This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.
General Instructions To Markers

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept / technique had been used.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.

5. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

6. In the marking scheme, marks are classified into the following three categories:

   - 'M' marks — awarded for applying correct methods
   - 'A' marks — awarded for the accuracy of the answers
   - Marks without 'M' or 'A' — awarded for correctly completing a proof or arriving at an answer given in the question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates' work in awarding 'M' marks.) However, 'A' marks for the corresponding answers should NOT be awarded, unless otherwise specified.

7. In the marking scheme, steps which can be skipped are enclosed by dotted rectangles, whereas alternative answers are enclosed by solid rectangles.

8. Marks may be deducted for poor presentation (pp). The symbol $\square_{pp}$ should be used to denote 1 mark deducted for pp.
   (a) At most deduct 1 mark for pp in each section.
   (b) In any case, do not deduct any marks for pp in those steps where candidates could not score any marks.

9. Marks may be deducted for numerical answers with inappropriate degree of accuracy (a). The symbol $\square_{a}$ should be used to denote 1 mark deducted for a.
   (a) At most deduct 1 mark for a in each section.
   (b) In any case, do not deduct any marks for a in those steps where candidates could not score any marks.

10. Marks entered in the Page Total Box should be the NET total scored on that page.
1. (a) \[ \frac{1}{1+\sqrt{1-x}} = \frac{1}{x(1-\sqrt{1-x})} \]

(b) \[ \frac{1}{1+\sqrt{1-x}} = \frac{1}{x} \left[ 1 - \left( 1- \frac{1}{2} x + \frac{1}{2} \frac{3}{2} x^2 - \frac{1}{3} \frac{3}{2} \frac{5}{2} x^3 + \ldots \right) \right] \]

\[ = \frac{1}{2} + \frac{1}{8} x + \frac{1}{16} x^2 + \ldots \quad \text{(where } -1 < x < 1 \text{)} \]

\[ \therefore I = \int \frac{1}{4} \left( 1-\sqrt{1-x} \right) \, dx \]

\[ \approx \int \frac{1}{4} \left( \frac{1}{2} + \frac{1}{8} x + \frac{1}{16} x^2 \right) \, dx \]

\[ = \left[ \frac{x}{2} + \frac{x^2}{16} + \frac{x^3}{48} \right] \]

\[ = \frac{427}{3072} \quad \text{(where } -1 < x < 1 \text{)} \]

(c) Since the expansion of \( \sqrt{1-x} \) is only valid for \( -1 < x < 1 \), we cannot use the same method in (b) to estimate the value of \( J \).

2. (a) \[ \frac{dX}{dt} = 6 \left( \frac{t}{0.2t^3 + 1} \right)^2 \]

\[ X = 6 \int \frac{t^2}{(0.2t^3 + 1)^2} \, dt \]

Let \( u = 0.2t^3 + 1 \), and therefore \( du = 0.6t^2 \, dt \).

\[ \therefore X = 6 \int \frac{1}{0.6u^2} \, du \]

\[ = -\frac{10}{u} + C \]

\[ = -\frac{10}{0.2t^3 + 1} + C \]

When \( t = 0 \), \( X = 4 \) and hence \( C = 14 \).

i.e. \( X = \frac{-10}{0.2t^3 + 1} + 14 \)

(b) \[ 13 = \frac{-10}{0.2t^3 + 1} + 14 \]

\( t = \sqrt[3]{45} \) months

(c) \( X = 14 - \frac{10}{0.2t^3 + 1} < 14 \) for any value of \( t \).

