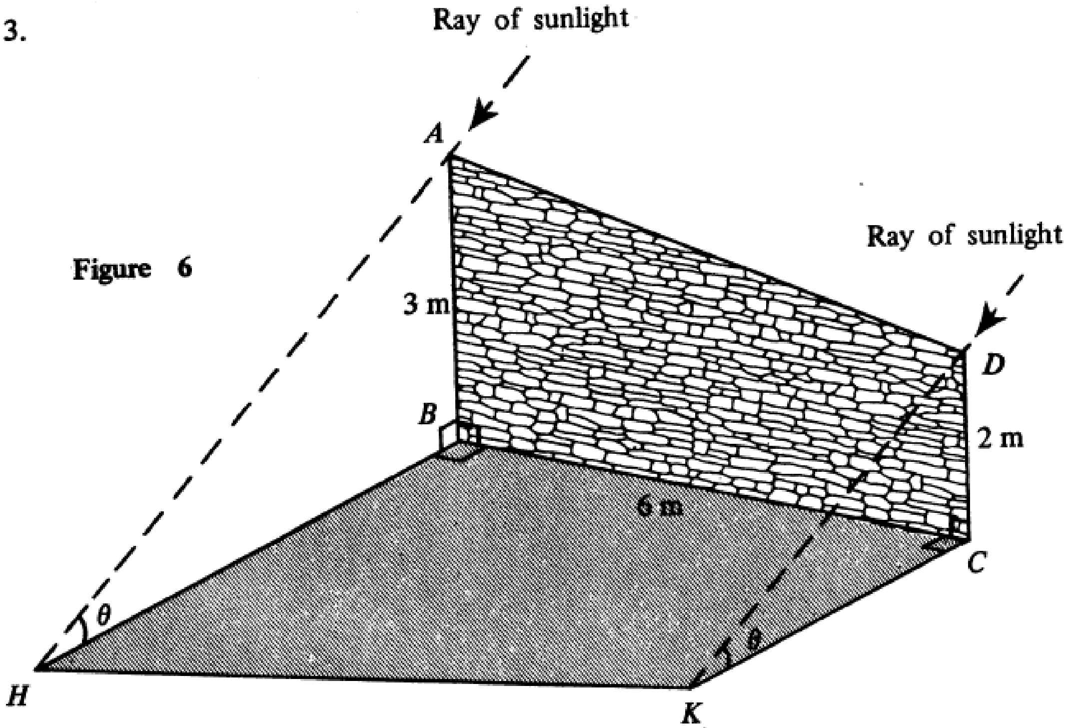
(9 marks)

(b) A student claims that the angle between *DE* and the horizontal ground is $\angle CDE$. Do you agree? Explain your answer. (2 marks)

Plane object under sunshine and its shadow

1988B13

13.



In Figure 6, $ABCD$ is a wall in the shape of a trapezium with AB and DC vertical. Rays of sunlight coming from the back of the wall cast a shadow $HBCK$ on the horizontal ground such that the edges HB and KC of the shadow are perpendicular to BC. Suppose the angle of elevation of the sun is θ, $AB = 3$ m, $CD = 2$ m and $BC = 6$ m.

(a) Express HB and KC in terms of θ.

(3 marks)

(b) (i) Find the area S_1 of the wall.

(ii) Find, in terms of θ, the area S_2 of the shadow.

Hence show that $\frac{S_1}{S_2} = \tan\theta$.

(3 marks)

(c) If $\theta = 30°$, find the length of the edge HK, leaving your answer in surd form.

