Marking Scheme

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

General Instructions To Markers

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits <u>all the marks</u> allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept / technique had been used.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.

5. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

6. In the marking scheme, marks are classified into the following three categories:

'M' marks	–	awarded for applying correct methods
'A' marks	–	awarded for the accuracy of the answers
Marks without 'M' or 'A'	–	awarded for correctly completing a proof or arriving at an answer given in the question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates' work in awarding 'M' marks.) However, 'A' marks for the corresponding answers should <u>NOT</u> be awarded, unless otherwise specified.

7. In the marking scheme, steps which can be skipped are enclosed by dotted rectangles, whereas alternative answers are enclosed by solid rectangles.

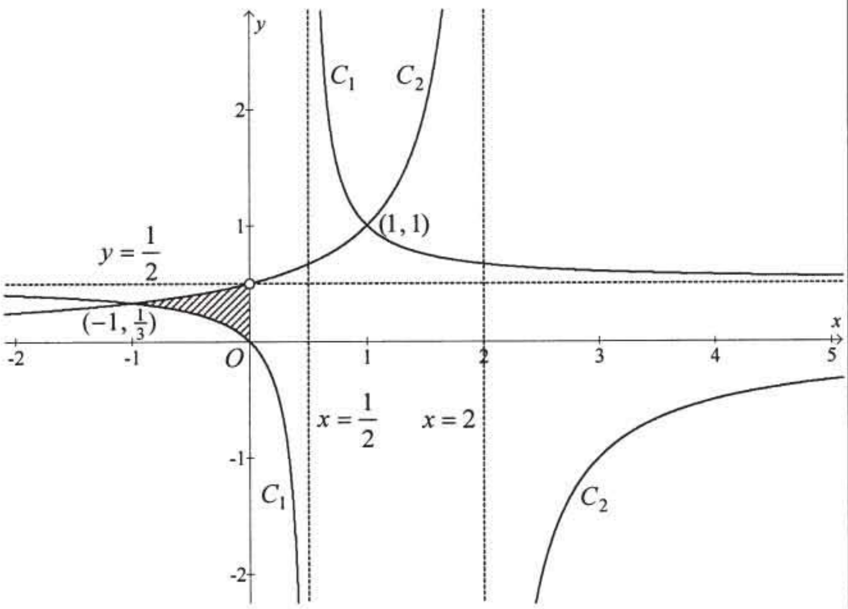
8. Marks may be deducted for poor presentation (*pp*). The symbol (*pp*-1) should be used to denote 1 mark deducted for *pp*.
   (a) At most deduct 1 mark for *pp* in each section.
   (b) In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.

9. Marks may be deducted for numerical answers with inappropriate degree of accuracy (*a*). The symbol (*a*-1) should be used to denote 1 mark deducted for *a*.
   (a) At most deduct 1 mark for *a* in each section.
   (b) In any case, do not deduct any marks for *a* in those steps where candidates could not score any marks.

Solution	Marks	Remarks
1. (a) $e^{\frac{x}{16}}=1+\frac{x}{16}+\frac{1}{2!}\left(\frac{x}{16}\right)^2+\cdots$		
$=1+\frac{x}{16}+\frac{x^2}{512}+\cdots$	1A	
$\therefore\ e^{\frac{8}{16}}=1+\frac{8}{16}+\frac{8^2}{512}+\cdots$		
i.e. $\sqrt{e}\approx 1.625$	1	
(b) $(1+x)^{\frac{-1}{2}}=1+\left(\frac{-1}{2}\right)x+\frac{1}{2!}\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)x^2+\cdots$	1A	
$=1-\frac{1}{2}x+\frac{3}{8}x^2+\cdots$	1A	
(c) (i) $\frac{e^{\frac{x}{16}}}{\sqrt{1+x}}=e^{\frac{x}{16}}(1+x)^{\frac{-1}{2}}$		
$=\left(1+\frac{x}{16}+\frac{x^2}{512}+\cdots\right)\left(1-\frac{1}{2}x+\frac{3}{8}x^2+\cdots\right)$	1M	
$\approx 1-\frac{7}{16}x+\frac{177}{512}x^2$	1	
(ii) Mary is wrong because she cannot put $x=8$ into the binomial series in (c)(i) since it is valid only for $-1<x<1$.	1A	
	(7)	
2. (a) $x=\ln\frac{1+t}{1-t}$		
$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1-t}{1+t}\cdot\frac{(1-t)(1)-(1+t)(-1)}{(1-t)^2}$	1M	
Alternative Solution $x=\ln(1+t)-\ln(1-t)$ $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{1+t}+\frac{1}{1-t}$	1M	
$=\frac{2}{1-t^2}$	1A	
(b) (i) $y=1+e^{-x}-e^{-2x}$		
$\frac{\mathrm{d}y}{\mathrm{d}x}=-e^{-x}+2e^{-2x}$	1A	

