香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2012年香港高級程度會考
HONG KONG ADVANCED LEVEL EXAMINATION 2012

數學及統計學 高級補充程度

MATHEMATICS AND STATISTICS AS-LEVEL

評卷參考

MARKING SCHEME

本評卷參考乃香港考試及評核局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

General Instructions To Markers

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits <u>all the marks</u> allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept / technique had been used.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.

5. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

6. In the marking scheme, marks are classified into the following three categories:

   'M' marks – awarded for applying correct methods
   'A' marks – awarded for the accuracy of the answers
   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates' work in awarding 'M' marks.) However, 'A' marks for the corresponding answers should <u>NOT</u> be awarded, unless otherwise specified.

7. In the marking scheme, steps which can be skipped are enclosed by dotted rectangles, whereas alternative answers are enclosed by solid rectangles.

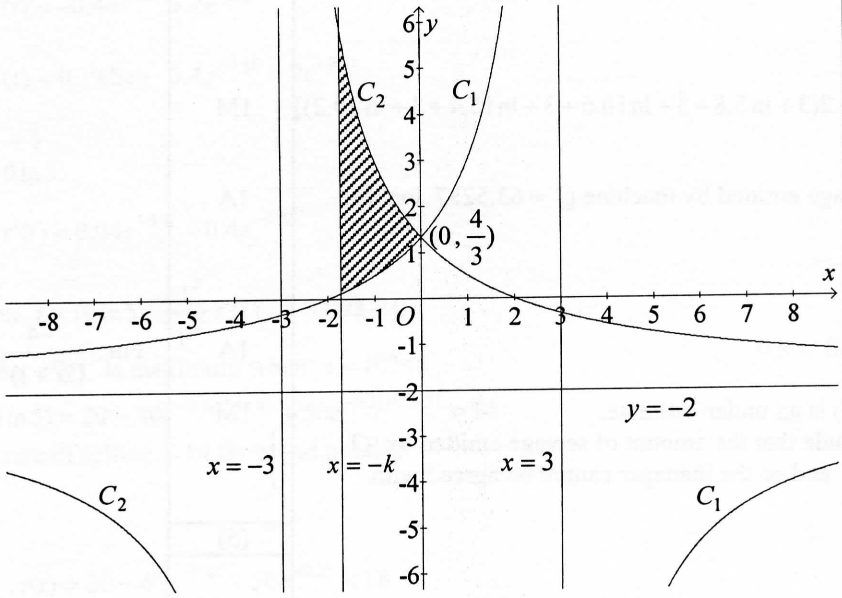
8. Marks may be deducted for poor presentation (*pp*). The symbol pp-1 should be used to denote 1 mark deducted for *pp*.
   (a) At most deduct 1 mark for *pp* in each section.
   (b) In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.

9. Marks may be deducted for numerical answers with inappropriate degree of accuracy (*a*). The symbol a-1 should be used to denote 1 mark deducted for *a*.
   (a) At most deduct 1 mark for *a* in each section.
   (b) In any case, do not deduct any marks for *a* in those steps where candidates could not score any marks.