Hence the plan can be run for a long time.
3. \( f(x) = x + \frac{3}{x} + \ln(x^2) \)

\[
f'(x) = 1 - \frac{3}{x^2} + \frac{2}{x}
\]

\[
f'(x) = 0 \text{ gives } \frac{x^2 + 2x - 3}{x^2} = 0
\]

i.e. \( x = 1 \) or \(-3\)

\( f(1) = 4 \) and \( f(-3) = -4 + \ln 9 \)

\[
f''(x) = \frac{6}{x^3} - \frac{2}{x^2}
\]

\[
f''(1) = 4 > 0 \text{ and } f''(-3) = -\frac{4}{9} < 0
\]

Hence \((1, 4)\) is a minimum point and \((-3, -4 + \ln 9)\) is a maximum point.

\[
f''(x) = 0 \text{ gives } \frac{6 - 2x}{x^3} = 0
\]

i.e. \( x = 3 \)

\( f(3) = 4 + \ln 9 \)

<table>
<thead>
<tr>
<th>( x &lt; 3 ) and ( x \neq 0 )</th>
<th>( x = 3 )</th>
<th>( x &gt; 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f''(x) )</td>
<td>+ve</td>
<td>0</td>
</tr>
</tbody>
</table>

Hence \((3, 4 + \ln 9)\) is a point of inflexion.

4. (a) \( P(\text{Susan wins}) = 0.75^2 (1 - 0.55^2) + C_1^2 0.75(1 - 0.75)(1 - 0.55)^2 \)

\[
= \frac{2997}{6400}
\]

(b) \( P(\text{Susan wins} | \text{Peter scores at least 1 point}) = 0.75^2 \cdot C_2^2 0.55 (1 - 0.55) \)

\[
= \frac{81}{232}
\]

5. (a) Total number of ways \( C_2^3 = 21 \)

(b) Total number of ways \( C_3^3 - C_2^3 \times C_1^3 = 26 \)

(c) \( P(A \text{ to } B \mid \text{traps at } T_1 \text{ & } T_2) = \frac{C_2^3 - C_1^2 \times C_1^1}{C_3^2} = \frac{9}{56} \)
### Solution

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution</th>
</tr>
</thead>
</table>
| 6. (a) | Mean = \( \frac{743 + (30 + a) + (40 + b)}{19} = 40 + b \)  
\( a = 18b - 53 \)  
Since \( a \) and \( b \) are integers where \( 0 \leq a \leq 2 \) and \( 0 \leq b \leq 3 \), \( b \) can only be 3 and hence \( a = 1 \).  
\( \sigma = 12.1915 \) minutes  
(c) Since the mean does not change, the two new data will be “equidistance” from the mean. Hence the two data should be 43, 43 for minimum \( \sigma \) and 26, 60 for maximum \( \sigma \) since the range does not change.  
Minimum \( \sigma = 11.5964 \) and maximum \( \sigma = 12.7279 \).  
i.e. \( 11.5964 \leq \sigma \leq 12.7279 \) | 1M  
1A  
1A  
1M  
1M  
1A  
(7) |
| 7. (a) | The equation of the vertical asymptote of \( C \) is \( x = \frac{-d}{c} \).
Since it is given that the vertical asymptote of \( C \) is \( x = \frac{1}{2} \), \( \frac{-d}{c} = \frac{1}{2} \).
i.e. \( d = \frac{-c}{2} \)  
(1)  
Similarly, the horizontal asymptote is \( y = \frac{a}{c} \) which is the same as \( y = -2 \).  
\( \therefore \frac{a}{c} = -2 \) which gives \( a = -2c \)  
(2)  
Since \( C \) passes through the origin, \( 0 = \frac{a(0) + b}{c(0) + d} \).  
i.e. \( b = 0 \)  
(3)  
Substituting (1), (2) and (3) into \( y = \frac{ax + b}{cx + d} \), we get the equation of \( C \) as \( y = \frac{-2cx + 0}{cx - \frac{c}{2}} \) where \( c \neq 0 \)  
i.e. \( y = \frac{-4x}{2x - 1} \)  
(4)  
(b) The equation of curve \( D \) is \( y = \frac{2x - 1}{-4x} \).  
Solving \( C \) and \( D \), we get \( \frac{-4x}{2x - 1} = \frac{2x - 1}{-4x} \).  
\( 16x^2 = 4x^2 - 4x + 1 \)  
\( 12x^2 + 4x - 1 = 0 \)  
\( x = \frac{1}{6} \) or \( \frac{-1}{2} \).  
When \( x = \frac{1}{6} \), \( y = 1 \); when \( x = \frac{-1}{2} \), \( y = -1 \).  
Hence the coordinates of all the intersecting points are \( \left( \frac{1}{6}, 1 \right) \) and \( \left( \frac{-1}{2}, -1 \right) \). | 1A  
1M  
1A  
For both  
1A  
(3)  
(4) |
(d) The area bounded by $C$, $D$ and the positive $x$-axis