Solution	Marks	Remarks
(ii) $\frac{dy}{dt}=\frac{dy}{dx}\cdot\frac{dx}{dt}$		
$=(-e^{-x}+2e^{-2x})\left(\frac{2}{1-t^2}\right)$	1M	
When $t=\frac{1}{2}$, $x=\ln 3$.	1A	
$\frac{dy}{dt}=(-e^{-\ln 3}+2e^{-2\ln 3})\left[\frac{2}{1-\left(\frac{1}{2}\right)^2}\right]$		
$=\frac{-8}{27}$	1A	OR -0.2963
	(6)	
3. (a) $R(t)=Ae^{-0.5t}+B$		
$R(t)\to 10$ when $t\to\infty$	1M	
$\therefore\ B=10$	1A	
$R(0)=500$		
$500=A+B$		
$\therefore\ A=490$	1A	
(b) $\int_0^5 P'(t)\,dt+R(5)-R(0)$	1M	
$=\int_0^5 600e^{-0.3t}dt+\left[490e^{-0.5(5)}+10\right]-500$		
$=[-2000e^{-0.3t}]_0^5+490e^{-2.5}-490$	1A	For $[-2000e^{-0.3t}]_0^5$
$=-2000e^{-1.5}+490e^{-2.5}+1510$		
≈ 1104	1A	
Hence Richard gains 1104 thousand dollars in the process.		
	(6)	
4. (a) $P(\text{a player wins})=\frac{4}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}+\frac{4}{4}\times\frac{3}{4}\times\frac{2}{4}\times\frac{1}{4}$	1M+1A	1M for additional law 1A for either term
$=\frac{7}{64}$	1A	OR 0.109375 OR 0.1094
(b) $P(\text{a player wins} \mid \text{first 2 balls in different slots})=\frac{\frac{4}{4}\times\frac{3}{4}\times\frac{2}{4}\times\frac{1}{4}}{\frac{4}{4}\times\frac{3}{4}}$	1M+1A	1M for conditional prob 1A for denominator
$=\frac{1}{8}$	1A	OR 0.125
	(6)	
5. (a) $P(A\cap B')=P(B'\mid A)\cdot P(A)$		
$=\frac{27}{32}a$	1A	

Solution	Marks	Remarks
(b) $P(A\cap B')=P(A\mid B')\cdot P(B')$		
$\frac{27}{32}a=\frac{27}{31}\cdot[1-P(B)]$	1M	
$P(B)=1-\frac{31}{32}a$	1A	
(c) (i) $P(A)=P(A\cap B)+P(A\cap B')$		
$a=0.1+\frac{27}{32}a$	1M	
$a=0.64$	1A	
(ii) $P(A)\cdot P(B)=(0.64)\left[1-\frac{31}{32}(0.64)\right]$		
$=0.2432$	1A	
$\neq P(A\cap B)$		
Hence A and B are not independent.	1	
	(7)	
6. (a) $E(X)=10\left(\frac{1}{4}\right)=2.5$	1A	
$Var(X)=10\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)=1.875$	1A	
$\therefore$ $(1+\theta)(2.5)=2.5+(0.1)(1.875)$		
$\theta=0.075$	1A	
(b) $Var(X)=10p(1-p)$		
$=-10\left[p^2-p+\left(\frac{1}{2}\right)^2-\frac{1}{4}\right]$	1M	
$=-10\left(p-\frac{1}{2}\right)^2+\frac{5}{2}$	1A	
Alternative Solution		
$\frac{d}{dp}Var(X)=10(1-2p)$	1A	
$\therefore$ $\frac{d}{dp}Var(X)=0$ when $p=\frac{1}{2}$		
$\frac{d^2}{dp^2}Var(X)=-20<0$	1M	
Hence Var(X) is greatest when $p=\frac{1}{2}$.	1	
(c) For Plan 1, $F=(1+0.075)\cdot 10\left(\frac{1}{2}\right)=5.375$.	1M	For both
For Plan 2, $F=10\left(\frac{1}{2}\right)+0.1\times 10\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=5.25$.		
Hence Plan 2 will give a lower game fee.	1	
	(8)	

