

香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2 0 1 1 年 香 港 高 級 程 度 會 考
HONG KONG ADVANCED LEVEL EXAMINATION 2011

數學及統計學 高級補充程度

MATHEMATICS AND STATISTICS AS-LEVEL

評 卷 參 考

MARKING SCHEME

本評卷參考乃香港考試及評核局專為今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視為標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

General Instructions To Markers

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.
2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept / technique had been used.
3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
4. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.
5. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.
6. In the marking scheme, marks are classified into the following three categories:

'M' marks	–	awarded for applying correct methods
'A' marks	–	awarded for the accuracy of the answers
Marks without 'M' or 'A'	–	awarded for correctly completing a proof or arriving at an answer given in the question.

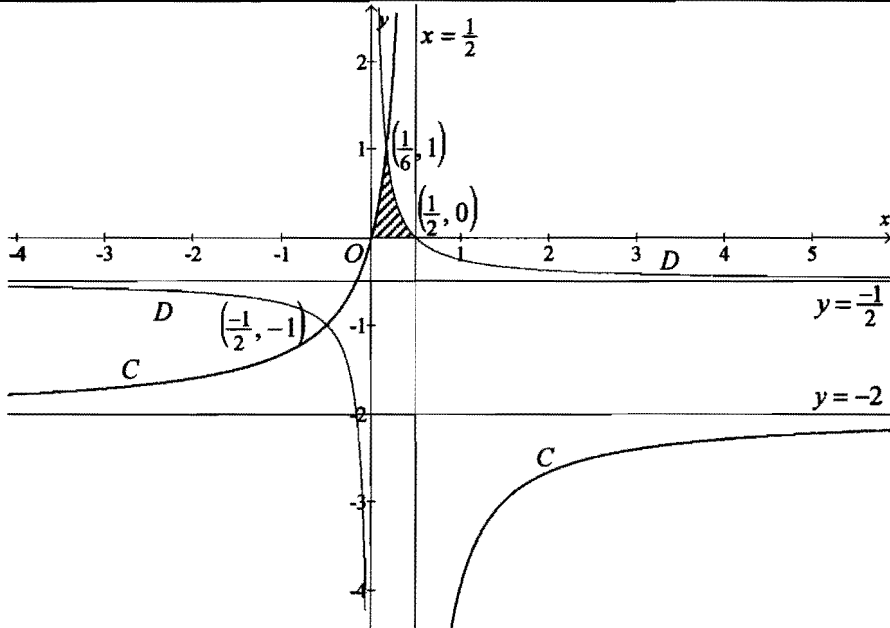
In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates' work in awarding 'M' marks.) However, 'A' marks for the corresponding answers should NOT be awarded, unless otherwise specified.

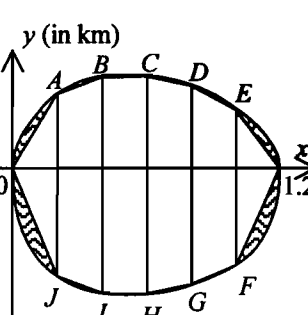
7. In the marking scheme, steps which can be skipped are enclosed by dotted rectangles, whereas alternative answers are enclosed by solid rectangles.
8. Marks may be deducted for poor presentation (*pp*). The symbol $\textcircled{pp-1}$ should be used to denote 1 mark deducted for *pp*.
 - (a) At most deduct 1 mark for *pp* in each section.
 - (b) In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.
9. Marks may be deducted for numerical answers with inappropriate degree of accuracy (*a*). The symbol $\textcircled{a-1}$ should be used to denote 1 mark deducted for *a*.
 - (a) At most deduct 1 mark for *a* in each section.
 - (b) In any case, do not deduct any marks for *a* in those steps where candidates could not score any marks.
10. Marks entered in the Page Total Box should be the NET total scored on that page.