(6 marks)

1989B10

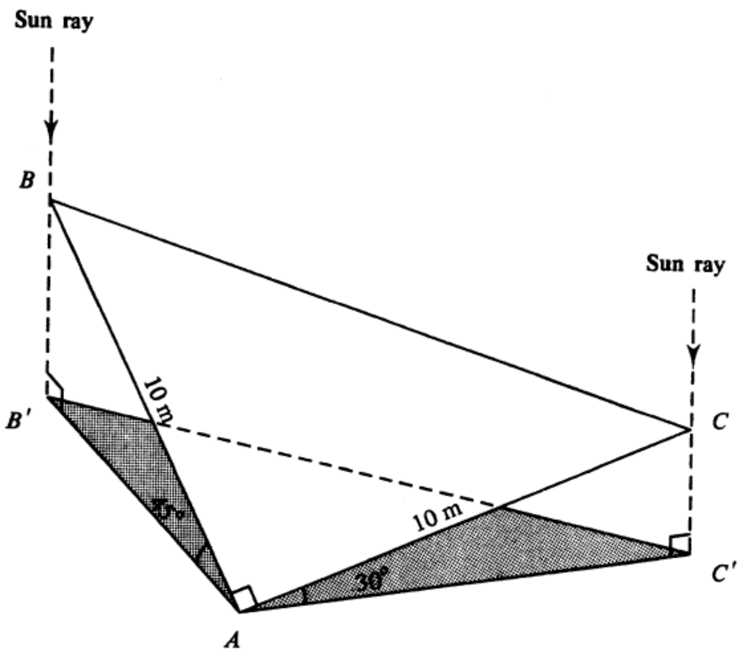


Figure 3

10. **Answers in this question should be given correct to at least 3 significant figures or in surd form.**

In Figure 3, a triangular board ABC, right-angled at A with $AB = AC = 10$ m, is placed with the vertex A on the horizontal ground. AB and AC make angles of 45° and 30° with the horizontal respectively. The sun casts a shadow $AB'C'$ of the board on the ground such that B' and C' are vertically below B and C respectively.

(a) Find the lengths of AB' and AC'.

(2 marks)

(b) Find the lengths of BC, BB' and CC'.

(3 marks)

(c) Using the results of (b), or otherwise, find the length of $B'C'$.

(3 marks)

(d) Find $\angle B'AC'$.

Hence find the area of the shadow.

(4 marks)

1995B15

15. Figure 9 shows a triangular road sign ABC attached to a vertical pole OAB standing on the horizontal ground. The plane ABC is vertical with $OA = 2$ m, $AB = 0.6$ m, $AC = 0.7$ m and $BC = 0.8$ m. D is a point on the horizontal ground vertically below C and is due north of the foot O of the pole.

The sun is due west. When its angle of elevation is $30°$, the shadow of the road sign on the horizontal ground is $A'B'C'$.

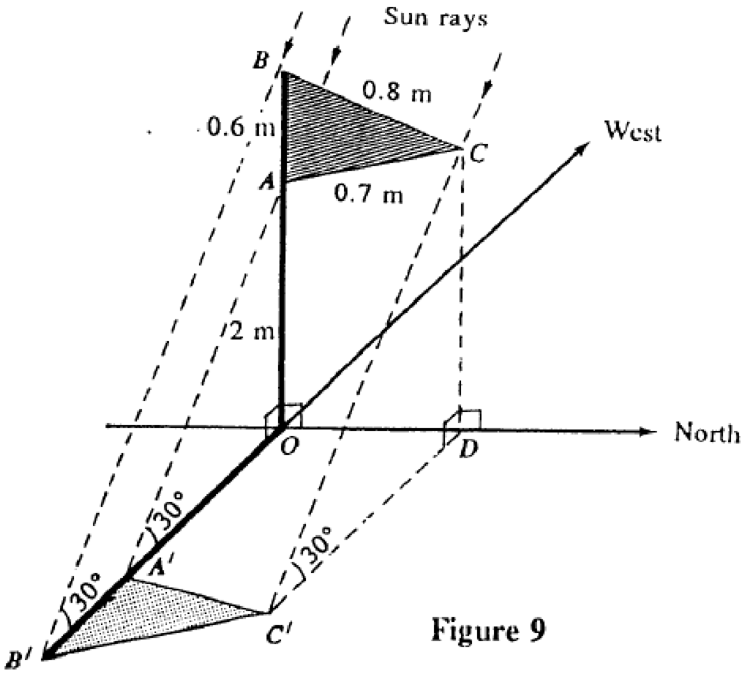


Figure 9

(a) Find the lengths of OA' and $A'B'$. (3 marks)

