This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.
General Instructions To Markers

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates’ work. In general, marks for a certain step should be awarded if candidates’ solution indicated that the relevant concept / technique had been used.

3. In marking candidates’ work, the benefit of doubt should be given in the candidates’ favour.

4. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.

5. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

6. In the marking scheme, marks are classified into the following three categories:

   'M' marks – awarded for applying correct methods
   'A' marks – awarded for the accuracy of the answers
   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

   In a question consisting of several parts each depending on the previous parts, ‘M’ marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates’ work in awarding ‘M’ marks.) However, ‘A’ marks for the corresponding answers should NOT be awarded, unless otherwise specified.

7. In the marking scheme, steps which can be skipped are enclosed by dotted rectangles \( \text{\textdottedangle} \), whereas alternative answers are enclosed by solid rectangles \( \text{\textsolidangle} \).

8. Marks may be deducted for poor presentation (\( pp \)). The symbol \( pp-\text{-1} \) should be used to denote 1 mark deducted for \( pp \).
   (a) At most deduct 1 mark for \( pp \) in each section.
   (b) In any case, do not deduct any marks for \( pp \) in those steps where candidates could not score any marks.

9. Marks may be deducted for numerical answers with inappropriate degree of accuracy (\( a \)). The symbol \( a-\text{-1} \) should be used to denote 1 mark deducted for \( a \).
   (a) At most deduct 1 mark for \( a \) in each section.
   (b) In any case, do not deduct any marks for \( a \) in those steps where candidates could not score any marks.
1. (a) \[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \]

(b) (i) \[
\begin{align*}
(1 + 4x)^2 &= 1 + \left( \frac{-1}{2} \right) (4x) + \frac{1}{2!} \left( \frac{-1}{2} \right) \left( \frac{-1}{2} \right) (4x)^2 + \frac{1}{3!} \left( \frac{-1}{2} \right) \left( \frac{-1}{2} \right) \left( \frac{-1}{2} \right) (4x)^3 + \ldots \\
n &= 1 - 2x + 6x^2 - 20x^3 + \ldots
\end{align*}
\]

\[
(1 + 64x^3)^{\frac{1}{2}}
\]

\[
= 1 + \left( \frac{1}{2} \right) (64x^3) + \ldots
\]

\[
= 1 + 32x^3 - \ldots
\]

(ii) \[
\begin{align*}
1 + 64x^3 &= (1 + 4x)(1 - 4x + 16x^2) \\
(1 - 4x + 16x^2)^2 &= (1 + 64x^3)^2 (1 + 4x)^2 \\
&= (1 + 32x^3 - \ldots)(1 - 2x + 6x^2 - 20x^3 + \ldots)
\end{align*}
\]

\[
= 1 - 2x + 6x^2 + 12x^3 + \ldots
\]

2. (a) \[ y = e^{x^2 + 4t + 4} \text{ and } x = \ln(2t + 4) \]

\[
\frac{dy}{dt} = e^{x^2+4t+4}(2t+4) \text{ and } \frac{dx}{dt} = \frac{1}{t+2}
\]

\[
\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = 2e^{x^2+4t+4}(t+2) \cdot \frac{1}{t+2}
\]

Alternative Solution

\[
\ln y = (t+2)^2 \text{ and } x = \ln 2 + \ln(t+2)
\]

\[
\ln y = (t+2)^2 \text{ and } x = \ln 2 + \ln(t+2)
\]

\[
\frac{dx}{dy} = \frac{1}{2} \cdot \frac{1}{y} = \frac{1}{2y}
\]

\[
\frac{dy}{dx} = 2y \ln y
\]

(b) \[
\frac{d^2 y}{dx^2} = \left( 2y \cdot \frac{1}{y} + 2 \ln y \right) \frac{dy}{dx}
\]

\[
= 4y \ln y(1 + \ln y)
\]

When \[ x = 0, \ t = \frac{-3}{2} \] and so \[ y = e^{\frac{1}{4}} \].

\[
\frac{d^2 y}{dx^2} = 4e^{\frac{1}{4}} \left( 1 + \frac{1}{4} \right)
\]

\[
= \frac{5}{4} e^{\frac{1}{4}}
\]

For both \[ OR \ldots \text{ and } t + 2 = \frac{1}{2} e^x \]

OR \[ \ln y = \frac{1}{4} e^{2x} \]

OR \[ \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{4} e^{2x} \cdot 2 \]