\[
\int_{-2}^{\frac{1}{2}} -4x \ dx + \int_{\frac{1}{2}}^{\frac{1}{4}} \frac{2x - 1}{6 - 4x} \ dx \\
\int_{0}^{\frac{1}{6}} \left( -2 - \frac{2}{2x - 1} \right) \ dx + \int_{\frac{1}{6}}^{\frac{1}{2}} \left( \frac{1}{2} + \frac{1}{4x} \right) \ dx \\
= \left[ -2x - \ln|2x| - 1 \right]\bigg|_{0}^{\frac{1}{6}} + \left[ -\frac{x}{2} + \frac{1}{4} \ln|x| \right]\bigg|_{\frac{1}{6}}^{\frac{1}{2}} \\
= \left( -\frac{1}{3} - \frac{1}{3} + \frac{1}{4} \right) + \left( -\frac{1}{4} + \frac{1}{2} + \frac{1}{12} - \frac{1}{4} \right) \\
= -\frac{1}{2} - \ln 2 + \ln 3 - \frac{1}{4} \ln 2 + \frac{1}{4} (\ln 2 + \ln 3) \\
= \frac{5}{4} \ln 3 - \ln 2 - \frac{1}{2}
\]

For the intercepts of $C$ & $D$
8. (a) \( e^{t^2+t} = 1 + (t^2 + t) + \frac{(t^2 + t)^2}{2} + \frac{(t^2 + t)^3}{3!} + \ldots \)
\[ = 1 + t^2 + t + \frac{2t^3 + t^2}{2} + \frac{t^3 + \ldots}{6} + \ldots \]
\[ = 1 + t + \frac{3t^2}{2} + \frac{7t^3}{6} + \ldots \]
\[
V = \int_0^1 \frac{1}{25} e^{t^2+t+2} \, dt
\]
\[ \approx \frac{e^2}{25} \left[ 1 + t + \frac{3t^2}{2} + \frac{7t^3}{6} \right]_0^1 \]
\[ = \frac{271}{9600} e^2 \text{ hundred thousand m}^3 \]
Since for \( t > 0 \), \( e^{t^2+t} = 1 + t + \frac{3t^2}{2} + \frac{7t^3}{6} \) + positive terms,
\[ e^{t^2+t} > 1 + t + \frac{3t^2}{2} + \frac{7t^3}{6} \]
Hence the estimation is an under-estimate.

(b) (i) Area \( \approx \frac{0.2}{2} \left[ 0 + 2(3.8 + 4.2 + 4.3 + 4.1 + 3.4) + 0 \right] \]
\[ = 3.96 \text{ km}^2 \]
Since the upper half of the curve is concave downwards and the lower half is concave upwards, the estimation is an under-estimate.

(ii) Thickness \( \approx \frac{20858.6896}{3.96 \times 1000^2} \text{ m} \)
\[ = 0.0053 \text{ m} \]
Since both the numerator and denominator are under-estimates, we cannot determine whether the thickness is an over- or under-estimate.
\[
\frac{dW}{dt} = \frac{-W + 1}{40}^{3/2}
\]
\[
\frac{dt}{dW} = -\frac{1}{40(W + 1)^{3/2}}
\]
\[
t = -40 \int (W + 1)^{3/2} dW
\]
\[
= -60(W + 1)^{5/2} + C
\]

When \( t = 0, W = \frac{271}{9600} e^2 \).

\[
0 = -60 \left( \frac{271}{9600} e^2 + 1 \right)^{5/2} + C
\]
\[
C = 68.07743296
\]

Hence \( t = -60(W + 1)^{5/2} + 68.07743296 \)

When \( W = 0, t \approx 8.0774 \).