(b) Calculate $\angle BAC$ and hence find the length of OD. (4 marks)

(c) Find the area of the shadow $A'B'C'$. (2 marks)

(d) If the angle of elevation of the sun is less than $30°$,

(i) state whether the shadow of AB is longer than, shorter than, or equal to $A'B'$ in (a); and hence

(ii) state with reasons whether the area of the shadow of the road sign ABC is larger than, smaller than, or equal to that of $A'B'C'$ in (c).

(3 marks)

1998B17

17. In Figure 10, triangular sign post ABC stands vertically on the horizontal ground along the east-west direction. $AC = 4$ m, $BC = 6$ m, $\angle ACB = 72°$ and F is the foot of the perpendicular from A to BC. When the sun shines from N50°W with an angle of elevation 35°, the shadow of the sign post on the horizontal ground is DBC.

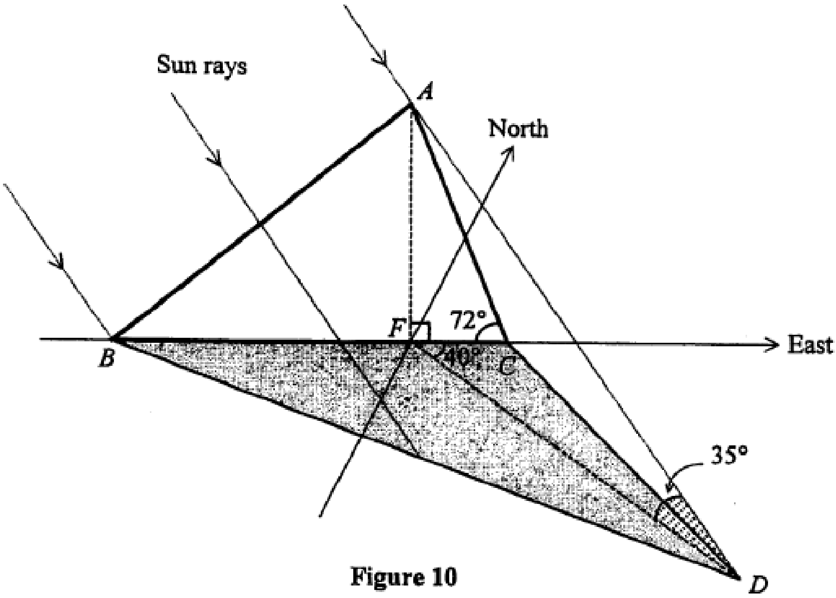


Figure 10

(a) Find AF and FD. (4 marks)

(b) Find the area of the shadow DBC. (5 marks)

(c) Suppose the sun shines from Nx°W, where $50 < x < 90$, but its angle of elevation is still 35°. State **with reasons** whether the area of the shadow of the sign post on the horizontal ground is greater than, smaller than or equal to the area obtained in (b). (2 marks)

2003B14

14.