For chain rule

OR \[ 1.6050 \]

Marks

<table>
<thead>
<tr>
<th>Marks</th>
<th>Remarks</th>
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</thead>
<tbody>
<tr>
<td>1A</td>
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<tr>
<td>1M</td>
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<td>1A</td>
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<td></td>
</tr>
</tbody>
</table>
3. (a) Let \( u = 1 + e^{-0.2t} \). 
\[
\frac{du}{dt} = -0.2e^{-0.2t} \]
\[
N = \int \frac{0.3e^{-0.2t}}{(1 + e^{-0.2t})^2} \, dt
\]
\[
N = \int \frac{0.3}{u^2} \cdot \frac{du}{-0.2} = \frac{3}{2u} + C
\]
\[
= \frac{3}{2(1 + e^{-0.2t})} + C
\]
When \( t = 0 \), \( N = 0.5 \).
\[
\therefore \quad C = \frac{-1}{4}
\]
i.e. \( N = \frac{3}{2(1 + e^{-0.2t})} - \frac{1}{4} \)

(b) \( N(t) - N(0) \)
\[
= \frac{3}{2(1 + e^{-0.2t})} - \frac{1}{4} - 0.5
\]
\[
= 0.284961721
\]
Hence the increase in the number of people is 285.

(c) \( \frac{dN}{dt} \) \( (1 + e^{-0.2t})^2 \) > 0 for all \( t \geq 0 \)
Hence \( N \) is always increasing.
\[
\lim_{t \to \infty} N = \lim_{t \to \infty} \left[ \frac{3}{2(1 + e^{-0.2t})} - \frac{1}{4} \right] = 1.25
\]
Hence the number of members will never reach 1300.

4. (a) Let \( X \) be the number of defective packs in a day.
\[
P(X \geq 1) = 1 - \frac{e^{-\lambda} \lambda^0}{0!}
\]
\[
\therefore \quad 1 - e^{-\lambda} = 1 - e^{-\lambda}
\]
i.e. \( \lambda = 2 \)

(b) \( P(\text{the company will have to inspect the production line in a given day}) \)
\[
P(X \geq 4) = 1 - e^{-2} - \frac{e^{-2}2!}{2!} - \frac{e^{-2}2^2}{3!}
\]
\[
= 0.142876539
\]

(c) \( (1 - 0.142876539)^n > 0.5 \)
\( n \ln(1 - 0.142876539) > \ln 0.5 \)
\( n < 4.495896098 \)
i.e. the greatest integral value of \( n \) is 4.

<table>
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<th>Solution</th>
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\frac{du}{dt} = -0.2e^{-0.2t} \]
\[
N = \int \frac{0.3e^{-0.2t}}{(1 + e^{-0.2t})^2} \, dt
\]
\[
N = \int \frac{0.3}{u^2} \cdot \frac{du}{-0.2} = \frac{3}{2u} + C
\]
\[
= \frac{3}{2(1 + e^{-0.2t})} + C
\]
When \( t = 0 \), \( N = 0.5 \).
\[
\therefore \quad C = \frac{-1}{4}
\]
i.e. \( N = \frac{3}{2(1 + e^{-0.2t})} - \frac{1}{4} \) | 1A |
| (b) \( N(t) - N(0) \)
\[
= \frac{3}{2(1 + e^{-0.2t})} - \frac{1}{4} - 0.5
\]
\[
= 0.284961721
\]
Hence the increase in the number of people is 285. | 1M |
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\( n < 4.495896098 \)
i.e. the greatest integral value of \( n \) is 4. | 1A |
5. (a) \[ P(A \cap B) = P(A)P(B \mid A) \]
\[ = \frac{3a}{8} \]
\[ P(B) = \frac{a}{2} \]

(b) \[ P(A \cap B) = P(B)P(A \mid B) \]
\[ = \frac{3a}{8} \times \frac{3}{4}P(B) \]

(c) (i) \[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
\[ = 1 - \frac{7}{16} = a + \frac{3a}{2} - \frac{8}{8} \]
\[ a = \frac{1}{2} \]

(ii) \[ P(A \mid B') = \frac{P(A \cap B')}{P(B')} \]
\[ = \frac{P(A) - P(A \cap B)}{1 - P(B)} \]
\[ = \frac{1}{2} \times \frac{3}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{1}{2} \]
\[ = \frac{5}{12} \]