Solution	Marks	Remarks
7. (a) $g(x) = f\left(\frac{1}{x}\right)$		
$= \dfrac{\frac{1}{x}}{k \cdot \frac{1}{x} - 1}$		
$= \dfrac{1}{k - x}$	1A	
Since the vertical asymptote is $x = 2$, we get $k = 2$.	1A	
	(2)	
(b) $f(x) = g(x)$		
$\dfrac{x}{2x-1} = \dfrac{1}{2-x}$		
$2x - x^2 = 2x - 1$		
$x = \pm 1$		
Hence the points of intersection are $(1, 1)$ and $\left(-1, \frac{1}{3}\right)$	1A+1A	
	(2)	
(c) 		
	1M+1M	For shapes of C_1 and C
	1A+1A	For asymptotes of C_1 and C
	1A	For intercepts of C_1 and C
	1A	For intersections of C_1 and C
	(6)	
(d) The area $= \displaystyle\int_{-1}^{0} \left(\frac{1}{2-x} - \frac{x}{2x-1} \right) dx$	1M	
$= \displaystyle\int_{-1}^{0} \left[\frac{1}{2-x} - \frac{1}{2}\left(1 + \frac{1}{2x-1}\right) \right] dx$	1M	
$= \left[-\ln\lvert 2-x \rvert - \frac{1}{2}\left(x + \frac{1}{2}\ln\lvert 2x-1 \rvert \right) \right]_{-1}^{0}$	1A+1A	
$= -\ln 2 - \left(-\ln 3 + \frac{1}{2} - \frac{1}{4}\ln 3 \right)$		
$= \frac{5}{4}\ln 3 - \ln 2 - \frac{1}{2}$	1A	
	(5)	

Solution	Marks	Remarks
8. (a) $\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{k-3t}{1+ae^{-bt}}$		
$\ln\left(\frac{k-3t}{\frac{\mathrm{d}P}{\mathrm{d}t}}-1\right)=-bt+\ln a$	1A	
$\because$ slope $=-0.3$		
$\therefore\ b=0.3$	1A	
$\because$ intercept on the horizontal axis $=0.32$		
$\therefore\ 0=-(0.3)(0.32)+\ln a$		
$a\approx 1.100759064$		
≈ 1.1008	1A	OR $e^{0.096}$
When $t=3$ P attains maximum and hence $\frac{\mathrm{d}P}{\mathrm{d}t}=0$.	1M	
$\frac{k-3(3)}{1+(1.100759064)e^{-(0.3)(3)}}=0$		
$k=9$	1A	
	(5)	
(b) (i) $P=\int_0^3\frac{9-3t}{1+1.100759064e^{-0.3t}}\,\mathrm{d}t$		
$\approx\frac{0.5}{2}[4.284165735+0+2(3.851225403+3.30494319+2.644142541$ $+1.870196654+0.986866929)]$	1M	
≈ 7.3997 million barrels	1A	
(ii) From the graph $\frac{\mathrm{d}^2P}{\mathrm{d}t^2}$ is decreasing for $0<t<3$.	1A	
Thus, $\frac{\mathrm{d}^3P}{\mathrm{d}t^3}<0$ for $0<t<3$ and hence the estimation is under-estimate.	1	
	(4)	
(c) (i) $y=\alpha^{\beta x}$		
$\ln y=\beta x\ln\alpha$	1M	
$\frac{1}{y}\cdot\frac{\mathrm{d}y}{\mathrm{d}x}=\beta\ln\alpha$		
$\frac{\mathrm{d}y}{\mathrm{d}x}=\beta\alpha^{\beta x}\ln\alpha$	1A	
(ii) $\therefore\ \int\alpha^{\beta x}\,\mathrm{d}x=\frac{1}{\beta\ln\alpha}\alpha^{\beta x}+C$ -------------------------------- (*)		
$D=\int_0^3 1.63^{2-0.1t}\,\mathrm{d}t$		
$=1.63^2\left[\frac{1}{-0.1\ln 1.63}1.63^{-0.1t}\right]_0^3$ by (*)	1M	
≈ 7.414075736		
≈ 7.4141 (million barrels)	1A	
(iii) The amount of oil production is approximately 7.3997 million barrels which is an underestimate. Compare with (c)(ii), we cannot conclude that whether the overall oil production meets the overall demand of oil.	1M 1	
	(6)	