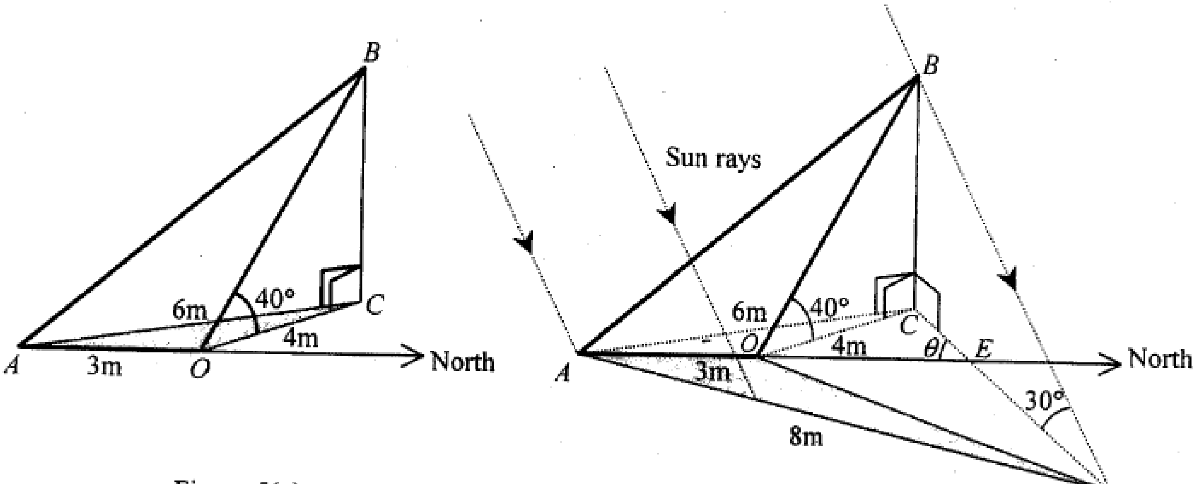


Figure 5(a) Figure 5(b)

Figure 5(a) shows a triangular metal plate OAB standing on the horizontal ground. The side OA lies along the north-south direction on the ground. OB is inclined at an angle of $40°$ to the horizontal. The overhead sun casts a shadow of the plate, OAC, on the ground. $OA = 3$ m, $OC = 4$ m and $AC = 6$ m.

(a) Find $\angle OAC$.

(2 marks)

(b) In Figure 5(b), OAD is the shadow of the plate cast on the horizontal ground when the sun shines from S θ W with an angle of elevation $30°$. AO is produced to cut CD at E. $AD = 8$ m.

(i) Find CD.

(ii) Find $\angle CAD$.

(iii) Using $CE + ED = CD$, or otherwise, find θ.

(9 marks)