6. (a) \[ \frac{(30 + a) + 52 + \cdots + (90 + b)}{30} = 71 \] and \( (90 + b) - (30 + a) = 56 \)
\[ a + b = 10 \text{ and } b - a = -4 \]
Solving, \( a = 7 \) and \( b = 3 \)
\[ \sigma = \sqrt{\frac{803}{5}} \]

(b) \[ P(3 \text{ of the excessive students will have scores higher than 80}) \]
\[ = \frac{C_3^2C_6^6}{C_4^{13}} \]
\[ = \frac{42}{143} \]

(c) Let \( x_i \ (i = 1, 2, \ldots, 30) \) be the original data.
The new standard deviation
\[ = \sqrt{\frac{1}{30 + 2n} \sum_{i=1}^{30} (x_i - \mu)^2 + n(\mu - \sigma - \mu)^2 + n(\mu + \sigma - \mu)^2} \]
\[ = \sqrt{\frac{1}{30 + 2n} - (30\sigma^2 + n\sigma^2 + n\sigma^2)} \]
\[ = \sigma \sqrt{\frac{803}{5}} \]
\[ OR 12.6728 \]
7. (a) \( f(x) = \frac{ax+b}{c-x} \) gives \( g(x) = \frac{-ax+b}{c+x} \)

Since the vertical asymptotes of \( C_2: y = g(x) \) is \( x = -3 \),
\( c = 3 \)

Since the \( y \)-intercept of \( C_1: y = f(x) \) is \( \frac{4}{3} \),
\( \frac{b}{c} = 4 \)
\( \therefore b = 4 \)

Since the \( x \)-intercept of \( C_2: y = g(x) \) is 2,
\( \frac{b}{a} = 2 \)
\( \therefore a = 2 \)

(b) (i) \( C_1: y = \frac{2x+4}{3-x} \)

The vertical asymptote is \( x = 3 \).
The horizontal asymptote is \( y = -2 \).

(ii)

\[ \text{The area} = \int_{-k}^{0} \left( \frac{-2x+4}{3+x} - \frac{2x+4}{3-x} \right) \, dx = \int_{-k}^{0} \left( -2 + \frac{10}{3+x} + \frac{10}{3-x} \right) \, dx = \left[ 10 \ln |3+x| + 10 \ln |3-x| \right]_{-k}^{0} = 10 \left[ \ln 3 + \ln 3 \ln 3 - \ln(3+k) - \ln(3-k) \right] \]
\( \therefore 10 \ln \frac{9}{9-k^2} = 10 \ln \frac{3}{2} \)
\( 6 = 9 - k^2 \)
\( k = \sqrt{3} \) or \( -\sqrt{3} \) (rejected).

(c) The area \( \int_{-k}^{0} \left( \frac{-2x+4}{3+x} - \frac{2x+4}{3-x} \right) \, dx \)

\( = \left[ 10 \ln |3+x| + 10 \ln |3-x| \right]_{-k}^{0} = 10 \left[ \ln 3 + \ln 3 \ln 3 - \ln(3+k) - \ln(3-k) \right] \)

\( \therefore 10 \ln \frac{9}{9-k^2} = 10 \ln \frac{3}{2} \)
\( 6 = 9 - k^2 \)
\( k = \sqrt{3} \) or \( -\sqrt{3} \) (rejected).
### Solution

#### (a)
Let \( u = 1 + 6t \).

\[
\frac{du}{dt} = 6 \quad \text{or} \quad du = 6 \, dt
\]

When \( t = 0 \), \( u = 1 \); when \( t = \frac{1}{2} \), \( u = 13 \).

\[
\int_0^{1/2} \left( 4.5 + 2t(1 + 6t)^{-2} \right) \, dt
\]

\[
= \int_1^{13} \left( 4.5 + \frac{u-1}{3} \, u^{-2} \right) \frac{du}{6}
\]

\[
= \int_1^{13} \left( \frac{3}{4} + \frac{1}{18} u^3 - \frac{1}{18} \, u^{-3} \right) \, du
\]

\[
= \left[ \frac{3u}{4} + \frac{1}{24} \, u^2 - \frac{1}{6} \, u^{-2} \right]_1^{13}
\]

\[
\approx 66.14060019
\]

\[
\therefore \text{the total amount of sewage emitted by machine } P \approx 66.1406 \text{ tonnes.}
\]

#### (b)

(i) \[ \int_0^2 \left[ 3 + \ln(2t+1) \right] \, dt \]

\[
= \left[ \frac{12 - 0}{2(5)} \right] [3 + \ln 1 + 3 + \ln 25 + 2(3 + \ln 5.8 + 3 + \ln 10.6 + 3 + \ln 15.4 + 3 + \ln 20.2)]
\]

\[
= 63.52367987
\]

\[
\therefore \text{the total amount of sewage emitted by machine } Q \approx 63.5237 \text{ tonnes.}
\]

(ii) \[ q''(t) = \frac{2}{2t+1}, \quad q'''(t) = -\frac{4}{(2t+1)^2} < 0 \quad \text{for all} \quad t \geq 0 \]

Hence the estimate in (b)(i) is an under-estimate.

