Learning Objectives of Module 2 (Algebra and Calculus)

Notes:

1. *Learning units are grouped under three areas ("Foundation Knowledge", "Algebra" and "Calculus") and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*
4. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*

Learning Unit	Learning Objective	Time	Remarks
Foundation Knowledge Area			
1 Surds	1.1 rationalise the denominators of expressions of the form $\dfrac{k}{\sqrt{a} \pm \sqrt{b}}$	1.5	This topic can be introduced when teaching limits and differentiation.

Learning Unit	Learning Objective	Time	Remarks
2. Mathematical induction	2.1 understand the principle of mathematical induction	3	Only the First Principle of Mathematical Induction is required. Applications to proving propositions related to the summation of a finite sequence are included. Proving propositions involving inequalities is **not** required.
3. Binomial Theorem	3.1 expand binomials with positive integral indices using the Binomial Theorem	3	Proving the Binomial Theorem is required. The use of the summation notation ($\sum$) should be introduced. The following are **not** required: • expansion of trinomials • the greatest coefficient, the greatest term and the properties of binomial coefficients • applications to numerical approximation

Learning Unit	Learning Objective	Time	Remarks
4. More about trigonometric functions	4.1 understand the concept of radian measure 4.2 find arc lengths and areas of sectors through radian measure 4.3 understand the functions cosecant, secant and cotangent and their graphs 4.4 understand the identities $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ 4.5 understand compound angle formulae and double angle formulae for the functions sine, cosine and tangent, and product-to-sum and sum-to-product formulae for the functions sine and cosine	11	Simplifying trigonometric expressions by identities is required. The following formulae are required: • $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ • $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ • $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ • $\sin 2A = 2 \sin A \cos A$ • $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$ • $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$ • $\sin^2 A = \dfrac{1}{2}(1 - \cos 2A)$

Learning Unit	Learning Objective	Time	Remarks
			• $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$ • $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$ • $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$ • $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$ • $\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$ • $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$ • $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$ • $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$ “Subsidiary angle form” is **not** required. $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$ and $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$ can be considered as formulae derived from the double angle formulae.

Learning Unit	Learning Objective	Time	Remarks
5. Introduction to the number e	5.1 recognise the definitions and notations of the number e and the natural logarithm	1.5	Two approaches for the introduction to e can be considered: • $e = \lim_{n \to \infty}(1 + \frac{1}{n})^n$ (proving the existence of this limit is **not** required) • $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ This section can be introduced when teaching Learning Objective 6.1.
	Subtotal in hours	20	

Learning Unit	Learning Objective	Time	Remarks
Calculus Area			
Limits and Differentiation			
6. Limits	6.1 understand the intuitive concept of the limit of a function	3	Students are **not** required to distinguish "continuous functions" and "discontinuous functions" from their graphs. The theorem on the limits of sum, difference, product, quotient, scalar multiple and composite functions should be introduced but the proofs are **not** required.

Learning Unit	Learning Objective	Time	Remarks
	6.2 find the limit of a function		The following formulae are required: • $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ • $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$ Finding the limit of a rational function at infinity is required.
7. Differentiation	7.1 understand the concept of the derivative of a function	14	Students should be able to find the derivatives of elementary functions, including C, x^n (n is a positive integer), $\sqrt{x}$, $\sin x$, $\cos x$, e^x, $\ln x$ from first principles. Notations including y', $f'(x)$ and $\frac{dy}{dx}$ should be introduced. Testing differentiability of functions is **not** required.
	7.2 understand the addition rule, product rule, quotient rule and chain rule of differentiation		The following rules are required:

Learning Unit	Learning Objective	Time	Remarks
	7.3 find the derivatives of functions involving algebraic functions, trigonometric functions, exponential functions and logarithmic functions		• $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$ • $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$ • $\frac{d}{dx}(\frac{u}{v})=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ • $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$ The following formulae are required: • $(C)' = 0$ • $(x^n)' = n x^{n-1}$ • $(\sin x)' = \cos x$ • $(\cos x)' = -\sin x$ • $(\tan x)' = \sec^2 x$ • $(\cot x)' = -\operatorname{cosec}^2 x$ • $(\sec x)' = \sec x \tan x$ • $(\operatorname{cosec} x)' = -\operatorname{cosec} x \cot x$ • $(e^x)' = e^x$ • $(\ln x)' = \frac{1}{x}$