Paper Folding

1992B15

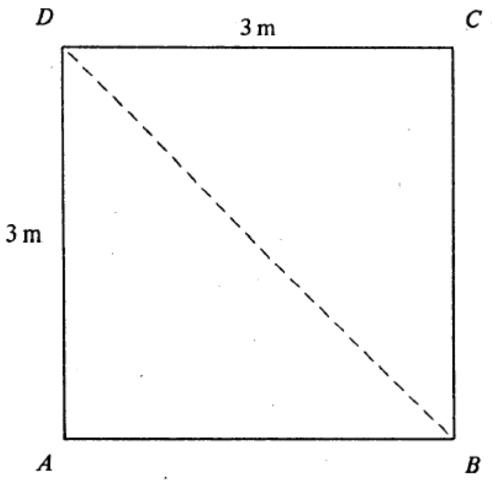


Figure 8a

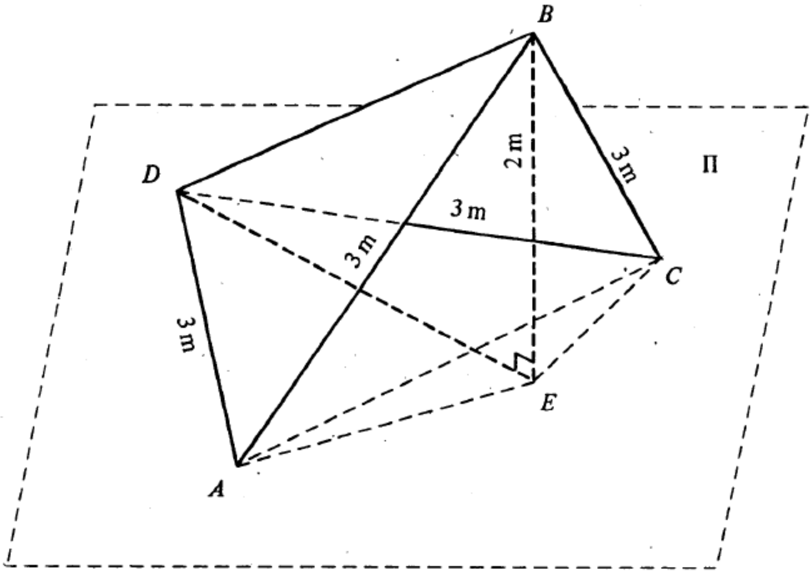


Figure 8b

15.(Cont'd)

In Figure 8a, *ABCD* is a thin square metal sheet of side three metres. The metal sheet is folded along *BD* and the edges *AD* and *CD* of the folded metal sheet are placed on a horizontal plane Π with *B* two metres vertically above the plane Π . *E* is the foot of the perpendicular from *B* to the plane Π . (See Figure 8b)

(a) Find the lengths of *BD* , *ED* and *AE* , leaving your answers in surd form.

(3 marks)

(b) Find $\angle ADE$.

(3 marks)

(c) Find the angle between *BD* and the plane Π .

(2 marks)

(d) Find the angle between the planes *ABD* and *CBD* .

(4 marks)

1999B18

18. In Figure 10, a paper card ABC in the shape of an equilateral triangle of side 24 cm is folded to form a paper aeroplane. D, E and F are points on edge BC so that $BD = DE = EF = FC$. The aeroplane is formed by folding the paper card along the lines AD, AE and AF so that AD and AF coincide. It is supported by two vertical sticks BM and CN of equal length so that A, B, D, F, C lie on the same plane and A, E, M, N lie on the same horizontal ground.

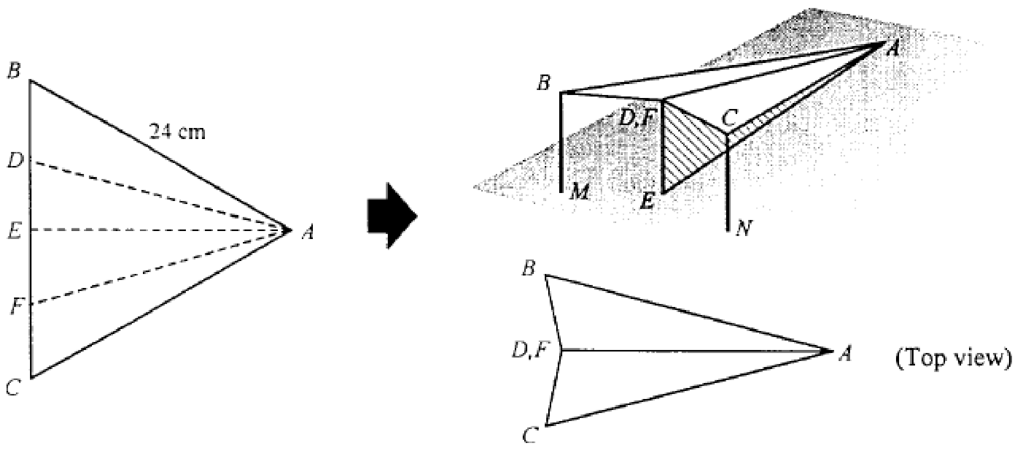


Figure 10

(a) Find the distance between the tips, B and C, of the wings of the aeroplane. (6 marks)

(b) Find the inclination of the wings of the aeroplane to the horizontal ground. (2 marks)

(c) Find the length of the stick CN. (3 marks)

2001B16

16. Figure 11 shows a piece of pentagonal cardboard $ABCDE$. It is formed by cutting off two equilateral triangular parts, each of side x cm, from an equilateral triangular cardboard AFG. AB is 6 cm long and the area of $BCDE$ is $5\sqrt{3}$ cm^2.