Thus all the oil spread will be cleaned up after 8.0774 days.

9. (a) (i) \[
P = \frac{ke^{-\lambda t}}{t^2}
\]
\[
\ln P + 2 \ln t = \ln k - \lambda t
\]

(ii) Intercept of vertical axis = 2.3 = \( \ln k \)

\[
\therefore k \approx 10 \text{ correct to the nearest integer}
\]

slope = \[
\frac{2.3 - 0}{0 - (-1.15)} = -\lambda
\]

\[
\therefore \lambda = -2
\]

\[
P = \frac{10e^{-2t}}{t^2}
\]

\[
\frac{dP}{dt} = \frac{10e^{-2t}2e^{2t} - e^{2t}2t}{t^4}
\]
\[
= \frac{20e^{2t}(t - 1)}{t^3}
\]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{t} & 0 < t < 1 & t = 1 & t > 1 \\
\hline
\frac{dP}{dt} & -ve & 0 & +ve \\
\hline
\end{array}
\]

Hence the minimum population size is attained when \( t = 1 \).

\[
P = \frac{10e^{2(1)}}{(1)^2}
\]
\[
= 74
\]

Hence the minimum population size is 74 hundred thousands.
(b) (i) \[
\frac{dE}{dt} = ate^{ht} - 1.2e^{ht} + 4.214
\]
\[
\frac{d^2E}{dt^2} = he^{ht} + h^2te^{ht} - 1.2he^{ht}
\]
\[
= he^{ht}(ht - 0.2)
\]
When \( t = 1 \), \( \frac{dE}{dt} \) is minimum and hence \( \frac{d^2E}{dt^2} = 0 \).

Thus, \( h = 0.2 \).

(ii) \[
\frac{d}{dt}(te^{0.2t}) = 0.2te^{0.2t} + e^{0.2t}
\]
\[
\therefore \; te^{0.2t} = 5 \frac{d}{dt}(te^{0.2t}) - 5e^{0.2t}
\]
\[
\int t e^{0.2t} dt = 5te^{0.2t} - 5 \int e^{0.2t} dt
\]
\[
= 5te^{0.2t} - 25e^{0.2t} + C_1
\]
\[
\frac{dE}{dt} = 0.2te^{0.2t} - 1.2e^{0.2t} + 4.214
\]
\[
E = 0.2 \int te^{0.2t} dt - 1.2 \int e^{0.2t} dt + \int 4.214 dt
\]
\[
= te^{0.2t} - 5e^{0.2t} - 6e^{0.2t} + 4.214t + C
\]
\[
= te^{0.2t} - 11e^{0.2t} + 4.214t + C
\]
When \( t = 0 \), \( E = 1 \).
Hence \( 1 = 0 - 11 + 0 + C \) which gives \( C = 12 \).
i.e. \( E = te^{0.2t} - 11e^{0.2t} + 4.214t + 12 \)
When \( t = 1 \), \( E = e^{0.2} - 11e^{0.2} + 4.214 + 12 \)
\approx 4
Thus the annual electricity consumption is 4 thousand terajoules per year.