Therefore we cannot conclude that the amount of sewage emitted by \( Q \) will be less than that by \( P \) and so the manager cannot be agreed with.

#### (c)

(i) \[ R = 16 - ae^{-bx} \]

\[
\ln(16 - R) = \ln a - bx
\]

(ii) \[
\begin{align*}
1 &= \ln a + 10b \\
0 &= \ln a - 90b
\end{align*}
\]

Solving, \( a = e^{0.9} \) and \( b = 0.01 \)

(iii) Total amount of sewage

\[
\approx 80 + 66.14060019 + 63.52367987
\]

\[
= 209.6642801
\]

Hence \[ R = 16 - e^{-0.9 \times 0.01(209.6642801)} \]

\[
\approx 15.69779292
\]

i.e. the tax paid is 15.6978 million dollars.
<table>
<thead>
<tr>
<th>Solution</th>
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<tbody>
<tr>
<td>9. (a) ( r(t) = 20 - 40e^{-at} + be^{-2at} )</td>
<td>1A</td>
</tr>
<tr>
<td>( \therefore r(0) = 20 - 40e^0 + be^0 = 30 )</td>
<td></td>
</tr>
<tr>
<td>( \therefore b = 50 )</td>
<td></td>
</tr>
<tr>
<td>(b) ( r'(t) &lt; 0 ) for 9 days</td>
<td>1M</td>
</tr>
<tr>
<td>( 40ae^{-at} - 100ae^{-2at} &lt; 0 ) for ( t &lt; 9 )</td>
<td></td>
</tr>
<tr>
<td>( 20ae^{-2at}(2e^{at} - 5) &lt; 0 )</td>
<td></td>
</tr>
<tr>
<td>( e^{at} &lt; 2.5 )</td>
<td></td>
</tr>
<tr>
<td>( t &lt; \frac{\ln 2.5}{a} )</td>
<td>1A</td>
</tr>
<tr>
<td>( \therefore \frac{\ln 2.5}{a} = 9 )</td>
<td></td>
</tr>
<tr>
<td>i.e. ( a \approx 0.1 ) (correct to 1 decimal place)</td>
<td></td>
</tr>
<tr>
<td>(c) The rate of change of the rate of selling of handbags is ( r'(t) = 4e^{-0.1t} - 10e^{-0.2t} ). ( \frac{dr}{dt} = -0.4e^{-0.1t} + 2e^{-0.2t} ) ( \frac{dr}{dt} = 0 ) when ( 0.4e^{-0.1t} = 2e^{-0.2t} ) ( e^{0.1t} = 5 ) ( t = 10\ln 5 ) ( \frac{d^2r}{dt^2} = 0.04e^{-0.1t} - 0.4e^{-0.2t} ) ( \text{When } t = 10\ln 5, \frac{d^2r}{dt^2} r'(t) = -0.008 &lt; 0 ) Hence ( r'(t) ) is maximum when ( t = 10\ln 5 ) ( r(10\ln 5) = 20 - 40e^{0.1(10\ln 5)} + 50e^{-2(10\ln 5)} = 14 ) The rate of selling = 14 thousand per day</td>
<td></td>
</tr>
<tr>
<td>(d) (i) ( r(t) = 20 - 40e^{-0.1t} + 50e^{-0.2t} &lt; 18 ) ( 25e^{-0.2t} - 20e^{-0.1t} + 1 &lt; 0 ) ( 0.053589838 &lt; e^{-0.1t} &lt; 0.746410161 ) ( 2.924800155 &lt; t &lt; 29.26395809 ) ( \therefore 29.26395809 - 2.924800155 = 26.33915794 ) ( \text{the 'sales warning' will last for 26 days.} )</td>
<td></td>
</tr>
<tr>
<td>(ii) Number of handbags sold (in thousand) during the 'sales warning' period ( = \int_{29.26395809}^{29.26395809} (20 - 40e^{-0.1t} + 50e^{-0.2t}) dt ) ( = [20 + 400e^{-0.1t} - 250e^{-0.2t}]_{29.26395809}^{29.26395809} = 388.2190941 ) ( 388.2190941 = 14.7392 ) ( 26.33915794 ) Hence the average number of handbags sold per day is 15 thousand.</td>
<td>1A</td>
</tr>
<tr>
<td>1A</td>
<td>OR 15000</td>
</tr>
</tbody>
</table>
10. Let $A$ and $B$ be the operation time of a randomly chosen battery $A$ and $B$ respectively.