Learning Unit	Learning Objective	Time	Remarks
			The following types of algebraic functions are required: • polynomial functions • rational functions • power functions x^{α} • functions formed from the above functions through addition, subtraction, multiplication, division and composition, for example $\sqrt{x^2+1}$
	7.4 find derivatives by implicit differentiation		Logarithmic differentiation is required.
	7.5 find the second derivative of an explicit function		Notations including y'', $f''(x)$ and $\frac{d^2y}{dx^2}$ should be introduced. Third and higher order derivatives are **not** required.
8. Applications of differentiation	8.1 find the equations of tangents and normals to a curve	14	
	8.2 find maxima and minima		Local and global extrema are required.

Learning Unit	Learning Objective	Time	Remarks
	8.3 sketch curves of polynomial functions and rational functions 8.4 solve the problems relating to rate of change, maximum and minimum		The following points are noteworthy in curve sketching: • symmetry of the curve • limitations on the values of x and y • intercepts with the axes • maximum and minimum points • points of inflexion • vertical, horizontal and oblique asymptotes to the curve Students may deduce the equation of the oblique asymptote to the curve of a rational function by division.
	Subtotal in hours	31	

Learning Unit	Learning Objective	Time	Remarks
Integration			
9. Indefinite integration	9.1 recognise the concept of indefinite integration 9.2 understand the properties of indefinite integrals and use the integration formulae of algebraic functions, trigonometric functions and exponential functions to find indefinite integrals	16	Indefinite integration as the reverse process of differentiation should be introduced. The following formulae are required: • $\int k\,dx = kx + C$ • $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, where $n \neq -1$ • $\int \frac{1}{x}\,dx = \ln\lvert x \rvert + C$ • $\int e^x\,dx = e^x + C$ • $\int \sin x\,dx = -\cos x + C$ • $\int \cos x\,dx = \sin x + C$ • $\int \sec^2 x\,dx = \tan x + C$ • $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$ • $\int \sec x \tan x\,dx = \sec x + C$ • $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$ For more complicated calculations, see Learning Objectives 9.4 to 9.6.

Learning Unit	Learning Objective	Time	Remarks
	9.3 understand the applications of indefinite integrals in real-life or mathematical contexts		Applications of indefinite integrals in some fields such as geometry and physics are required.
	9.4 use integration by substitution to find indefinite integrals		
	9.5 use trigonometric substitutions to find the indefinite integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$ or $\sqrt{a^2 + x^2}$		Notations including $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$ and their related principal values should be introduced.
	9.6 use integration by parts to find indefinite integrals		$\int \ln x \, dx$ can be used as an example to illustrate the method of integration by parts. The use of integration by parts is limited to at most two times in finding an integral.

Learning Unit	Learning Objective	Time	Remarks
10. Definite integration	10.1 recognise the concept of definite integration	11	The definition of the definite integral as the limit of a sum and finding a definite integral from the definition should be introduced. The use of dummy variables, including $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$, is required. Using definite integration to find the sum to infinity of a sequence is **not** required.
	10.2 understand the properties of definite integrals		The following properties are required: • $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ • $\int_a^a f(x)\,dx = 0$ • $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ • $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$ • $\int_a^b [f(x) \pm g(x)]\,dx$ $= \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

Learning Unit	Learning Objective	Time	Remarks
	10.3 find definite integrals of algebraic functions, trigonometric functions and exponential functions		Fundamental Theorem of Calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $\frac{d}{dx}F(x) = f(x)$, should be introduced.
	10.4 use integration by substitution to find definite integrals		
	10.5 use integration by parts to find definite integrals		The use of integration by parts is limited to at most two times in finding an integral.
	10.6 understand the properties of the definite integrals of even, odd and periodic functions		The following properties are required: • $\int_{-a}^{a} f(x)\,dx = 0$ if f is odd • $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$ if f is even • $\int_0^{nT} f(x)\,dx = n\int_0^T f(x)\,dx$ if $f(x+T) = f(x)$, i.e. f is periodic