Solution	Marks	Remarks
1. (a) $a^3+b^3=(a+b)(a^2-ab+b^2)$	1A	
(b) (i) $(1+4x)^{\frac{-1}{2}}$		
$=1+\left(\frac{-1}{2}\right)(4x)+\frac{1}{2!}\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)(4x)^2+\frac{1}{3!}\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)\left(\frac{-1}{2}-2\right)(4x)^3+\cdots$	1M	
$=1-2x+6x^2-20x^3+\cdots$	1A	
$(1+64x^3)^{\frac{1}{2}}$		
$=1+\left(\frac{1}{2}\right)(64x^3)+\cdots$		
$=1+32x^3-\cdots$	1A	
(ii) $\because\ 1+64x^3=(1+4x)(1-4x+16x^2)$		
$\therefore\ (1-4x+16x^2)^{\frac{1}{2}}=(1+64x^3)^{\frac{1}{2}}(1+4x)^{\frac{-1}{2}}$	1M	
$=(1+32x^3-\cdots)(1-2x+6x^2-20x^3+\cdots)$		
$=1-2x+6x^2+12x^3+\cdots$	1A	
	(6)	
2. (a) $y=e^{t^2+4t+4}$ and $x=\ln(2t+4)$		
$\frac{dy}{dt}=e^{t^2+4t+4}(2t+4)$ and $\frac{dx}{dt}=\frac{1}{t+2}$	1A	For both
$\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$		
$=2e^{t^2+4t+4}(t+2)\cdot(t+2)$	1M	
Alternative Solution		
$\ln y=(t+2)^2$ and $x=\ln 2+\ln(t+2)$		OR $\cdots$ and $t+2=\frac{1}{2}e^x$
$\therefore\ x=\ln 2+\frac{1}{2}\ln(\ln y)$	1A	OR $\ln y=\frac{1}{4}e^{2x}$
$\frac{dx}{dy}=\frac{1}{2}\cdot\frac{1}{\ln y}\cdot\frac{1}{y}$	1M	OR $\frac{1}{y}\cdot\frac{dy}{dx}=\frac{1}{4}e^{2x}\cdot 2$
$\frac{dy}{dx}=2y\ln y$	1A	
(b) $\frac{d^2y}{dx^2}=\left(2y\cdot\frac{1}{y}+2\ln y\right)\frac{dy}{dx}$	1M	For chain rule
$=4y\ln y(1+\ln y)$		
When $x=0$, $t=\frac{-3}{2}$ and so $y=e^{\frac{1}{4}}$.	1A	
$\therefore\ \frac{d^2y}{dx^2}=4e^{\frac{1}{4}}\left(\frac{1}{4}\right)\left(1+\frac{1}{4}\right)$		
$=\frac{5}{4}e^{\frac{1}{4}}$	1A	OR 1.6050
	(6)	

Solution	Marks	Remarks
3. (a) Let $u = 1 + e^{-0.2t}$.		
$du = -0.2e^{-0.2t}dt$	1A	
$N = \int \frac{0.3e^{-0.2t}}{(1+e^{-0.2t})^2} dt$		
$N = \int \frac{0.3}{u^2} \cdot \frac{du}{-0.2}$		
$= \frac{3}{2u} + C$	1A	
$= \frac{3}{2(1+e^{-0.2t})} + C$		
When $t = 0$, $N = 0.5$.		
$\therefore\ C = \frac{-1}{4}$		
i.e. $N = \frac{3}{2(1+e^{-0.2t})} - \frac{1}{4}$	1A	
(b) $N(4) - N(0)$		
$= \frac{3}{2(1+e^{-0.2\times 4})} - \frac{1}{4} - 0.5$	1M	
≈ 0.284961721		
Hence the increase in the number of people is 285 .	1A	
(c) $\frac{dN}{dt} = \frac{0.3e^{-0.2t}}{(1+e^{-0.2t})^2} > 0$ for all $t \geq 0$		Withhold the last mark if this argument is missing
Hence N is always increasing.		
$\lim_{t\to\infty} N = \lim_{t\to\infty}\left[\frac{3}{2(1+e^{-0.2t})} - \frac{1}{4}\right]$		OR by arguing that $e^{-0.2t} > 0 \Rightarrow N < \frac{3}{2} - \frac{1}{4}$
$= 1.25$	1A	
Hence the number of members will never reach 1300 .	1	OR by arguing that $\frac{3}{2(1+e^{-0.2t})} - \frac{1}{4} = 1.3$ has no real solution
	(7)	
4. (a) Let X be the number of defective packs in a day.		
$P(X \geq 1) = 1 - \frac{e^{-\lambda}\lambda^0}{0!}$	1A	
$\therefore\ 1 - e^{-2} = 1 - e^{-\lambda}$		
i.e. $\lambda = 2$	1A	
(b) P(the company will have to inspect the production line in a given day)		
$= P(X \geq 4)$		
$= 1 - e^{-2} - \frac{e^{-2}2^1}{1!} - \frac{e^{-2}2^2}{2!} - \frac{e^{-2}2^3}{3!}$	1M	
≈ 0.142876539		
≈ 0.1429	1A	
(c) $(1 - 0.142876539)^n > 0.5$	1M	
$n\ln(1 - 0.142876539) > \ln 0.5$		
$n < 4.495896098$		
i.e. the greatest integral value of n is 4 .	1A	
	(6)	