Solution	Marks	Remarks
9. (a) (i) Let $u=2t+1$.	1A	
$\therefore\quad t=\frac{u-1}{2}$		
$dt=\frac{1}{2}du$		
$\therefore\quad \int\frac{t^2}{2t+1}\,dt=\int\frac{1}{u}\left(\frac{u-1}{2}\right)^2\frac{1}{2}du$		
$=\frac{1}{8}\int\left(u-2+\frac{1}{u}\right)du$	1A	
$=\frac{u^2}{16}-\frac{u}{4}+\frac{1}{8}\ln\lvert u\rvert+C$		
$=\frac{(2t+1)^2}{16}-\frac{2t+1}{4}+\frac{1}{8}\ln\lvert 2t+1\rvert+C$	1A	
(ii) $\frac{d}{dt}[t^2\ln(2t+1)]=2t\ln(2t+1)+\frac{2t^2}{2t+1}$	1A	
(iii) $\therefore\quad t\ln(2t+1)=\frac{1}{2}\cdot\frac{d}{dt}t^2\ln(2t+1)-\frac{t^2}{2t+1}$		
$\int t\ln(2t+1)\,dt=\frac{1}{2}t^2\ln(2t+1)-\int\frac{t^2}{2t+1}\,dt$	1M	
$=\frac{1}{2}t^2\ln(2t+1)-\frac{(2t+1)^2}{16}+\frac{2t+1}{4}-\frac{1}{8}\ln\lvert 2t+1\rvert-C$ by (a)(i)	1M	
$N\rvert_{t=5}-N\rvert_{t=0}=\int_0^5 t\ln(2t+1)\,dt$	1M	
$N\rvert_{t=5}-21=\left[\frac{1}{2}t^2\ln(2t+1)-\frac{(2t+1)^2}{16}+\frac{2t+1}{4}-\frac{1}{8}\ln\lvert 2t+1\rvert\right]_0^5$		
$N\rvert_{t=5}\approx 45.673954$		
Hence the population of the culture of bacteria is approximately 46 trillions.	1A	
	(8)	
(b) (i) By (a)(iii), $45.673954=40e^{-2\lambda(5-5)}-20e^{-\lambda(5-5)}+K$		
$K\approx 25.673954$		
≈ 26	1A	
$27=40e^{-2\lambda(11-5)}-20e^{-\lambda(11-5)}+25.673954$	1M	
$40e^{-12\lambda}-20e^{-6\lambda}-1.326046=0$		
$e^{-6\lambda}=0.559275201$ or -0.059275201 (rejected)		
$\lambda\approx 0.1$	1A	
(ii) $M=40e^{-0.2(t-5)}-20e^{-0.1(t-5)}+26$		
$M'=-8e^{-0.2(t-5)}+2e^{-0.1(t-5)}$	1M	
$=-2e^{-0.2(t-5)}[4-e^{0.1(t-5)}]$		
<0 since $e^{0.1(t-5)}\le e^{1.3}<4$ for $t\le 18$	1M	
Thus, M is always decreasing for $t\le 18$.	1	
Since we have $M\approx 23.5203$ when $t=18$, the population of the bacteria will not drop to 23 trillion.	1	
	(7)	