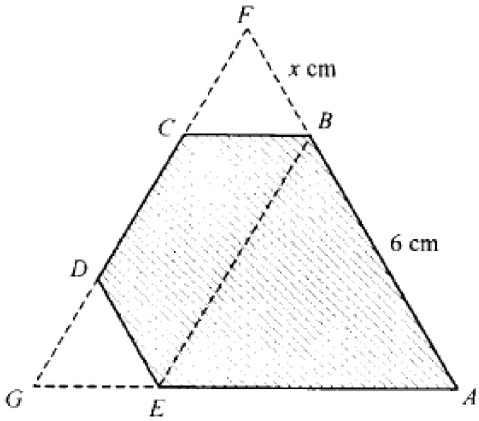


Figure 11

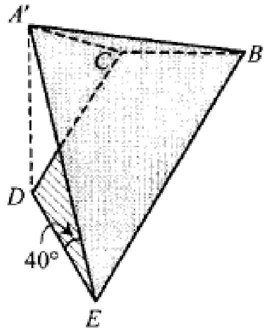


Figure 12

(a) Show that $x^2 - 12x + 20 = 0$.

Hence find x.

(4 marks)

(b) The triangular part ABE in Figure 11 is folded up along the line BE until the vertex A comes to the position A' (as shown in Figure 12) such that $\angle A'ED = 40°$.

(i) Find the length of $A'D$.

(ii) Find the angle between the planes $BCDE$ and $A'BE$.

(iii) If A', B, C, D, E are the vertices of a pyramid with base $BCDE$, find the volume of the pyramid.

(7 marks)

2006B17

In Figure 1, ABC is a triangular paper card. D is a point lying on AC such that BD is perpendicular to AC. It is known that AB = 40cm, BC = 60cm and AC = 90cm.

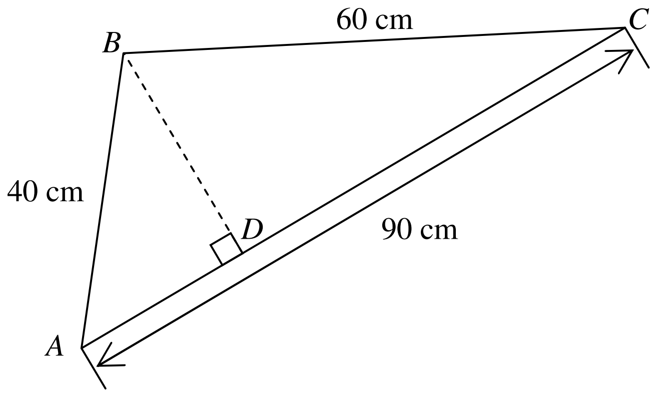


Figure 1

(a) Find AD.

(2 marks)

(b) The triangular paper card in the figure is folded along BD such that AB and BC lie on a horizontal plane as shown in Figure 2.

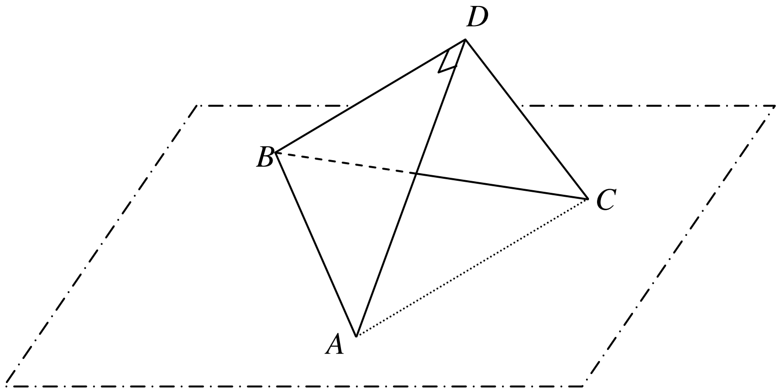


Figure 2

(i) Suppose $\angle DAC = 62°$.