(iii) \[
F = \frac{6}{1 - 5e^r + 3e^{2r}} + 2
\]
\[
\frac{6}{1 - 5e^r + 3e^{2r}} + 2 \approx 4
\]
\[
3e^{2r} - 5e^r - 2 \approx 0
\]
\[
e^r \approx 2 \quad \text{or} \quad -\frac{1}{3} \quad \text{(rejected)}
\]
\[
r = \ln 2
\]
\[
1A \quad \text{OR} \quad 0.6931
\]

(9)
10. (a) \[ P(160 \leq Y < K) = 78.88\% \]
\[
P\left(\frac{160 - 165}{4} \leq Z < \frac{K - 165}{4}\right) = 0.7888
\]
\[
0.3944 + P\left(0 \leq Z < \frac{K - 165}{4}\right) = 0.7888
\]
\[\frac{K - 165}{4} = 1.25\]
\[K = 170\]

(b) \[ P(\text{score } 30) = P(170 \leq Y < 174) \]
\[
= P(1.25 \leq Z < 2.25)
\]
\[
= 0.4878 - 0.3944
\]
\[= 0.0934\]

(c) \[ P(\text{6th game is the 3rd Bingo}) = C_3^2(0.2112)^3(0.7888)^3 \]
\[= 0.0462\]

(d) The number of "Bingo" in \( n \) games \( \sim B(n, 0.7888) \).
\[
\therefore n(0.7888)(0.2112) \leq 2.3
\]
\[n \leq 13.80597302\]
Thus the largest value of \( n \) is 13.

(e) (i) \[ P(\text{score } 20) = P(-2.75 \leq Z < -1.25) = 0.1026\]
\[
\therefore P(\text{win a prize})
\]
\[
= P(\text{total score in 4 games } \geq 160)
\]
\[
= (0.7888)^4 + \binom{4}{1}(0.7888)^3(0.0934 + 0.1026) + \binom{4}{2}(0.7888)^2(0.0934)^2
\]
\[= 0.804490478
\]
\[= 0.8045\]

(ii) \[ P(\text{win a prize and average score in the first 2 games } \geq 40) \]
\[
= P(\text{total score in 4 games } \geq 160 \text{ and total score in first 2 games } \geq 80)
\]
\[
= (0.7888)^4 + \binom{4}{1}(0.7888)^3(0.0934) + \binom{4}{2}(0.7888)^2(0.0934)^2 + \binom{4}{3}(0.7888)(0.0934)^3 + \binom{4}{4}(0.0934)^4
\]
\[= 0.804490478 - 0.698351364 - 0.6984
\]
\[= 0.1319\]

Alternative Solution
\[
P(\text{total score in 4 games } \geq 160) - P(\text{total score in 4 games } \geq 160 \text{ and total score in first 2 games } < 80)
\]
\[= 0.804490478 - (0.7888)^2(0.0934)^2 - \binom{4}{2}(0.7888)^2(0.0934)^2(0.1026)
\]
\[= 0.698351364
\]
\[= 0.6984\]

(iii) \[ P(\text{average score in the first 2 games } < 40 \mid \text{ win a prize}) \]
\[
= \frac{0.804490478 - 0.698351364}{0.804490478 - 0.6984}
\]
\[= 0.1319\]

(7)
11. (a) (i) \( P(\text{lift is full at G/F}) \)
\[
= 1 - e^{-4} \left( 1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!} + \frac{4^5}{5!} \right)
\]
\[
\approx 0.214869613
\]
\[
\approx 0.2149
\]
1M

(ii) \( P(4 \text{ persons gets into the lift and it stops at each floor}) \)
\[
= e^{-4} \cdot 4! \cdot \frac{1}{4!}
\]
\[
\approx 0.0183
\]
1M

(iii) \( P(\text{lift stops at each floor}) \)
\[
= \frac{e^{-4} \cdot 4^4}{4!} + \frac{e^{-4} \cdot 4^5}{5!} + \frac{e^{-4} \cdot 4^6}{6!} + 0.214869613 \cdot \frac{C_3^6 \cdot 4! + C_2^6 \cdot 4! \cdot \frac{4^2}{2!}}{4^6}
\]
\[
\approx 0.1368
\]
1M+1M

(7)

