Learning Objectives of Module 1 (Calculus and Statistics)

Notes:

1. *Learning units are grouped under three areas ("Foundation Knowledge", "Calculus" and "Statistics") and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*
4. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*

Learning Unit	Learning Objective	Time	Remarks
Foundation Knowledge Area			
1. Binomial expansion	1.1 recognise the expansion of $(a+b)^n$, where n is a positive integer	3	The use of the summation notation ($\sum$) should be introduced. The following are **not** required: • expansion of trinomials • the greatest coefficient, the greatest term and the properties of binomial coefficients • applications to numerical approximation

Learning Unit	Learning Objective	Time	Remarks
2. Exponential and logarithmic functions	2.1 recognise the definition of the number *e* and the exponential series $$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$	7	
	2.2 recognise exponential functions and logarithmic functions		The following functions are required: • $y = e^x$ • $y = \ln x$
	2.3 use exponential functions and logarithmic functions to solve problems		Students are expected to know how to solve problems including those related to compound interest, population growth and radioactive decay.

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	2.4 transform $y = kx^n$ and $y = ka^x$ to linear relations, where a, n and k are real numbers, $a > 0$ and $a \neq 1$		When experimental values of x and y are given, students can plot the graph of the corresponding linear relation from which they can determine the values of the unknown constants by considering its slope and intercept.
	Subtotal in hours	10	
Calculus Area			
Differentiation and Its Applications			
3. Derivative of a function	3.1 recognise the intuitive concept of the limit of a function	5	The concepts of continuous function and discontinuous function are **not** required. Theorems on the limits of sum, difference, product, quotient, scalar multiplication of functions and the limits of composite functions should be stated without proof.

Learning Unit	Learning Objective	Time	Remarks
	3.2 find the limits of algebraic functions, exponential functions and logarithmic functions		The following types of algebraic functions are required: ● polynomial functions ● rational functions ● power functions x^{α} ● functions derived from the above ones through addition, subtraction, multiplication, division and composition, for example, $\sqrt{x^2+1}$
	3.3 recognise the concept of the derivative of a function from first principles		Students are **not** required to find the derivatives of functions from first principles. Notations including y', $f'(x)$ and $\frac{dy}{dx}$ should be introduced.
	3.4 recognise the slope of the tangent of the curve $y = f(x)$ at a point $x = x_0$		Notations including $f'(x_0)$ and $\left.\frac{dy}{dx}\right\|_{x=x_0}$ should be introduced.

Learning Unit	Learning Objective	Time	Remarks
4. Differentiation of a function	4.1 understand the addition rule, product rule, quotient rule and chain rule of differentiation	7	The following rules are required: • $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$ • $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$ • $\frac{d}{dx}(\frac{u}{v})=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ • $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$

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	4.2 find the derivatives of algebraic functions, exponential functions and logarithmic functions		The following formulae are required: • $(C)'=0$ • $(x^n)'=nx^{n-1}$ • $(e^x)'=e^x$ • $(\ln x)'=\frac{1}{x}$ • $(\log_a x)'=\frac{1}{x\ln a}$ • $(a^x)'=a^x\ln a$ Implicit differentiation is **not** required. Logarithmic differentiation is **not** required.

Learning Unit	Learning Objective	Time	Remarks
5. Second derivative	5.1 recognise the concept of the second derivative of a function 5.2 find the second derivative of an explicit function	2	Notations including y'', $f''(x)$ and $\frac{d^2y}{dx^2}$ should be introduced. Third and higher order derivatives are **not** required.
6. Applications of differentiation	6.1 use differentiation to solve problems involving tangent, rate of change, maximum and minimum	9	Local and global extrema are required.
	Subtotal in hours	23	
Integration and Its Applications			
7. Indefinite integrals and their applications	7.1 recognise the concept of indefinite integration	10	Indefinite integration as the reverse process of differentiation should be introduced.

Learning Unit	Learning Objective	Time	Remarks
	7.2 understand the basic properties of indefinite integrals and basic integration formulae		The notation $\int f(x)\,dx$ should be introduced. The following properties are required: • $\int k\,f(x)dx = k\int f(x)\,dx$ • $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$ The following formulae are required and the meaning of the constant of integration C should be explained: • $\int k\,dx = kx + C$ • $\int x^n\ dx = \frac{x^{n+1}}{n+1} + C$, where $n \neq -1$ • $\int \frac{1}{x}\ dx = \ln\lvert x \rvert + C$ • $\int e^x\ dx = e^x + C$

Learning Unit	Learning Objective	Time	Remarks
	7.3 use basic integration formulae to find the indefinite integrals of algebraic functions and exponential functions		
	7.4 use integration by substitution to find indefinite integrals		Integration by parts is **not** required.
	7.5 use indefinite integration to solve problems		
8. Definite integrals and their applications	8.1 recognise the concept of definite integration	12	The definition of the definite integral as the limit of a sum of the areas of rectangles under a curve should be introduced. The notation $\int_a^b f(x)\,dx$ should be introduced. The knowledge of dummy variables, i.e. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$ is required.

