2012-ASL
M & S

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG ADVANCED LEVEL EXAMINATION 2012

MATHEMATICS AND STATISTICS AS-LEVEL

8.30 am – 11.30 am (3 hours)

This paper must be answered in English

1. This paper consists of Section A and Section B.
2. Answer **ALL** questions in Section A, **using the AL(E) answer book.**
3. Answer any **FOUR** questions in Section B, **using the AL(C) answer book.**
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

Not to be taken away before the end of the examination session

Section A (40 marks)
Answer **ALL** questions in this section.
Write your answers in the AL(E) answer book.

1. (a) Factorise a^3+b^3 .

   (b) Let $|x|<\frac{1}{4}$.

   (i) Expand $(1+4x)^{\frac{-1}{2}}$ and $(1+64x^3)^{\frac{1}{2}}$ respectively in ascending powers of x as far as the term in x^3 .

   (ii) Using (a) and (b)(i), or otherwise, expand $(1-4x+16x^2)^{\frac{1}{2}}$ in ascending powers of x as far as the term in x^3 .

   (6 marks)

2. It is given that $\begin{cases} x=\ln(2t+4) \\ y=e^{t^2+4t+4} \end{cases}$, where $t>-2$.

   (a) Express $\frac{dy}{dx}$ in terms of y only.

   (b) Find the value of $\frac{d^2y}{dx^2}$ when $x=0$.

   (6 marks)

3. An advertising company starts a media advertisement to recruit new members for a club. Past experience shows that the rate of change of the number of members N (in thousand) is given by

$$\frac{dN}{dt}=\frac{0.3e^{-0.2t}}{(1+e^{-0.2t})^2} ,$$

   where t (≥ 0) is the number of weeks elapsed after the launch of the advertisement. The club has 500 members before the launch of the advertisement.

   (a) Using the substitution $u=1+e^{-0.2t}$, express N in terms of t .

   (b) Find the increase in the number of members of the club 4 weeks after the launch of the advertisement. Correct your answer to the nearest integer.

   (c) Will the number of members of the club ever reach 1300 after the launch of the advertisement? Explain your answer.

   (7 marks)

4. Soft drinks are produced in packs by a production line in a company. Assume that the number of defective packs in a day follows a Poisson distribution with mean λ. The company has decided to inspect the production line whenever 4 or more defective packs are found in a day. It is known that the probability that at least 1 defective pack found in a day is $1-e^{-2}$.

(a) Find the value of λ.

(b) Find the probability that the company will have to inspect the production line in a given day.

(c) It is given that the probability that the production line will not be inspected for n consecutive days is greater than 0.5. Find the greatest integral value of n.

(6 marks)

5. Let A and B be two events. It is given that $\mathrm{P}(A \mid B)=\frac{3}{4}$, $\mathrm{P}(B \mid A)=\frac{3}{8}$ and $\mathrm{P}(A)=a$.

(a) Find $\mathrm{P}(A \cap B)$ in terms of a.

(b) Find $\mathrm{P}(B)$ in terms of a.

(c) It is given that $\mathrm{P}(A' \cap B')=\frac{7}{16}$.

(i) Find the value of a.

(ii) Find the value of $\mathrm{P}(A \mid B')$.

(7 marks)

6. An educational psychologist adopts the Internet Addiction Test to measure the students' level of Internet addiction. The scores of a random sample of 30 students are presented in the following stem-and-leaf diagram.

Tens	Units
3	a
4	
5	2 4 6 8
6	0 1 3 5 6 7 8 8 9
7	1 2 2 4 5 5 6 8
8	0 2 3 5 8
9	0 2 b

Let σ be the standard deviation of the scores. It is known that the mean of the scores is 71 and the range of the scores is 56.

(a) Find the values of a, b and σ.

(b) The psychologist classifies those scoring between 73 and 100 as excessive Internet users. If 4 students are selected randomly from the excessive Internet users among the students, find the probability that 3 of them will have scores higher than 80.

(c) If n scores of $(71-\sigma)$ and n scores of $(71+\sigma)$ are also included in the data set, find the new standard deviation of the $(30+2n)$ scores.

(8 marks)

Section B (60 marks)
Answer any **FOUR** questions in this section. Each question carries 15 marks.
Write your answers in the AL(C) answer book.

7. Let $f(x)=\dfrac{ax+b}{c-x}$ for all $x \neq c$, where a , b and c are constants, and $g(x)=f(-x)$ for all $x \neq -c$. Let C_1 and C_2 be the curves of $y=f(x)$ and $y=g(x)$ respectively. It is given that the vertical asymptotes of C_2 is $x=-3$, the y-intercept of C_1 is $\dfrac{4}{3}$ and the x-intercept of C_2 is 2 .