(a) (i) $P(A < 152 \text{ or } A > 184) = P \left( Z < \frac{152 - 168}{32} \text{ or } Z > \frac{184 - 168}{32} \right) = P(Z < -0.5 \text{ or } Z > 0.5) = 0.617$

(ii) $P(A > k) = 0.05$
\[
\begin{align*}
  k - 168 &= 1.645 \\
  k &= 220.64
\end{align*}
\]

(iii) $P(B > 188) = 0.33$ and $P(B < 213.2) = 0.877$
\[
\begin{align*}
  P \left( Z > \frac{188 - \mu}{\sigma} \right) &= 0.33 \quad \text{and} \quad P \left( Z < \frac{213.2 - \mu}{\sigma} \right) = 0.877 \\
  \frac{188 - \mu}{\sigma} &= 0.44 \quad \text{and} \quad \frac{213.2 - \mu}{\sigma} = 1.16 \\
  \text{Solving,} \quad \mu &= 172.6 \quad \text{and} \quad \sigma = 35.
\end{align*}
\]

(iv) $P(B < 146) = P \left( Z < \frac{146 - 172.6}{35} \right) = P(Z < -0.76) = 0.2236$

(b) (i) $\lambda_A = 1500 \times \frac{1}{3} \times P(A < 104)$
\[
\begin{align*}
  &= 500 \times P \left( Z < \frac{104 - 168}{32} \right) \\
  &= 500 \times P(Z < -2) \\
  &= 11.4 \quad \text{(correct to 1 d.p.)}
\end{align*}
\]

$\lambda_B = 1500 \times \frac{2}{3} \times P(B < 104)$
\[
\begin{align*}
  &= 1000 \times P \left( Z < \frac{104 - 172.6}{35} \right) \\
  &= 1000 \times P(Z < -1.96) \\
  &= 25.0 \quad \text{(correct to 1 d.p.)}
\end{align*}
\]

(ii) $P(4 \leq \text{number of 'faulty' batteries $A$ produced} \leq 6)$
\[
\begin{align*}
  &= \frac{e^{-11.4}11.4^4}{4!} + \frac{e^{-11.4}11.4^5}{5!} + \frac{e^{-11.4}11.4^6}{6!} \\
  &\approx 0.0600
\end{align*}
\]

(iii) The required probability
\[
\begin{align*}
  &= \frac{e^{-11.4}11.4^4 \times e^{-25}25^6}{4! \times 6!} + \frac{e^{-11.4}11.4^5 \times e^{-25}25^5}{5! \times 5!} + \frac{e^{-11.4}11.4^6 \times e^{-25}25^4}{6! \times 4!} \\
  &\approx 0.8815
\end{align*}
\]
11. (a) (i) \[ \frac{e^{-\lambda} \lambda^8}{8!} = \frac{12.39}{120} \quad \text{and} \quad \frac{e^{-\lambda} \lambda^9}{9!} = \frac{8.26}{120} \]

Dividing, we have \[ \frac{e^{-\lambda} \lambda^8}{8!} \cdot \frac{9!}{e^{-\lambda} \lambda^8} = \frac{8.26}{120} \cdot \frac{120}{12.39} \]

i.e. \( \lambda = 6 \)

\[ C_1^9 p^7 (1-p)^3 = \frac{25.80}{120} \quad \text{and} \quad C_8^9 p^8 (1-p)^2 = \frac{14.51}{120} \]

Dividing, we have \[ \frac{C_8^9 p^8 (1-p)^2}{C_1^9 p^7 (1-p)^3} = \frac{14.51}{120} \cdot \frac{120}{25.80} \]

\[ \frac{3p}{8(1-p)} = \frac{1451}{2580} \]

\( p \approx 0.6 \) (correct to 1 decimal place)