Learning Unit	Learning Objective	Time	Remarks
	8.2 recognise the Fundamental Theorem of Calculus and understand the properties of definite integrals		The Fundamental Theorem of Calculus refers to $\int_a^b f(x)\,dx = F(b) - F(a)$, where $\frac{d}{dx}F(x) = f(x)$. The following properties are required: • $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ • $\int_a^a f(x)\,dx = 0$ • $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ • $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$ • $\int_a^b [f(x) \pm g(x)]\,dx$ $= \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

Learning Unit	Learning Objective	Time	Remarks
	8.3 find the definite integrals of algebraic functions and exponential functions 8.4 use integration by substitution to find definite integrals 8.5 use definite integration to find the areas of plane figures 8.6 use definite integration to solve problems		Students are **not** required to use definite integration to find the area between a curve and the *y*-axis and the area between two curves.
9. Approximation of definite integrals using the trapezoidal rule	9.1 understand the trapezoidal rule and use it to estimate the values of definite integrals	4	Error estimation is **not** required.
	Subtotal in hours	26	

Learning Unit	Learning Objective	Time	Remarks
Statistics Area			
Further Probability			
10. Conditional probability and independence	10.1 understand the concepts of conditional probability and independent events 10.2 use the laws $P(A \cap B) = P(A)\,P(B \mid A)$ and $P(D \mid C) = P(D)$ for independent events C and D to solve problems	3	
11. Bayes' theorem	11.1 use Bayes' theorem to solve simple problems	4	
	Subtotal in hours	7	
Binomial, Geometric and Poisson Distributions and Their Applications			
12. Discrete random variables	12.1 recognise the concept of a discrete random variable	1	
13. Probability distribution, expectation and variance	13.1 recognise the concept of discrete probability distribution and its representation in the form of tables, graphs and mathematical formulae 13.2 recognise the concepts of expectation $E(X)$ and variance $\mathrm{Var}(X)$ and use them to solve simple problems	5	

Learning Unit	Learning Objective	Time	Remarks
	13.3 use the formulae $E(aX+b)=aE(X)+b$ and $\operatorname{Var}(aX+b)=a^2\operatorname{Var}(X)$ to solve simple problems		
14. Binomial distribution	14.1 recognise the concept and properties of the binomial distribution 14.2 calculate probabilities involving the binomial distribution	5	Bernoulli distribution should be introduced. The mean and variance of the binomial distribution should be introduced (proofs are **not** required). Use of the binomial distribution table is **not** required.
15. Geometric distribution	15.1 recognise the concept and properties of the geometric distribution 15.2 calculate probabilities involving the geometric distribution	4	The mean and variance of geometric distribution should be introduced (proofs are **not** required).
16. Poisson distribution	16.1 recognise the concept and properties of the Poisson distribution	4	The mean and variance of Poisson distribution should be introduced (proofs are **not** required).

Learning Unit	Learning Objective	Time	Remarks
	16.2 calculate probabilities involving the Poisson distribution		Use of the Poisson distribution table is **not** required.
17. Applications of binomial, geometric and Poisson distributions	17.1 use binomial, geometric and Poisson distributions to solve problems	5	
	Subtotal in hours	24	
Normal Distribution and Its Applications			
18. Basic definition and properties	18.1 recognise the concepts of continuous random variables and continuous probability distributions, with reference to the normal distribution	3	Derivations of the mean and variance of the normal distribution are **not** required. The formulae written in Learning Objective 13.3 are also applicable to continuous random variables.

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	18.2 recognise the concept and properties of the normal distribution		Properties of the normal distribution include: • the curve is bell-shaped and symmetrical about the mean • the mean, mode and median are equal • the dispersion can be determined by the value of σ • the area under the curve is 1
19. Standardisation of a normal variable and use of the standard normal table	19.1 standardise a normal variable and use the standard normal table to find probabilities involving the normal distribution	2	

Learning Unit	Learning Objective	Time	Remarks
20. Applications of the normal distribution	20.1 find the values of $P(X > x_1)$, $P(X < x_2)$, $P(x_1 < X < x_2)$ and related probabilities, given the values of x_1, x_2, μ and σ, where $X \sim N(\mu,\ \sigma^2)$ 20.2 find the values of x, given the values of $P(X > x)$, $P(X < x)$, $P(a < X < x)$, $P(x < X < b)$ or a related probability, where $X \sim N(\mu,\ \sigma^2)$ 20.3 use the normal distribution to solve problems	7	
	Subtotal in hours	12	
Point and Interval Estimation			
21. Sampling distribution and point estimates	21.1 recognise the concepts of sample statistics and population parameters 21.2 recognise the sampling distribution of the sample mean from a random sample of size n	7	If the population mean is μ and population variance is σ^2, then the mean of the sample mean is μ and the variance of the sample mean is $\frac{\sigma^2}{n}$.

Learning Unit	Learning Objective	Time	Remarks
	21.3 recognise the concept of point estimates including the sample mean, sample variance and sample proportion 21.4 recognise Central Limit Theorem		The concept of "estimator" should be introduced. If the population mean is μ and the population size is N, then the population variance is $\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$. If the sample mean is $\bar{x}$ and the sample size is n, then the sample variance is $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$. Recognising the concept of "unbiased estimator" is required.
22. Confidence interval for a population mean	22.1 recognise the concept of confidence interval 22.2 find the confidence interval for a population mean	6	• a $100(1-\alpha)\%$ confidence interval for the mean μ of a normal population with known variance σ^2 is given by

Learning Unit	Learning Objective	Time	Remarks
			$(\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}})$ ● when the sample size n is sufficiently large, a 100(1 − α)% confidence interval for the mean μ of a population with unknown variance is given by $(\bar{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}},\ \bar{x} + z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}})$, where s is the sample standard deviation
23. Confidence interval for a population proportion	23.1 find an approximate confidence interval for a population proportion	3	For a random sample of size n, where n is sufficiently large, drawn from a Bernoulli distribution, a 100(1 − α)% confidence interval for the population proportion p is given by $(\hat{p} - z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}})$, where $\hat{p}$ is an unbiased estimator of the population proportion.
	Subtotal in hours	16	

Learning Unit	Learning Objective	Time	Remarks
Further Learning Unit			
24. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	7	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.
	Subtotal in hours	7	

Grand total: 125 hours