(1) Find the distance between A and C on the horizontal plane.

(2) Using Heron's formula, or otherwise, find the area of ΔABC on the horizontal plane.

(3) Find the height of the tetrahedron $ABCD$ from the vertex D to the base ΔABC.

(ii) Describe how the volume of the tetrahedron $ABCD$ varies when $\angle ADC$ increases from $30°$ to $150°$. Explain your answer.

(9 marks)

Rectangular inclined plane
1996B15

15. In Figure 8, the rectangular plane $ABCD$ is a hillside with inclination $30°$. C' and O' are vertically below C and O respectively so that A, B, C', O' are on the same horizontal plane. BO is a straight path on the hillside which makes an angle $60°$ with BC, and OT is a vertical tower. $AB = 2\,000$ m, $BO = 1\,000$ m and $OT = 50$ m.

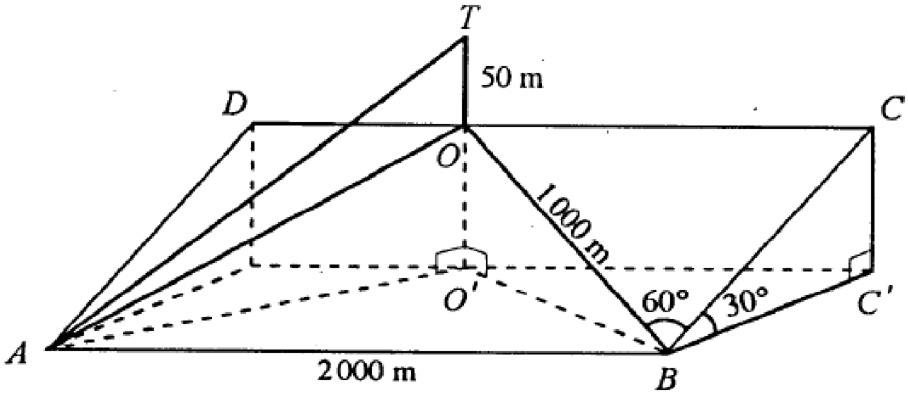


Figure 8

(a) Find BC and CC'. (2 marks)

(b) Find the inclination of BO with the horizontal. (2 marks)

(c) Find AT. (5 marks)

(d) There are cable cars going directly from A to T. A man wants to go to T from B and he can do this by taking either one of the following two routes:

Route I : Walking uphill along BO at an average speed of 0.3 m/s and taking a lift in the tower for 1 minute from O to T.

Route II: Walking along BA at an average speed of 0.8 m/s and taking a cable car from A to T at an average speed of 3.2 m/s.

Determine which route takes a shorter time.

(3 marks)

2004B17

17.