(b) (i) \( P(3 \text{ persons from different floor waits for the lift}) \)
\[
= C_3^4 \left( e^{-3} \right)^3 \left( e^{-3} \right)
\]
\[
\approx 0.0007
\]
1M

(ii) \( P(2 \text{ persons waits for the lift}) \)
\[
= C_1^4 \left( e^{-3} \right)^3 \left( e^{-3} \right) + C_2^4 \left( e^{-3} \right)^2 \left( e^{-3} \right)^2
\]
\[
\approx 0.0004
\]
1M

(iii) Let the number of persons waiting above 62/F be \( X \).
\( P(3 \text{ persons get into the lift at the 62/F | 3 persons wait at the 62/F}) \)
\[
P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)
\]
\[
= (e^{-3})^3 + C_1^3 (e^{-3})^2 + C_2^3 (e^{-3})^2 + \left[ C_1^3 \frac{e^{-3}^2}{2!} \right] (e^{-3})^2 + \left[ C_2^3 \frac{e^{-3}^2}{2!} \right] (e^{-3})^3
\]
\[
\approx 0.1512
\]
1M+1M+1M

(8)

12. (a) \( P(Y < 100) = 0.121 \)
\[
P\left( \frac{100 - \mu}{\sigma} \leq Z < 0 \right) = 0.379
\]
\[
\frac{100 - \mu}{\sigma} = -1.17 \quad \text{---------------------------------------- (1)}
\]
1A

\( P(Y \geq 200) = 0.0918 \)
\[
P\left( 0 < Z < \frac{200 - \mu}{\sigma} \right) = 0.4082
\]
\[
\frac{200 - \mu}{\sigma} = 1.33 \quad \text{---------------------------------------- (2)}
\]
1A

Solving (1) and (2), we get \( \mu = 146.8 \) and \( \sigma = 40 \)
1A
(b) \( P(\text{High level}) \)
\[= P(150 \leq Y < 200) \]
\[= P(0.08 \leq Z < 1.33) \]
\[\approx 0.4082 - 0.0319 \]
\[= 0.3763 \]

(c) \( P(\text{High} \mid \text{rainfall exceeds 100 mm}) \)
\[\approx \frac{0.3763}{1 - 0.121} \]
\[\approx 0.4281 \]

(d) (i) \( P(\text{Severe} \mid \text{rainfall exceeds 100 mm}) \)
\[\approx \frac{0.0918}{1 - 0.121} \]
\[\approx 0.1044 \]

\( P(\text{Medium} \mid \text{rainfall exceeds 100 mm}) \)
\[\approx \frac{1 - 0.121 - 0.0918 - 0.3763}{1 - 0.121} \]
\[\approx 0.4674 \]

\( P(\text{job will NOT be postponed} \mid \text{rainfall exceeds 100 mm}) \)
\[\approx (0.4674\cdot e^{-1} + (0.4281\cdot e^{-3}) + (0.1044\cdot e^{-6}) \cdot e^{-0}) \]
\[\approx 0.1935 \]

(ii) \( P(\text{job will be postponed for 1 day} \mid \text{rainfall exceeds 100 mm}) \)
\[\approx (0.4674\cdot e^{-1} + (0.4281\cdot e^{-3}) + (0.1044\cdot e^{-6}) \cdot e^{-1}) \]
\[\approx 0.2375 \]

(iii) \( P(\text{job will be postponed for 2 days} \mid \text{rainfall exceeds 100 mm}) \)
\[\approx (0.4674\cdot e^{-1}2 + (0.4281\cdot e^{-3}3) + (0.1044\cdot e^{-6}2) \cdot e^{-2}) \]
\[\approx 0.1865 \]

\( P(\text{High level} \mid \text{job will be postponed for at least 3 days}) \)
\[\approx \frac{0.4281\cdot e^{-3} - e^{-3}3 - e^{-3}32}{1 - 0.1935\cdot e^{-1} - 0.2375\cdot e^{-3} - 0.1865\cdot e^{-6}} \]
\[\approx 0.6457 \]