Solution	Marks	Remarks
10. Let X be the speed of a randomly selected vehicle.		
(a) $P(X > 82.64) = 0.123$ and $P(X < 75.2) = 0.242$		
$\frac{82.64-\mu}{\sigma} = 1.16$ and $\frac{75.2-\mu}{\sigma} = -0.7$	1M	
Dividing the equations, we have $\frac{82.64-\mu}{75.2-\mu} = \frac{1.16}{-0.7}$.		
i.e. $\mu = 78$	1A	
$\therefore \sigma = 4$	1A	
	(3)	
(b) (i) P(a notice will be issued) $= P(X > 80)$	1M	
$= P\left(Z > \frac{80-78}{4}\right)$		
$= P(Z > 0.5)$		
≈ 0.3085	1A	
(ii) P(at most 2 notices will be issued for the 10 vehicles)		
$\approx (1-0.3085)^{10} + C_1^{10}(0.3085)(1-0.3085)^9 + C_2^{10}(0.3085)^2(1-0.3085)^8$	1M	
≈ 0.3604	1A	
	(4)	
(c) (i) (I) P(a notice will be issued but the speed of vehicle is not over 80 if $\theta = 1$)		
= P(speed of vehicle $\leq$ 80 and (speed of vehicle + error) > 80)		
$= P(78 < X \leq 80 \mid Y = 2)P(Y = 2) + P(77 < X \leq 80 \mid Y = 3)P(Y = 3)$	1M+1A	1A for either term
$= P(0 < Z \leq 0.5)(0.5) + P(-0.25 < Z \leq 0.5)(0.5)$		
$\approx (0.1915)(0.5) + (0.0987 + 0.1915)(0.5)$		
≈ 0.2409	1A	Accept 0.24085
(II) P(a notice will be issued but the speed of vehicle is not over 80 if $\theta = -3$)		
= P(speed of vehicle $\leq$ 80 and (speed of vehicle + error) > 80)		
$= P(78 < X \leq 80 \mid Y = 2)P(Y = 2) + 0 \cdot P(Y = -1)$	1A	
$= P(0 < Z \leq 0.5)(0.5)$		
$\approx (0.1915)(0.5)$		
≈ 0.0958	1A	Accept 0.09575
(ii) We need $2 + \theta < 0$ for the scenario happens.		
P(a notice will not be issued but the speed of vehicle is over 80) ≤ 0.07125		
P(speed of vehicle > 80 and (speed of vehicle + error) $\leq$ 80) ≤ 0.07125	1M	
$P(80 < X \leq 80-(2+\theta) \mid Y = 2+\theta)\ P(Y = 2+\theta) \leq 0.07125$		
$P\left(0.5 < Z \leq \frac{78-\theta-78}{4}\right)(0.5) \leq 0.07125$		
$P\left(0 < Z \leq \frac{-\theta}{4}\right) \leq 0.1425 + 0.1915$		
$\frac{-\theta}{4} \leq 0.97$	1A	For 0.97
$\theta \geq -3.88$		
Hence the range is $-3.88 \leq \theta < -2$.	1A	
	(8)	