Solution	Marks	Remarks
1. (a) $\frac{1}{1+\sqrt{1-x}} = \frac{1}{1+\sqrt{1-x}} \cdot \frac{1-\sqrt{1-x}}{1-\sqrt{1-x}}$ $= \frac{1}{x}(1-\sqrt{1-x})$	1M 1	
(b) $\therefore \frac{1}{1+\sqrt{1-x}} = \frac{1}{x} \left[1 - \left(1 - \frac{1}{2}x + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}x^2 - \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}x^3 + \dots \right) \right]$ $= \frac{1}{2} + \frac{1}{8}x + \frac{1}{16}x^2 + \dots \quad (\text{where } -1 < x < 1)$	1M 1A	
$\therefore I = \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{x}(1-\sqrt{1-x}) dx$ $\approx \int_{\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{2} + \frac{1}{8}x + \frac{1}{16}x^2 \right) dx$ $= \left[\frac{x}{2} + \frac{x^2}{16} + \frac{x^3}{48} \right]_{\frac{1}{4}}^{\frac{1}{2}}$ $= \frac{427}{3072}$	1M 1A	OR 0.1390
(c) Since the expansion of $\sqrt{1-x}$ is only valid for $-1 < x < 1$, we cannot use the same method in (b) to estimate the value of J .	1A	
(7)		
2. (a) $\frac{dX}{dt} = 6 \left(\frac{t}{0.2t^3 + 1} \right)^2$ $X = 6 \int \frac{t^2}{(0.2t^3 + 1)^2} dt$ Let $u = 0.2t^3 + 1$, and therefore $du = 0.6t^2 dt$. $\therefore X = 6 \int \frac{1}{0.6u^2} du$ $= \frac{-10}{u} + C$ $= \frac{-10}{0.2t^3 + 1} + C$ When $t = 0$, $X = 4$ and hence $C = 14$. i.e. $X = \frac{-10}{0.2t^3 + 1} + 14$	1M 1M 1A 1A	For $u = 0.2t^3 + 1$ or $u = (0.2t^3 + 1)^2$
(b) $13 = \frac{-10}{0.2t^3 + 1} + 14$ $t = \sqrt[3]{45}$ months	1A	OR 3.5569 months
(c) $X = 14 - \frac{10}{0.2t^3 + 1} < 14$ for any value of t . Hence the plan can be run for a long time.	1M 1A	
(7)		

Solution	Marks	Remarks															
<p>3. $f(x) = x + \frac{3}{x} + \ln(x^2)$</p> <p>$f'(x) = 1 - \frac{3}{x^2} + \frac{2}{x}$</p> <p>$f'(x) = 0$ gives $\frac{x^2 + 2x - 3}{x^2} = 0$</p> <p>i.e. $x = 1$ or -3</p> <p>$f(1) = 4$ and $f(-3) = -4 + \ln 9$</p> <p>$f''(x) = \frac{6}{x^3} - \frac{2}{x^2}$</p> <p>$f''(1) = 4 > 0$ and $f''(-3) = \frac{-4}{9} < 0$</p> <p>Hence $(1, 4)$ is a minimum point and $(-3, -4 + \ln 9)$ is a maximum point.</p> <p>$f''(x) = 0$ gives $\frac{6-2x}{x^3} = 0$</p> <p>i.e. $x = 3$</p> <p>$f(3) = 4 + \ln 9$</p> <table border="1" data-bbox="144 884 632 963"> <tr> <td>x</td> <td>$x < 3$ and $x \neq 0$</td> <td>$x = 3$</td> <td>$x > 3$</td> </tr> <tr> <td>$f''(x)$</td> <td>+ve</td> <td>0</td> <td>-ve</td> </tr> </table> <p>Hence $(3, 4 + \ln 9)$ is a point of inflexion.</p>	x	$x < 3$ and $x \neq 0$	$x = 3$	$x > 3$	$f''(x)$	+ve	0	-ve	<p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>(7)</p>	<p>For $f'(x) = 0$</p> <p>For both</p> <p>OR using sign test</p> <p>For both max pt also: $(-3, -1.8028)$</p> <p>OR $(3, 6.1972)$</p>							
x	$x < 3$ and $x \neq 0$	$x = 3$	$x > 3$														
$f''(x)$	+ve	0	-ve														
<p>4. (a) $P(\text{Susan wins})$ $= 0.75^2(1 - 0.55^2) + C_1^2 0.75(1 - 0.75)(1 - 0.55)^2$ $= \frac{2997}{6400}$</p> <p>(b) $P(\text{Susan wins} \mid \text{Peter scores at least 1 point})$ $= \frac{0.75^2 \cdot C_1^2 \cdot 0.55(1 - 0.55)}{1 - (1 - 0.55)^2}$ $= \frac{81}{232}$</p>	<p>1M+1M</p> <p>1A</p> <p>1M+1M</p> <p>1A</p> <p>(6)</p>	<p>1M for the (2,0) and (2,1) cases 1M for the (1,0) case</p> <p>OR 0.4683</p> <p>1M for conditional prob 1M for numerator</p> <p>OR 0.3491</p>															
<p>5. (a) Total number of ways $= C_2^7 = 21$</p> <p>(b) Total number of ways $= C_3^8 - C_2^5 \times C_1^3 = 26$</p> <p>(c) $P(A \text{ to } B \mid \text{traps at } T_1 \text{ \& } T_2) = \frac{C_2^7 - C_1^4 \times C_1^3}{C_3^8} = \frac{9}{56}$</p>	<p>1M+1A</p> <p>1M+1A</p> <p>1M+1A</p> <p>(6)</p>	<p>OR $= C_3^8 - C_3^7 = 21$</p> <table border="1" data-bbox="1198 1713 1470 1881"> <tr> <td></td> <td></td> <td></td> <td></td> <td>B</td> </tr> <tr> <td></td> <td></td> <td>T_2</td> <td></td> <td></td> </tr> <tr> <td>A</td> <td>T_1</td> <td></td> <td></td> <td></td> </tr> </table> <p>OR 0.1607</p>					B			T_2			A	T_1			
				B													
		T_2															
A	T_1																