Solution	Marks	Remarks
5. (a) $\mathrm{P}(A\cap B)=\mathrm{P}(A)\mathrm{P}(B\mid A)$		
$=\dfrac{3a}{8}$	1A	
(b) $\mathrm{P}(A\cap B)=\mathrm{P}(B)\mathrm{P}(A\mid B)$		
$\dfrac{3a}{8}=\dfrac{3}{4}\mathrm{P}(B)$	1M	
$\mathrm{P}(B)=\dfrac{a}{2}$	1A	
(c) (i) $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$		
$1-\dfrac{7}{16}=a+\dfrac{a}{2}-\dfrac{3a}{8}$	1M	
$a=\dfrac{1}{2}$	1A	
(ii) $\mathrm{P}(A\mid B')=\dfrac{\mathrm{P}(A\cap B')}{\mathrm{P}(B')}$		
$=\dfrac{\mathrm{P}(A)-\mathrm{P}(A\cap B)}{1-\mathrm{P}(B)}$		
$=\dfrac{\frac{1}{2}-\frac{3}{8}\times\frac{1}{2}}{1-\frac{1}{2}\times\frac{1}{2}}$	1M	For numerator
$=\dfrac{5}{12}$	1A	
	(7)	
6. (a) $\dfrac{(30+a)+52+\cdots+(90+b)}{30}=71$ and $(90+b)-(30+a)=56$	1M+1M	
$a+b=10$ and $b-a=-4$		
Solving, $a=7$ and $b=3$	1A	For both
$\sigma=\sqrt{\dfrac{803}{5}}$	1A	OR 12.6728
(b) P(3 of the excessive students will have scores higher than 80)		
$=\dfrac{C_3^7C_1^6}{C_4^{13}}$	1M	OR $4\cdot\dfrac{7}{13}\cdot\dfrac{6}{12}\cdot\dfrac{5}{11}\cdot\dfrac{6}{10}$
$=\dfrac{42}{143}$	1A	OR 0.2937
(c) Let x_i $(i=1,2,\ldots,30)$ be the original data.		
The new standard deviation		
$=\sqrt{\dfrac{1}{30+2n}\left[\sum_{i=1}^{30}(x_i-\mu)^2+n(\mu-\sigma-\mu)^2+n(\mu+\sigma-\mu)^2\right]}$	1M	
$=\sqrt{\dfrac{1}{30+2n}(30\sigma^2+n\sigma^2+n\sigma^2)}$		
$=\sigma$	1A	
$=\sqrt{\dfrac{803}{5}}$		OR 12.6728
	(8)	

Solution	Marks	Remarks
7. (a) $\mathrm{f}(x)=\dfrac{ax+b}{c-x}$ gives $\mathrm{g}(x)=\dfrac{-ax+b}{c+x}$		
Since the vertical asymptotes of $C_2: y=\mathrm{g}(x)$ is $x=-3$,		
$c=3$	1A	
Since the y-intercept of $C_1: y=\mathrm{f}(x)$ is $\dfrac{4}{3}$,		
$\dfrac{b}{c}=\dfrac{4}{3}$ ---------- (1)	1M	For either (1) or (2)
$\therefore\ b=4$		
Since the x-intercept of $C_2: y=\mathrm{g}(x)$ is 2,		
$\dfrac{b}{a}=2$ ---------- (2)		OR $0=\dfrac{-a\cdot 2+b}{c+2}$
$\therefore\ a=2$	1A	For both b and a
	(3)	
(b) (i) $C_1:\ y=\dfrac{2x+4}{3-x}$		
The vertical asymptote is $x=3$.	1A	
The horizontal asymptote is $y=-2$.	1A	
(ii) 	1A	For shape of C_1
	1A	For $x=3$ and $y=-2$
	1A	For shape of C_2
	1A	For $x=-3$ and $y=-2$
	1A	For $(\pm 2, 0)$ and $\left(0,\dfrac{4}{3}\right)$
	(7)	
(c) The area $=\displaystyle\int_{-k}^{0}\left(\frac{-2x+4}{3+x}-\frac{2x+4}{3-x}\right)\mathrm{d}x$	1M	
$=\displaystyle\int_{-k}^{0}\left(-2+\frac{10}{3+x}+2-\frac{10}{3-x}\right)\mathrm{d}x$	1M	OR $\displaystyle\int_{x=-k}^{0}\frac{10}{9-x^2}\,\mathrm{d}(9-x^2)$
$=\big[10\ln\lvert 3+x\rvert+10\ln\lvert 3-x\rvert\big]_{-k}^{0}$	1A	For primitive function
$=10[\ln 3+\ln 3-\ln(3-k)-\ln(3+k)]$		OR $10[\ln 9-\ln(9-k^2)]$
$\therefore\ 10\ln\dfrac{9}{9-k^2}=10\ln\dfrac{3}{2}$	1A	
$6=9-k^2$		
$k=\sqrt{3}$ or $-\sqrt{3}$ (rejected)	1A	
	(5)	