(ii) \( a = \frac{e^{-6} 6^7}{7!} \cdot 120 \approx 16.52 \)

\( b = \frac{e^{-6} 6^{10}}{10!} \cdot 120 \approx 4.96 \)

\( c = C_9^8 (0.6)^9 (0.4) \cdot 120 \approx 4.84 \)

\( d = (0.6)^{10} \cdot 120 \approx 0.73 \)

(iii) For the number of new born babies diagnosed with congenital diseases greater than 10, the expected frequency by Po(6) is

\[ 120 - 72.76 - 16.52 - 12.39 - 8.26 - 4.96 = 5.11 \]

The sum of errors for model fitted by Po(6) is

\[ E_1 = 74 - 72.76 + 20 - 16.52 + 14 - 12.39 + 8 - 8.26 + 4 - 4.96 + 0 - 5.11 \]

\[ = 12.66 \]

The sum of errors for model fitted by B(10, 0.6) is

\[ E_2 = 74 - 74.13 + 20 - 25.80 + 14 - 14.51 + 8 - 4.84 + 4 - 0.73 \]

\[ = 12.87 \]

Since \( E_1 < E_2 \), Po(6) fits the observed data better.

(b) (i) \( P(\text{a new born baby has congenital diseases}) = 0.45 \times 0.025 + 0.55 \times 0.01 \)

\[ = 0.01675 \]

(ii) \( P(\text{a baby is born to a non-local mother | the baby has congenital diseases}) = \frac{0.55 \times 0.01}{0.01675} \)

\[ = \frac{22}{67} \]

(iii) \( P(\text{7 babies are born to non-local mothers | 8 to 10 babies have congenital diseases}) \)

\[ = \frac{e^{-6} 6^8}{8!} \left( \sum_{k=8}^{10} \frac{C_7^8 8^7 6^{10-k}}{7!} \right) \]

\[ \approx 0.0061 \]
12. (a) (i) \( P(\text{the centre needs to give out 2 or 3 coupons}) = P(10 \text{ or } 11 \text{ customers show up}) \)
\[
= \binom{12}{10} \left( \frac{2}{3} \right)^{10} \left( \frac{1}{3} \right)^2 + \binom{12}{11} \left( \frac{2}{3} \right)^{11} \left( \frac{1}{3} \right)^1
\]
\[
= 10240 + 59049
\]
\[
= 70290
\]

(ii) \( P(\text{every customer with booking who shows up can be assigned a trainer}) = P(\text{at most 8 customers show up}) \)
\[
= 1 - \binom{12}{10} \left( \frac{2}{3} \right)^9 \left( \frac{1}{3} \right)^3 - \binom{12}{11} \left( \frac{2}{3} \right)^{10} \left( \frac{1}{3} \right)^2 - \binom{12}{12} \left( \frac{2}{3} \right)^{12} \left( \frac{1}{3} \right)^0
\]
\[
= 107515
\]
\[
= 177147
\]

(b) If the centre accepts 10 bookings, then
\[ P(\text{every customer who have made a booking can be assigned a trainer}) \approx 0.8960 \]
If the centre accepts 11 bookings, then
\[ P(\text{every customer who have made a booking can be assigned a trainer}) \approx 0.7659 \]
Hence the centre can accepts 10 bookings at most.

(c) (i) The expected income in that evening
\[ = \$ (0.5 \times 3800 + 0.3 \times 2800 + 0.2 \times 1800) \times 8 \]
\[ = \$ 24800 \]

(ii) \( P(\text{the 8th customer is the first one to select Jade programs}) = (0.8)^7 (0.2) \)
\[ = 0.16384 \]
\[ = 390625 \]

(iii) \( P(\text{all programs are selected and exactly 3 are Diamond programs}) \)
\[
= \frac{8!}{3! 2! 1!} (0.5)^3 (0.3)^4 (0.2)^1 + \frac{8!}{3! 3! 2!} (0.5)^3 (0.3)^3 (0.2)^2
\]
\[ + \frac{8!}{3! 2! 2!} (0.5)^3 (0.3)^2 (0.2)^3 \]
\[ = 0.1995 \]

(iv) The required probability
\[
= \frac{1}{0.1995} \left[ \frac{8!}{3! 4! 1!} (0.5)^3 (0.3)^4 (0.2)^1 + \frac{8!}{3! 3! 2!} (0.5)^3 (0.3)^3 (0.2)^2 \right]
\]
\[ \approx 0.6632 \]