Learning Unit	Learning Objective	Time	Remarks
11. Applications of definite integration	11.1 understand the application of definite integrals in finding the area of a plane figure	4	
	11.2 understand the application of definite integrals in finding the volume of a solid of revolution about a coordinate axis or a line parallel to a coordinate axis		Only “disc method” is required. Finding the volume of a hollow solid is required.
	Subtotal in hours	31	
Algebra Area			
Matrices and Systems of Linear Equations			
12. Determinants	12.1 recognise the concept and properties of determinants of order 2 and order 3	3	The following properties are required: • $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ • $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = -\begin{vmatrix} c_1 & b_1 & a_1 \\ c_2 & b_2 & a_2 \\ c_3 & b_3 & a_3 \end{vmatrix}$ • $\begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} = 0$

Learning Unit	Learning Objective	Time	Remarks
			• $\begin{vmatrix} a_1 & kb_1 & c_1 \\ a_2 & kb_2 & c_2 \\ a_3 & kb_3 & c_3 \end{vmatrix} = k\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ • $\begin{vmatrix} a_1 & b_1 & kb_1 \\ a_2 & b_2 & kb_2 \\ a_3 & b_3 & kb_3 \end{vmatrix} = 0$ • $\begin{vmatrix} a_1 + a_1' & b_1 & c_1 \\ a_2 + a_2' & b_2 & c_2 \\ a_3 + a_3' & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1' & b_1 & c_1 \\ a_2' & b_2 & c_2 \\ a_3' & b_3 & c_3 \end{vmatrix}$ • $\begin{vmatrix} a_1 + kb_1 & b_1 & c_1 \\ a_2 + kb_2 & b_2 & c_2 \\ a_3 + kb_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ • $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$ Notations including $\lvert A \rvert$ and $\det(A)$ should be introduced.
13. Matrices	13.1 understand the concept, operations and properties of matrices	9	The addition, scalar multiplication and multiplication of matrices are required. The following properties are required: • $A + B = B + A$

Learning Unit	Learning Objective	Time	Remarks
			• $A + (B + C) = (A + B) + C$ • $(\lambda + \mu)A = \lambda A + \mu A$ • $\lambda(A + B) = \lambda A + \lambda B$ • $A(BC) = (AB)C$ • $A(B + C) = AB + AC$ • $(A + B)C = AC + BC$ • $(\lambda A)(\mu B) = (\lambda\mu)AB$ • $\lvert AB \rvert = \lvert A \rvert \lvert B \rvert$
	13.2 understand the concept, operations and properties of inverses of square matrices of order 2 and order 3		The following properties are required: • the inverse of A is unique • $(A^{-1})^{-1} = A$ • $(\lambda A)^{-1} = \lambda^{-1}A^{-1}$ • $(A^n)^{-1} = (A^{-1})^n$ • $(A^{t})^{-1} = (A^{-1})^{t}$ • $\lvert A^{-1} \rvert = \lvert A \rvert^{-1}$ • $(AB)^{-1} = B^{-1}A^{-1}$ where A and B are invertible matrices and λ is a non-zero scalar.

Learning Unit	Learning Objective	Time	Remarks
14. Systems of linear equations	14.1 solve the systems of linear equations of order 2 and order 3 by Cramer's rule, inverse matrices and Gaussian elimination	6	The following theorem is required: • A system of homogeneous linear equations in three unknowns has nontrivial solutions if and only if the coefficient matrix is singular The wording "necessary and sufficient conditions" could be introduced to students.
	Subtotal in hours	18	
Vectors			
15. Introduction to vectors	15.1 understand the concepts of vectors and scalars	5	The concepts of magnitudes of vectors, zero vector and unit vectors are required. Students should recognise some common notations of vectors in printed form (including $\mathbf{a}$ and $\overrightarrow{AB}$) and in written form (including $\vec{a}$, $\overrightarrow{AB}$ and $\underline{a}$); and some notations for magnitude (including $\lvert \mathbf{a} \rvert$ and $\lvert \vec{a} \rvert$).