Solution	Marks	Remarks
8. (a) Let $u = 1 + 6t$.	1A	
$du = 6dt$		
When $t = 0$, $u = 1$; when $t = 12$, $u = 73$		
$\int_0^{12} \left[4.5 + 2t(1+6t)^{\frac{-2}{3}} \right] dt$		
$= \int_1^{73} \left(4.5 + \frac{u-1}{3} u^{\frac{-2}{3}} \right) \frac{du}{6}$	1M	For integrand
$= \int_1^{73} \left(\frac{3}{4} + \frac{1}{18} u^{\frac{1}{3}} - \frac{1}{18} u^{\frac{-2}{3}} \right) du$		
$= \left[\frac{3u}{4} + \frac{1}{24} u^{\frac{4}{3}} - \frac{1}{6} u^{\frac{1}{3}} \right]_1^{73}$	1A	For primitive function
≈ 66.14060019		
$\therefore$ the total amount of sewage emitted by machine $P \approx 66.1406$ tonnes.	1A	OR $\frac{433 + 23\sqrt[3]{73}}{8}$ tonnes
	(4)	
(b) (i) $\int_0^{12} [3 + \ln(2t+1)] \, dt$		
$= \frac{12-0}{2(5)} [3 + \ln 1 + 3 + \ln 25 + 2(3 + \ln 5.8 + 3 + \ln 10.6 + 3 + \ln 15.4 + 3 + \ln 20.2)]$	1M	
≈ 63.52367987		
$\therefore$ the total amount of sewage emitted by machine $Q \approx 63.5237$ tonnes.	1A	
(ii) $q''(t) = \frac{2}{2t+1}$		
$q'''(t) = \frac{-4}{(2t+1)^2} < 0$ for all $t \geq 0$	1A	For $\frac{-4}{(2t+1)^2}$
Hence the estimate in (b)(i) is an under-estimate.	1M	
Therefore we cannot conclude that the amount of sewage emitted by Q will be less than that by P and so the manager cannot be agreed with.	} 1	
	(5)	
(c) (i) $R = 16 - ae^{-bx}$		
$\ln(16 - R) = \ln a - bx$	1A	
(ii) $\begin{cases} 1 = \ln a + 10b \\ 0 = \ln a - 90b \end{cases}$	1M	
Solving, $a = e^{0.9}$ and $b = 0.01$	1A+1A	OR $a \approx 2.4596$
(iii) Total amount of sewage		
$\approx 80 + 66.14060019 + 63.52367987$	1M	
$= 209.6642801$		
Hence $R = 16 - e^{0.9} e^{-0.01(209.6642801)}$		
≈ 15.69779292		
i.e. the tax paid is 15.6978 million dollars.	1A	
	(6)	