(a) Find the values of a , b and c .

(3 marks)

(b) (i) Find the equations of the vertical and horizontal asymptotes of C_1 .

(ii) Sketch the graphs of C_1 and C_2 on the same diagram. Indicate the asymptotes, intercepts and their intersecting point(s).

(7 marks)

(c) If the area of the region bounded by C_1 , C_2 and the line $x=-k$ (where $0<k<3$) is $10\ln\dfrac{3}{2}$, find the value of k .

(5 marks)

8. A textile factory has bought two new dyeing machines P and Q. The two machines start to operate at the same time and will emit sewage into a lake near the factory. The manager of the factory estimates the amount of sewage emitted (in tonnes) by the two machines and finds that the rates of emission of sewage by the two machines P and Q can be respectively modelled by

$$p'(t) = 4.5 + 2t(1+6t)^{\frac{-2}{3}} \quad \text{and}$$
$$q'(t) = 3 + \ln(2t+1) \ ,$$

where $t\ (\geq 0)$ is the number of months that the machines have been in operation.

(a) By using a suitable substitution, find the total amount of sewage emitted by machine P in the first year of operation.

(4 marks)

(b) (i) By using the trapezoidal rule with 5 sub-intervals, estimate the total amount of sewage emitted by machine Q in the first year of operation.

(ii) The manager thinks that the amount of sewage emitted by machine Q will be less than that emitted by machine P in the first year of operation. Do you agree? Explain your answer.

(5 marks)

(c) The manager studies the relationship between the environmental protection tax R (in million dollars) paid by the factory and the amount of sewage x (in tonnes) emitted by the factory. He uses the following model:

$$R = 16 - ae^{-bx} \ ,$$

where a and b are constants.

(i) Express $\ln(16-R)$ as a linear function of x.

(ii) Given that the graph of the linear function in (c)(i) passes through the point $(-10, 1)$ and the x-intercept of the graph is 90, find the values of a and b.

(iii) In addition to the sewage emitted by the machines P and Q, the other operations of the factory emit 80 tonnes of sewage annually. Using the model suggested by the manager and the values of a and b found in (c)(ii), estimate the tax paid by the factory in the first year of the operation of machines P and Q.

(6 marks)

9. The current rate of selling of a certain kind of handbags is 30 thousand per day. The sales manager decides to raise the price of the handbags. After the price of the handbags has been raised for t days, the rate of selling of handbags $r(t)$ (in thousand per day) can be modelled by

$$r(t) = 20 - 40e^{-at} + be^{-2at} \qquad (t \geq 0),$$

where a and b are positive constants. From past experience, it is known that after the increase in the price of the handbags, the rate of selling of handbags will decrease for 9 days.

(a) Find the value of b.

(1 mark)

(b) Find the value of a correct to 1 decimal place.

(3 marks)

(c) The sales manager will start to advertise when the rate of change of the rate of selling of handbags reaches a maximum. Use the results obtained in (a) and (b) to find the rate of selling of handbags when the sales manager starts to advertise.

(4 marks)

(d) When the rate of selling of handbags drops below 18 thousand per day, the sales manager will give a 'sales warning' to his team. Use the results obtained in (a) and (b) to find

(i) the duration of the 'sales warning' period correct to the nearest day,

(ii) the average number of handbags sold per day during the 'sales warning' period correct to the nearest thousand.

(7 marks)

10. A manufacturer produces batteries A and B for notebook computers. After fully charged, the operation times (in minutes) of batteries A are normally distributed with mean 168 minutes and standard deviation 32 minutes, and those of batteries B are normally distributed with mean μ minutes and standard deviation σ minutes. Past data revealed that 33% of batteries B have operation times longer than 188 minutes, while 87.7% have operation times shorter than 213.2 minutes.

(a) (i) Find the probability that a randomly chosen battery A has an operation time shorter than 152 minutes or longer than 184 minutes.

(ii) If the probability that a randomly chosen battery A has an operation time longer than k minutes is 5%, find the value of k.

(iii) Find the values of μ and σ.

(iv) Find the probability of a randomly chosen battery B having an operation time shorter than 146 minutes.

(7 marks)

(b) The manufacturer produces 1500 batteries per day. One-third of them are A and the rest are B. A battery is regarded as 'faulty' when the operation time is shorter than 104 minutes. Let λ_A and λ_B be respectively the mean numbers of 'faulty' batteries of A and B produced per day. Assume that the numbers of 'faulty' batteries A and B produced per day can be approximately modelled by Poisson distributions with means λ_A and λ_B respectively.