Learning Unit	Learning Objective	Time	Remarks
	15.2 understand the operations and properties of vectors		The addition, subtraction and scalar multiplication of vectors are required. The following properties are required: • $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$ • $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$ • $\mathbf{a} + \mathbf{0} = \mathbf{a}$ • $0\,\mathbf{a} = \mathbf{0}$ • $\lambda(\mu\mathbf{a}) = (\lambda\mu)\mathbf{a}$ • $(\lambda + \mu)\mathbf{a} = \lambda\mathbf{a} + \mu\mathbf{a}$ • $\lambda(\mathbf{a} + \mathbf{b}) = \lambda\mathbf{a} + \lambda\mathbf{b}$ • If $\alpha\mathbf{a} + \beta\mathbf{b} = \alpha_1\mathbf{a} + \beta_1\mathbf{b}$ ($\mathbf{a}$ and $\mathbf{b}$ are non-zero and are not parallel to each other), then $\alpha = \alpha_1$ and $\beta = \beta_1$
	15.3 understand the representation of a vector in the rectangular coordinate system		The following formulae are required: • $\lvert\overrightarrow{OP}\rvert = \sqrt{x^2 + y^2 + z^2}$ in $\mathbf{R}^3$ • $\sin\theta = \dfrac{y}{\sqrt{x^2 + y^2}}$ and

Learning Unit	Learning Objective	Time	Remarks
			$\cos\theta = \dfrac{x}{\sqrt{x^2+y^2}}$ in $\mathbf{R}^2$ The representation of vectors in the rectangular coordinate system can be used to discuss those properties listed in the Remarks against Learning Objective 15.2. The concept of direction cosines is **not** required.
16. Scalar product and vector product	16.1 understand the definition and properties of the scalar product (dot product) of vectors	5	The following properties are required: • $\mathbf{a}\cdot\mathbf{b}=\mathbf{b}\cdot\mathbf{a}$ • $\mathbf{a}\cdot(\lambda\,\mathbf{b})=\lambda\,(\mathbf{a}\cdot\mathbf{b})$ • $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c})=\mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{c}$ • $\mathbf{a}\cdot\mathbf{a}=\lvert\,\mathbf{a}\,\rvert^2\geq 0$ • $\mathbf{a}\cdot\mathbf{a}=0$ if and only if $\mathbf{a}=\mathbf{0}$ • $\lvert\,\mathbf{a}\,\rvert\,\lvert\,\mathbf{b}\,\rvert\geq\lvert\,\mathbf{a}\cdot\mathbf{b}\,\rvert$ • $\lvert\,\mathbf{a}-\mathbf{b}\,\rvert^2=\lvert\,\mathbf{a}\,\rvert^2+\lvert\,\mathbf{b}\,\rvert^2-2(\mathbf{a}\cdot\mathbf{b})$
	16.2 understand the definition and properties of the vector product (cross product) of vectors in $\mathbf{R}^3$		The following properties are required: • $\mathbf{a}\times\mathbf{a}=\mathbf{0}$

Learning Unit	Learning Objective	Time	Remarks
			• $\mathbf{b} \times \mathbf{a} = -(\mathbf{a} \times \mathbf{b})$ • $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$ • $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$ • $(\lambda\, \mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (\lambda\, \mathbf{b}) = \lambda\, (\mathbf{a} \times \mathbf{b})$ • $\lvert \mathbf{a} \times \mathbf{b} \rvert^2 = \lvert \mathbf{a} \rvert^2 \lvert \mathbf{b} \rvert^2 - (\mathbf{a} \cdot \mathbf{b})^2$ The following properties of scalar triple products should be introduced: • $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ • $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = (\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a} = (\mathbf{c} \times \mathbf{a}) \cdot \mathbf{b}$
17. Applications of vectors	17.1 understand the applications of vectors	8	Division of a line segment, parallelism and orthogonality are required. Finding angles between two vectors, the projection of a vector onto another vector, the volume of a parallelepiped and the area of a triangle are required.
	Subtotal in hours	18	

Learning Unit	Learning Objective	Time	Remarks
Further Learning Unit			
18. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	7	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.
	Subtotal in hours	7	

Grand total: 125 hours