Solution	Marks	Remarks
9. (a) $r(t)=20-40e^{-at}+be^{-2at}$		
$\because\ r(0)=20-40e^{0}+be^{0}=30$		
$\therefore\ b=50$	1A	
	(1)	
(b) $r'(t)<0$ for 9 days		
$40ae^{-at}-100ae^{-2at}<0$ for $t<9$	1M	
$20ae^{-2at}(2e^{at}-5)<0$		
$e^{at}<2.5$		
$t<\dfrac{\ln 2.5}{a}$	1A	
$\therefore\ \dfrac{\ln 2.5}{a}=9$		
i.e. $a\approx 0.1$ (correct to 1 decimal place)	1A	
	(3)	
(c) The rate of change of the rate of selling of handbags is $r'(t)=4e^{-0.1t}-10e^{-0.2t}$.		
$\dfrac{d}{dt}r'(t)=-0.4e^{-0.1t}+2e^{-0.2t}$		
$\dfrac{d}{dt}r'(t)=0$ when $0.4e^{-0.1t}=2e^{-0.2t}$	1M	
$e^{0.1t}=5$		
$t=10\ln 5$	1A	OR 16.0944
$\dfrac{d^2}{dt^2}r'(t)=0.04e^{-0.1t}-0.4e^{-0.2t}$		
When $t=10\ln 5$, $\dfrac{d^2}{dt^2}r'(t)=-0.008<0$	1M	OR by using sign test
Hence $r'(t)$ is maximum when $t=10\ln 5$		
$r(10\ln 5)=20-40e^{-0.1(10\ln 5)}+50e^{-0.2(10\ln 5)}=14$		
The rate of selling $=14$ thousand per day	1A	OR 14000 per day
	(4)	
(d) (i) $r(t)=20-40e^{-0.1t}+50e^{-0.2t}<18$	1M	
$25e^{-0.2t}-20e^{-0.1t}+1<0$		
$0.053589838<e^{-0.1t}<0.746410161$		
$2.924800155<t<29.26395809$	1A	
$\because\ 29.26395809-2.924800155=26.33915794$		
$\therefore$ the 'sales warning' will last for 26 days.	1A	
(ii) Number of handbags sold (in thousand) during the 'sales warning' period		
$=\displaystyle\int_{2.92480155}^{29.26395809}(20-40e^{-0.1t}+50e^{-0.2t})\,dt$	1M	
$=\left[20t+400e^{-0.1t}-250e^{-0.2t}\right]_{2.924800155}^{29.26395809}$		
≈ 388.2190941	1A	Accept 388.2191
$\dfrac{388.2190941}{26.33915794}\approx 14.7392$	1M	
Hence the average number of handbags sold per day is 15 thousand.	1A	OR 15000
	(7)	

Solution	Marks	Remarks
10. Let A and B be the operation time of a randomly chosen battery A and B respectively.		
(a) (i) $P(A<152 \text{ or } A>184)$		
$=P\left(Z<\frac{152-168}{32} \text{ or } Z>\frac{184-168}{32}\right)$		
$=P(Z<-0.5 \text{ or } Z>0.5)$		
$=0.617$	1A	
(ii) $P(A>k)=0.05$		
$\frac{k-168}{32}=1.645$	1M	Accept 1.64 or 1.65
$k=220.64$	1A	Accept 220.48 or 220.8
(iii) $P(B>188)=0.33$ and $P(B<213.2)=0.877$		
$P\left(Z>\frac{188-\mu}{\sigma}\right)=0.33$ and $P\left(Z<\frac{213.2-\mu}{\sigma}\right)=0.877$		
$\frac{188-\mu}{\sigma}=0.44$ and $\frac{213.2-\mu}{\sigma}=1.16$	1M	
Solving, $\mu=172.6$ and $\sigma=35$.	1A+1A	
(iv) $P(B<146)=P\left(Z<\frac{146-172.6}{35}\right)$		
$=P(Z<-0.76)$		
$=0.2236$	1A	
	(7)	
(b) (i) $\lambda_A=1500\times\frac{1}{3}\times P(A<104)$	1M	Either one
$=500\times P\left(Z<\frac{104-168}{32}\right)$		
$=500\times P(Z<-2)$		
$=11.4$ (correct to 1 d.p.)	1A	
$\lambda_B=1500\times\frac{2}{3}\times P(B<104)$		
$=1000\times P\left(Z<\frac{104-172.6}{35}\right)$		
$=1000\times P(Z<-1.96)$		
$=25.0$ (correct to 1 d.p.)	1A	Accept 25
(ii) $P(4\le \text{number of 'faulty' batteries } A \text{ produced} \le 6)$		
$=\frac{e^{-11.4}11.4^4}{4!}+\frac{e^{-11.4}11.4^5}{5!}+\frac{e^{-11.4}11.4^6}{6!}$	1M	
≈ 0.0600	1A	
(iii) The required probability		
$=\dfrac{\frac{e^{-11.4}11.4^4}{4!}\times\frac{e^{-25}25^6}{6!}+\frac{e^{-11.4}11.4^5}{5!}\times\frac{e^{-25}25^5}{5!}}{\frac{e^{-11.4}11.4^4}{4!}\times\frac{e^{-25}25^6}{6!}+\frac{e^{-11.4}11.4^5}{5!}\times\frac{e^{-25}25^5}{5!}+\frac{e^{-11.4}11.4^6}{6!}\times\frac{e^{-25}25^4}{4!}}$	1M+1M	1M for numerator 1M for denominator
≈ 0.8815	1A	
	(8)	