Solution	Marks	Remarks
11. (a) (i) P(the air-conditioners are switched on for not more than one day on two consecutive school days) $= q^2 + C_1^2 q(1-q)$		OR $1-(1-q)^2$
$= 2q - q^2$	1	
(ii) $2q - q^2 = \frac{7}{16}$		
$16q^2 - 32q + 7 = 0$		
$q = 0.25$ or 1.75 (rejected)	1A	
	(2)	
(b) (i) P(the fifth week is the second week that the air-conditioners are *fully engaged*)		
$= C_1^4(0.75^5)(1-0.75^5)^3 \cdot (0.75^5)$	1M+1M	1M for Binomial prob 1M for Geometric prob
≈ 0.0999	1A	
(ii) Expected number of consecutive weeks $= \frac{1}{0.75^5} - 1$	1M	For $\frac{1}{0.75^5}$
$= 3\frac{52}{243}$	1A	OR 3.2140
	(5)	
(c) (i) P(all conditioners are switched off) $= 0.25^5$		
$= \frac{1}{1024}$	1A	OR 0.0010
(ii) P(exactly 2 classrooms with no air-conditioners being switched off and at most 1 classroom with exactly 1 air-conditioner being switched off)		
$= C_2^5(0.45)^2[0.25^3 + C_1^3(0.25)^2(0.3)]$	1M+1A	
$= \frac{1863}{12800}$	1A	OR 0.1455
(iii) P(at least 1 classroom has no air-conditioners being switched off)		
$= \dfrac{\frac{5!}{2!2!1!}(0.25)^2(0.3)^2(0.45) + C_2^5(0.45)^2(0.25)^3}{C_1^5(0.25)(0.3)^4 + \frac{5!}{2!2!1!}(0.25)^2(0.3)^2(0.45) + C_2^5(0.45)^2(0.25)^3}$	1M+1M+1A	1M for conditional prob 1M for cases in numerator 1A for numerator
$= \frac{85}{93}$	1A	OR 0.9140
	(8)	

Solution	Marks	Remarks
12. (a) P(three consecutive mini-buses with at least one empty seat) = 0.6465		
$(1-e^{-\lambda})^3 = 0.6465$	1M	
$\lambda = -\ln(1-\sqrt[3]{0.6465})$		
≈ 2 (correct to the nearest integer)	1A	
	(2)	
(b) (i) P(the 5 members cannot get on the first arriving mini-bus together)		
$= e^{-2} + \frac{2e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!} + \frac{2^3 e^{-2}}{3!} + \frac{2^4 e^{-2}}{4!}$	1M	
$= 7e^{-2}$	1A	OR 0.9473
(ii) P(the 5 members will have to wait for more than two mini-buses)		
$= (7e^{-2})^2$	1M	
$= 49e^{-4}$	1A	OR 0.8975
	(4)	
(c) (i) P(the group of 2 gets on the first mini-bus and the group of 3 gets on the next mini-bus)		
$= \frac{2^2 e^{-2}}{2!}\left[1-\left(e^{-2}+\frac{2e^{-2}}{1!}+\frac{2^2 e^{-2}}{2!}\right)\right]$	1M	
$= 2e^{-2}(1-5e^{-2})$	1A	OR 0.0875
(ii) P(none of the members have to wait for more than two mini-buses)		
$= \left(e^{-2}+\frac{2e^{-2}}{1!}\right)(1-7e^{-2}) + 2e^{-2}(1-5e^{-2})$ $+\left(\frac{2^3 e^{-2}}{3!}+\frac{2^4 e^{-2}}{4!}\right)\left[1-\left(e^{-2}+\frac{2e^{-2}}{1!}\right)\right]+1-7e^{-2}$ by (b)(i) & (c)(i)	1M+1M	1M for using (c)(i) 1M for any other one case
$= 1-37e^{-4}$	1A	OR 0.3223
(iii) P(the group of 2 go first | some members have to wait for more than two mini-buses)		
$= \dfrac{\frac{2^2 e^{-2}}{2!}\cdot e^{-2}\left(1+2+\frac{2^2}{2!}\right)+e^{-2}(1+2)\cdot\frac{2^2 e^{-2}}{2!}+[e^{-2}(1+2)]^2\cdot\frac{2^2 e^{-2}}{2!}+\cdots}{1-(1-37e^{-4})}$	1M+1A	1A for any one case
$= \dfrac{10e^{-4}+\frac{6e^{-4}}{1-3e^{-2}}}{37e^{-4}}$	1M	For sum of geometric series
$= \dfrac{2(8e^2-15)}{37(e^2-3)}$	1A	OR 0.5433
	(9)	