(i) Find λ_A and λ_B correct to 1 decimal place.

(ii) Find the probability that the number of 'faulty' batteries A produced on a certain day is between 4 and 6 inclusively.

(iii) Given that the total number of 'faulty' batteries A and B produced on a certain day is 10 and the number of 'faulty' batteries A produced is between 4 and 6 inclusively, find the probability that the number of 'faulty' batteries B produced is more than 4.

(8 marks)

11. In the maternity ward of a public hospital, the number of new born babies diagnosed as having congenital diseases in 120 days is shown in the following table and is fitted by a Poisson distribution and a binomial distribution.

Number of new born babies x diagnosed as having congenital diseases	Observed number of days	Expected number of days	
		Poisson distribution $\text{Po}(\lambda)$	Binomial distribution $B(10, p)$
$0 \le x \le 6$	74	72.76	74.13
7	20	a	25.80
8	14	12.39	14.51
9	8	8.26	c
10	4	b	d
Total	120		

(a) (i) Find the values of λ and p correct to 1 decimal place.

(ii) Write down the values of a, b, c and d correct to 2 decimal places.

(iii) Suppose the absolute values of the differences between observed and expected frequencies are defined as errors. The distribution with a smaller sum of errors is regarded as fitting the data better. Which distribution fits the data better?

(10 marks)

(b) In the maternity ward, 45% of expectant mothers are local mothers and the rest are non-local. Statistics indicated that 2.5% of new born babies of local mothers have congenital diseases, and that of non-local mothers, the proportion is 1%. Assume new born babies having congenital diseases are independent.

(i) Find the probability of a new born baby having congenital diseases.

(ii) Given that a new born baby has congenital diseases, find the probability that the new born baby is born to a non-local mother.

(iii) The distribution that fits the observed data better in (a)(iii) is adopted. Given that 8 to 10 new born babies inclusively have congenital diseases, find the probability that exactly 7 of them are born to non-local mothers.

(5 marks)

12. A fitness centre has 8 certified personal trainers providing personal training programmes to its customers in evenings. A trainer can only train one customer each evening.

The customers have to book the service in advance. Assume all bookings are made independently. Past data revealed that 'no show' bookings account for one-third of the bookings and therefore the fitness centre accepts over-bookings every evening. Trainers are assigned based on a first-come-first-serve basis. If a customer has made a booking but cannot get training due to over-booking, the customer will be given a coupon for compensation.

(a) Suppose there are 12 bookings in a particular evening.

(i) Find the probability that the fitness centre needs to give out 2 or 3 coupons.

(ii) Find the probability that every customer with booking who shows up can be assigned a trainer.

(4 marks)

(b) Find the largest number of bookings the fitness centre can accept for an evening so that at least 80% of customers who have made a booking can be assigned a trainer.

(3 marks)

(c) The centre provides three kinds of personal training programmes for its customers in each evening as follows:

Personal training programmes	Fee per programme
Diamond	\$ 3800
Platinum	\$ 2800
Jade	\$ 1800

It is known that 50%, 30% and 20% of the customers select Diamond, Platinum and Jade programmes respectively. In a particular evening, all trainers are assigned customers.

(i) Find the expected income of the centre in that evening.

(ii) Find the probability that the 8th customer is the first one to select Jade programme.

(iii) Find the probability that all programmes are selected and exactly 3 are Diamond programmes.

(iv) It is given that all programmes are selected and exactly 3 are Diamond programmes. Find the probability that more than 2 customers select Platinum programmes.

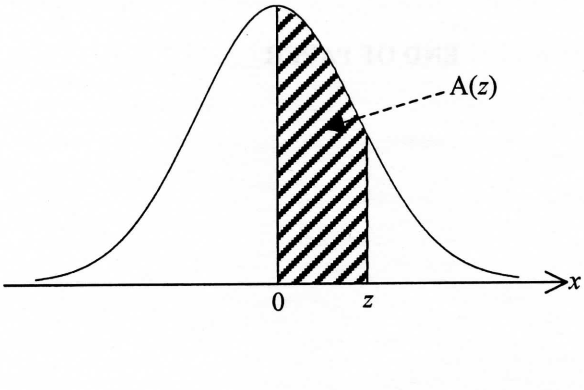
(8 marks)

END OF PAPER

Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ $(z \geq 0)$. Areas for negative values of z can be obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$