Solution	Marks	Remarks
11. (a) (i) $\frac{e^{-\lambda}\lambda^8}{8!}=\frac{12.39}{120}$ and $\frac{e^{-\lambda}\lambda^9}{9!}=\frac{8.26}{120}$	1M	
Dividing, we have $\frac{e^{-\lambda}\lambda^9}{9!}\cdot\frac{8!}{e^{-\lambda}\lambda^8}=\frac{8.26}{120}\cdot\frac{120}{12.39}$		
i.e. $\lambda=6$	1A	OR 6.0
$C_7^{10}p^7(1-p)^3=\frac{25.80}{120}$ and $C_8^{10}p^8(1-p)^2=\frac{14.51}{120}$	1M	
Dividing, we have $\frac{C_8^{10}p^8(1-p)^2}{C_7^{10}p^7(1-p)^3}=\frac{14.51}{120}\cdot\frac{120}{25.80}$		
$\frac{3p}{8(1-p)}=\frac{1451}{2580}$		
$p\approx 0.6$ (correct to 1 decimal place)	1A	$\frac{2902}{4837}$ not acceptable
(ii) $a=\frac{e^{-6}6^7}{7!}\cdot 120\approx 16.52$		
$b=\frac{e^{-6}6^{10}}{10!}\cdot 120\approx 4.96$	1A	1A for any two correct
$c=C_9^{10}(0.6)^9(0.4)\cdot 120\approx 4.84$	1A	1A for the other two correct
$d=(0.6)^{10}\cdot 120\approx 0.73$		
(iii) For the number of new born babies diagnosed with congenital diseases greater than 10, the expected frequency by $\mathrm{Po}(6)$ is		
$120-72.76-16.52-12.39-8.26-4.96\approx 5.11$	1M	
The sum of errors for model fitted by $\mathrm{Po}(6)$ is		
$E_1=\lvert 74-72.76\rvert+\lvert 20-16.52\rvert+\lvert 14-12.39\rvert+\lvert 8-8.26\rvert+\lvert 4-4.96\rvert+\lvert 0-5.11\rvert$		
$=12.66$	1A	
The sum of errors for model fitted by $\mathrm{B}(10, 0.6)$ is		
$E_2=\lvert 74-74.13\rvert+\lvert 20-25.80\rvert+\lvert 14-14.51\rvert+\lvert 8-4.84\rvert+\lvert 4-0.73\rvert$		
$=12.87$	1A	Accept 12.88
Since $E_1<E_2$, $\mathrm{Po}(6)$ fits the observed data better.	1	
	(10)	
(b) (i) P(a new born baby has congenital diseases)		
$=0.45\times 0.025+0.55\times 0.01$		
$=0.01675$	1A	Accept 0.0168
(ii) P(a baby is born to a non-local mother | the baby has congenital diseases)		
$=\frac{0.55\times 0.01}{0.01675}$		
$=\frac{22}{67}$	1A	OR 0.3284
(iii) P(7 babies are born to non-local mothers | 8 to 10 babies have congenital diseases)		
$=\dfrac{\frac{e^{-6}6^8}{8!}C_7^8\left(\frac{22}{67}\right)^7\left(\frac{45}{67}\right)+\frac{e^{-6}6^9}{9!}C_7^9\left(\frac{22}{67}\right)^7\left(\frac{45}{67}\right)^2+\frac{e^{-6}6^{10}}{10!}C_7^{10}\left(\frac{22}{67}\right)^7\left(\frac{45}{67}\right)^3}{\frac{e^{-6}6^8}{8!}+\frac{e^{-6}6^9}{9!}+\frac{e^{-6}6^{10}}{10!}}$	1M+1M	1M for numerator 1M for denominator
≈ 0.0061	1A	
	(5)	