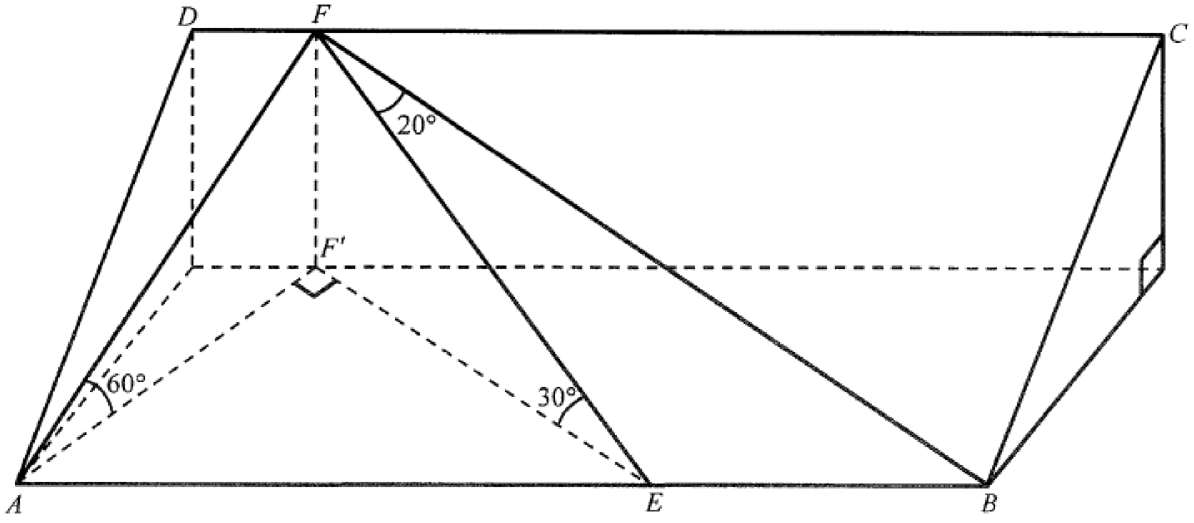


Figure 9

In Figure 9, $ABCD$ is a rectangular inclined plane. E and F are points on the straight lines AB and CD respectively. F' is vertically below F. A, E, B and F' are on the same horizontal ground. $\angle AF'E = 90°$, $\angle FAF' = 60°$, $\angle FEF' = 30°$, $\angle EFB = 20°$ and $EF = 20$ m.

(a) Find

(i) FF' and AE,

(ii) $\angle AEF$.

(7 marks)

(b) A small red toy car goes straight from E to B at an average speed of 2 m/s while a small yellow toy car goes straight from F to B at an average speed of 3 m/s. The two toy cars start going at the same time. Will the yellow toy car reach B before the red one? Explain your answer. (4 marks)

Laser projection

2000B17

17. Figure 10 shows a circle with centre O and radius 10 m on a vertical wall which stands on the horizontal ground. A, B and C are three points on the circumference of the circle such that A is vertically below O, $\angle AOB = 90°$ and $\angle AOC = 20°$. A laser emitter D on the ground shoots a laser beam at B. The laser beam then sweeps through an angle of $30°$ to shoot at A. The angles of elevation of B and A from D are $60°$ and $30°$ respectively.

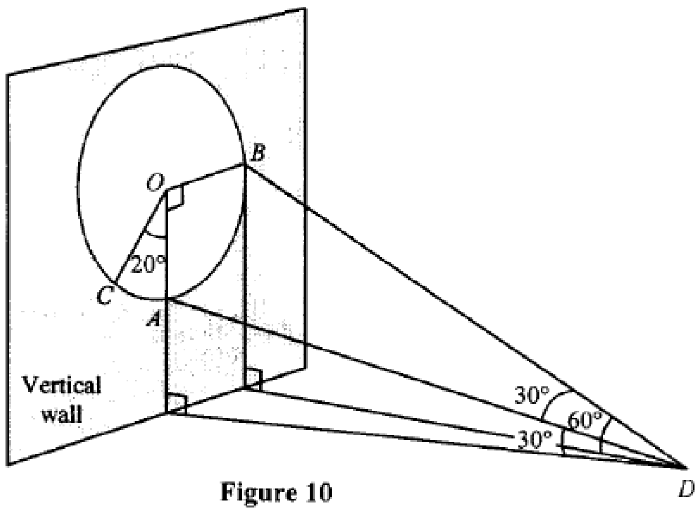


Figure 10

(a) Let A be h m above the ground.

(i) Express AD and BD in terms of h.

(ii) Find h.

(7 marks)

(b) Another laser emitter E on the ground shoots a laser beam at A with angle of elevation $25°$. The laser beam then sweeps through an angle of $5°$ to shoot at C. Find $\angle ACE$.

(4 marks)