Ratio of area of triangles

1. Similar Triangles

If two similar triangles are given, their ratio of area is given by $\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$, where ℓ_1 is the corresponding sides of ℓ_2 .

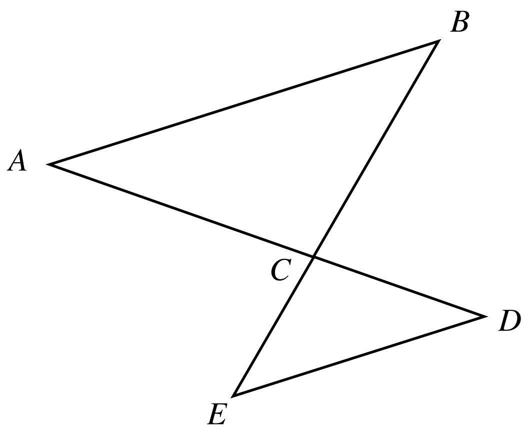
Example 1

It is given that $AB//DE$ and C is the intersection point of two straight lines ACD and BCE,

(a) Prove that $\Delta ABC \sim \Delta DEC$.

(b) If $AC=9$ cm , $CD=3$ cm and the area of ΔABC is 27 cm^2, find the area of ΔDEC .

Solution



(a) $\angle ACB=\angle DCE$ (vert. opp. $\angle$s eq.)

$\angle ABC=\angle CED$ (alt. $\angle$s, $AB//DE$)

$\angle BAC=\angle EDC$ (alt. $\angle$s, $AB//DE$)

$\therefore \Delta ABC \sim \Delta DEC$ (A.A.A.)

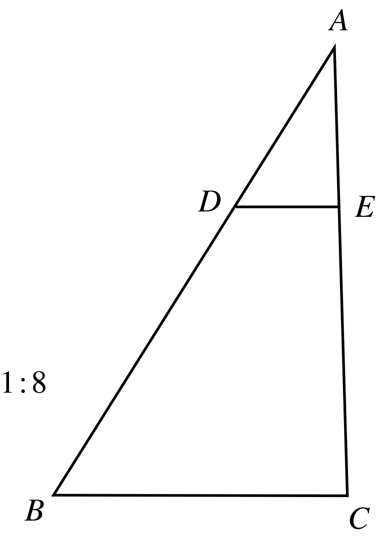
(b) As $\Delta ABC \sim \Delta DEC$ (proved),

$$\frac{A}{27}=\left(\frac{3}{9}\right)^2$$

$$A=3\text{ cm}^2$$

Example 2

It is given that D and E are two points lie on two straight lines ACD and BCE such that $BC//DE$. If $AE:AC=1:3$ and the area of the trapezium $BCED$ is 64 cm^2, find the area of ΔADE .



Solution

As $\Delta ADE \sim \Delta ABC$ (A.A.A.)

So area of ΔADE : area of ΔABC $=\left(\frac{1}{3}\right)^2=\frac{1}{9}$

Area of ΔADE : area of the trapezium $BCED$

= area of ΔADE : area of $\Delta ABC-$ area of $\Delta ADE=1:(9-1)=1:8$

Thus, the area of required $=\frac{64}{8}=8\text{ cm}^2$

2. Triangles with same base/height

Consider the ratio of area of two triangles.

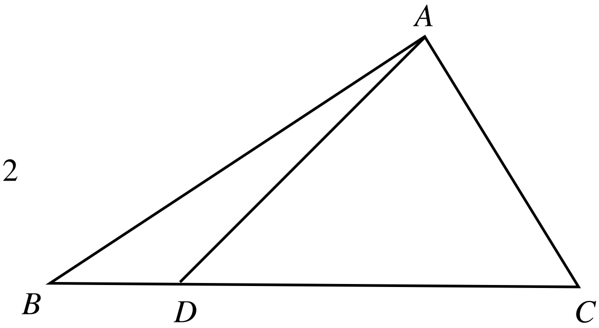
$$\begin{aligned}&A_1 : A_2\\&=\frac{1}{2}b_1h_1 : \frac{1}{2}b_2h_2\\&=b_1h_1 : b_2h_2\end{aligned}$$

If the two triangles have the same base, then the ratio of their area becomes $h_1 : h_2$.

Similarly, the ratio of their area is $b_1 : b_2$ when they have the same height.

Example 3

It is given that D is a point lies on the straight line BDC such that $BD:DC = 2:5$. If the area of ΔABC is 35 cm^2, find the area of ΔABD .



Solution

As ΔABC and ΔABD have the same height, so

area of ΔABC : area of ΔABD $=(2+5):2=7:2$

Area of $\Delta ABD = 35\times\frac{2}{7}=10 \text{ cm}^2$

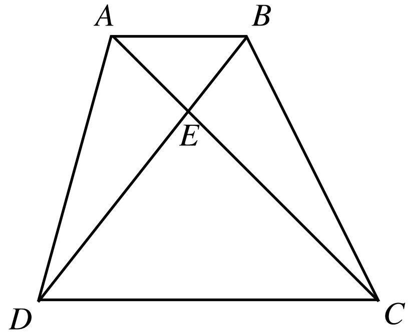
3. More complexed problems

The example below consists of using both methods mentioned above.

Example 4

It is given that $ABCD$ is a trapezium where $AB:CD=1:3$ and the area of ΔAEB is 12 cm^2 .

Find the area of the trapezium $ABCD$.



Solution

As $\Delta ABE \sim \Delta CED$,

area of $\Delta CED = 12\times 3^2 = 108 \text{ cm}^2$

Since ΔAEB and ΔBEC share the same height,

area of $\Delta BEC = 12\times 3 = 36 \text{ cm}^2$

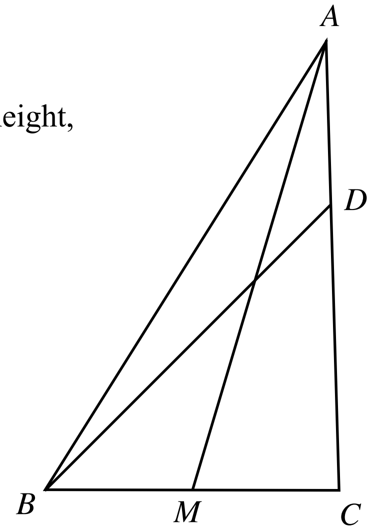
Similarly, area of $\Delta AED = 12\times 3 = 36 \text{ cm}^2$

Thus, the area of the trapezium $ABCD$ $=12+108+36+36=192 \text{ cm}^2$

When the ratio of area of two triangles do not seemed to have the above two direct relationships, we can express their areas in terms of an unknown. Consider the example below.

Example 5

It is given that M is a point lies on BC such that AM is a median of ΔABC. D is a point lies on AC such that $AD:DC=2:3$. Find area of ΔABM : area of ΔABD.



Solution

Let x be the area of ΔABC.

Note that $BM=\frac{1}{2}BC$. As ΔABC and ΔABC share the same height,

Area of $\Delta ABM=\frac{1}{2}x$

As ΔABD and ΔABC share the same height,

Area of $\Delta ABM=\frac{2}{2+3}x=\frac{2}{5}x$

Thus, area of ΔABM : area of ΔABD $=\frac{1}{2}x:\frac{2}{5}x=5:4$

Remainders

- When dealing with ratio of area of triangles, we must be very careful with the ratio of corresponding sides.
- Before applying $\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$, make sure that the two triangles are similar!
- Try to find the pair of triangles with same base/height such that you can use ratio of area = ratio of height/base.
- If necessary, add straight lines to the diagram to facilitate calculations.