Solution	Marks	Remarks
12. (a) (i) P(the centre needs to give out 2 or 3 coupons)		
$= \text{P}(\ 10 \text{ or } 11 \text{ customers show up})$		
$= C_{10}^{12}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{2} + C_{11}^{12}\left(\frac{2}{3}\right)^{11}\left(\frac{1}{3}\right)$	1M	
$= \frac{10240}{59049}$	1A	OR 0.1734
(ii) P(every customer with booking who shows up can be assigned a trainer)		
$= \text{P}(\text{at most } 8 \text{ customers show up})$		
$= 1 - C_{9}^{12}\left(\frac{2}{3}\right)^{9}\left(\frac{1}{3}\right)^{3} - C_{10}^{12}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{2} - C_{11}^{12}\left(\frac{2}{3}\right)^{11}\left(\frac{1}{3}\right) - \left(\frac{2}{3}\right)^{12}$	1M	
$= \frac{107515}{177147}$	1A	OR 0.6069
	(4)	
(b) If the centre accepts 10 bookings, then		
P(every customer who have made a booking can be assigned a trainer)		
$= 1 - C_{9}^{10}\left(\frac{2}{3}\right)^{9}\left(\frac{1}{3}\right) - \left(\frac{2}{3}\right)^{10}$		
≈ 0.8960	1A	
> 0.8		
If the centre accepts 11 bookings, then		
P(every customer who have made a booking can be assigned a trainer)		
$= 1 - C_{9}^{11}\left(\frac{2}{3}\right)^{9}\left(\frac{1}{3}\right)^{2} - C_{10}^{11}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right) - \left(\frac{2}{3}\right)^{11}$		
≈ 0.7659	1A	
< 0.8		
Hence the centre can accepts 10 bookings at most.	1A	
	(3)	
(c) (i) The expected income in that evening		
$= \$\,(0.5 \times 3800 + 0.3 \times 2800 + 0.2 \times 1800) \times 8$	1M	
$= \$24800$	1A	
(ii) P(the 8th customer is the first one to select Jade programs)		
$= (0.8)^{7}(0.2)$		
$= \frac{16384}{390625}$	1A	OR 0.0419
(iii) P(all programs are selected and exactly 3 are Diamond programs)		
$= \frac{8!}{3!\,4!\,1!}(0.5)^{3}(0.3)^{4}(0.2)^{1} + \frac{8!}{3!\,3!\,2!}(0.5)^{3}(0.3)^{3}(0.2)^{2}$		OR $C_{3}^{8}(0.5)^{3}[(0.5)^{5} - (0.3)^{5} - (0.2)^{5}]$
$+ \frac{8!}{3!\,2!\,3!}(0.5)^{3}(0.3)^{2}(0.2)^{3} + \frac{8!}{3!\,1!\,4!}(0.5)^{3}(0.3)^{1}(0.2)^{4}$	1M+1A	
$= 0.1995$	1A	
(iv) The required probability		
$= \frac{1}{0.1995}\left[\frac{8!}{3!\,4!\,1!}(0.5)^{3}(0.3)^{4}(0.2)^{1} + \frac{8!}{3!\,3!\,2!}(0.5)^{3}(0.3)^{3}(0.2)^{2}\right]$	1M	OR $\frac{C_{3}^{8}(0.5)^{3}[C_{1}^{5}(0.3)^{4}(0.2) + C_{3}^{5}(0.3)^{3}(0.2)^{2}]}{0.1995}$
≈ 0.6632	1A	
	(8)	