## $\mathcal{H K C E E}$ Paper 1 Section $\mathcal{B}$ sorted by topic <br> Question <br> (1990-2009)

Quadratic functions, Rate, Variations \& Polynomials
13. When other conditions remain the same, the quality of a cup of a particular kind of Chinese tea depends on the amount of time, $t$ seconds, that tea leaves are soaked in water and the temperature, $x^{\circ} \mathrm{C}$, of the water. It is proposed that the quality of a cup of this kind of tea can be measured by the indicator $Q$ as follows:

$$
Q=20000+F,
$$

where $F$ consists of two parts with one part varying jointly as $x$ and $t$, and the other part varying as the square of $t$. The greater the value of $Q$, the better is the quality of the tea.

It is known that $Q=30600$ when $t=40, x=85$; and $Q=28100$ when $t=60, x=75$.
(a) Show that $Q=20000+5 x t-4 t^{2}$.
(b) (i) Find the value of $Q$ when the tea leaves are soaked in water for 45 seconds at a temperature of $82^{\circ} \mathrm{C}$.
(ii) When the temperature of water is $78^{\circ} \mathrm{C}$, is it possible to achieve the same value of $Q$ in (b)(i) by changing the amount of time that the tea leaves are soaked in water? Explain your answer briefly.
(4 marks)
(c) Suppose the temperature of water is $80^{\circ} \mathrm{C}$. Using the method of completing the square, find the amount of time the tea leaves need to be soaked in the water in order to achieve the best quality of the tea.
(3 marks)
13. Miss Lee makes and sells handmade leather belts and handbags. She finds that if a batch of $x$ belts is made, where $1 \leq x \leq 11$, the cost per belt $\$ B$ is given by $B=x^{2}-20 x+120$. Page 8 shows the graph of the function $y=x^{2}-20 x+120$.
(a) Use the given graph to write down the number(s) of belts in a batch that will make the cost per belt
(i) a minimum,
(ii) less than $\$ 90$.
(3 marks)
(b) Miss Lee also finds that if a batch of $x$ handbags is made, where $1 \leq x \leq 8$, the cost per handbag $\$ H$ is given by $H=x^{2}-17 x+c$ ( $c$ is a constant). When a batch of 3 handbags is made, the cost per handbag is $\$ 144$.
(i) Find $c$.
(ii) By adding a suitable straight line on the given graph, find the number of handbags in a batch that will make the cost per handbag $\$ 120$.
(iii) Miss Lee made a batch of 10 belts and a batch of 6 handbags. She managed to sell 6 belts at $\$ 100$ each and 4 handbags at $\$ 300$ each while the remaining belts and handbags sold at half of their respective cost. Find her gain or loss.


1999P1Q15(b) (Rate \& cubic equation)
Service reservoir $Y$ is at first empty. Three water pipes $A, B$ and $C$ with constant filling rates can be used to fill this reservoir completely. If the pipes are used individually, using pipe A to fill up the reservoir takes 3 days more than pipe $B$ while using pipe $C$ takes 2 days less than pipe $B$. If the three pipes are used simultaneously, filling up the reservoir takes 4 days. Suppose pipe $B$ alone takes $x$ days to fill up this reservoir.

Show that $x^{3}-11 x^{2}-14 x+24=0$.
Hence find $x$ by factorizing $x^{3}-11 x^{2}-14 x+24$.

2006P1Q15 (Variations \& quadratic equation)
15. The cost of a souvenir of surface area $A \mathrm{~cm}^{2}$ is $\$ C$. It is given that $C$ is the sum of two parts, one part varies directly as $A$ while the other part varies directly as $A^{2}$ and inversely as $n$, where $n$ is the number of souvenirs produced. When $A=50$ and $n=500, C=350$; when $A=20$ and $n=400, C=100$.
(a) Express $C$ in terms of $A$ and $n$.
(b) The selling price of a souvenir of surface area $A \mathrm{~cm}^{2}$ is $\$ 8 A$ and the profit in selling the souvenir is $\$ P$.
(i) Express $P$ in terms of $A$ and $n$.
(ii) Suppose $P: n=5: 32$. Find $A: n$.
(iii) Suppose $n=500$. Can a profit of $\$ 100$ be made in selling a souvenir? Explain your answer.
(iv) Suppose $n=400$. Using the method of completing the square, find the greatest profit in selling a souvenir.
(8 marks)

2007P1Q14 (Variations \& cubic equation)
14. (a) Let $\mathrm{f}(x)=4 x^{3}+k x^{2}-243$, where $k$ is a constant. It is given that $x+3$ is a factor of $\mathrm{f}(x)$.
(i) Find the value of $k$.
(ii) Factorize $\mathrm{f}(x)$.
(b) Let $\$ C$ be the cost of making a cubical handicraft with a side of length $x \mathrm{~cm}$. It is given that $C$ is the sum of two parts, one part varies as $x^{3}$ and the other part varies as $x^{2}$. When $x=5.5, C=7381$ and when $x=6, C=9072$.
(i) Express $C$ in terms of $x$.
(ii) If the cost of making a cubical handicraft is $\$ 972$, find the length of a side of the handicraft.