Solution	Marks	Remarks
<p>6. (a) Mean = $\frac{743+(30+a)+(40+b)}{19} = 40+b$ $a = 18b - 53$ Since a and b are integers where $0 \leq a \leq 2$ and $0 \leq b \leq 3$, b can only be 3 and hence $a = 1$.</p> <p>(b) $\sigma = 12.1915$ minutes</p> <p>(c) Since the mean does not change, the two new data will be "equidistance" from the mean. Hence the two data should be 43, 43 for minimum σ and 26, 60 for maximum σ since the range does not change. Minimum $\sigma = 11.5964$ and maximum $\sigma = 12.7279$. i.e. $11.5964 \leq \sigma \leq 12.7279$</p>	<p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>(7)</p>	<p>For both</p>
<p>7. (a) The equation of the vertical asymptote of C is $x = \frac{-d}{c}$.</p> <p>Since it is given that the vertical asymptote of C is $x = \frac{1}{2}$, $\frac{-d}{c} = \frac{1}{2}$.</p> <p>i.e. $d = \frac{-c}{2}$ ----- (1)</p> <p>Similarly, the horizontal asymptote is $y = \frac{a}{c}$ which is the same as $y = -2$.</p> <p>$\therefore \frac{a}{c} = -2$ which gives $a = -2c$ ----- (2)</p> <p>Since C passes through the origin, $0 = \frac{a(0)+b}{c(0)+d}$.</p> <p>i.e. $b = 0$ ----- (3)</p> <p>Substituting (1), (2) and (3) into $y = \frac{ax+b}{cx+d}$, we get the equation of C as</p> <p>$y = \frac{-2cx+0}{cx-\frac{c}{2}}$ where $c \neq 0$</p> <p>i.e. $y = \frac{-4x}{2x-1}$</p>	<p>1</p> <p>1M</p> <p>1M</p> <p>1A</p> <p>(4)</p>	
<p>(b) The equation of curve D is $y = \frac{2x-1}{-4x}$.</p> <p>Solving C and D, we get $\frac{-4x}{2x-1} = \frac{2x-1}{-4x}$.</p> <p>$16x^2 = 4x^2 - 4x + 1$</p> <p>$12x^2 + 4x - 1 = 0$</p> <p>$x = \frac{1}{6}$ or $\frac{-1}{2}$</p> <p>When $x = \frac{1}{6}$, $y = 1$; when $x = \frac{-1}{2}$, $y = -1$.</p> <p>Hence the coordinates of all the intersecting points are $\left(\frac{1}{6}, 1\right)$ and $\left(\frac{-1}{2}, -1\right)$.</p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>(3)</p>	<p>For both</p>

Solution	Marks	Remarks
<p>(c)</p> 	<p>1A 1A 1A 1A</p>	<p>For the shape of C For the shape of D For the asymptotes of D For the intercepts of C & D</p>
	(4)	
<p>(d) The area bounded by C, D and the positive x-axis</p> $= \int_0^{\frac{1}{6}} \frac{-4x}{2x-1} dx + \int_{\frac{1}{6}}^{\frac{1}{2}} \frac{2x-1}{-4x} dx$ $= \int_0^{\frac{1}{6}} \left(-2 - \frac{2}{2x-1} \right) dx + \int_{\frac{1}{6}}^{\frac{1}{2}} \left(\frac{-1}{2} + \frac{1}{4x} \right) dx$ $= \left[-2x - \ln 2x-1 \right]_0^{\frac{1}{6}} + \left[\frac{-x}{2} + \frac{1}{4} \ln x \right]_{\frac{1}{6}}^{\frac{1}{2}}$ $= \frac{-1}{3} - \ln \frac{2}{3} - \frac{1}{4} + \frac{1}{4} \ln \frac{1}{2} + \frac{1}{12} - \frac{1}{4} \ln \frac{1}{6}$ $= \frac{-1}{2} - \ln 2 + \ln 3 - \frac{1}{4} \ln 2 + \frac{1}{4} (\ln 2 + \ln 3)$ $= \frac{5}{4} \ln 3 - \ln 2 - \frac{1}{2}$	<p>1M 1M 1A 1A</p>	<p>For primitive functions</p> <p>OR $\frac{-1}{2} - \ln \frac{2}{3} + \frac{1}{4} \ln \frac{1}{2} - \frac{1}{4} \ln \frac{1}{6}$</p>
	(4)	

Solution	Marks	Remarks
<p>8. (a) $e^{t^2+t} = 1 + (t^2+t) + \frac{(t^2+t)^2}{2} + \frac{(t^2+t)^3}{3!} + \dots$</p> $= 1 + t^2 + t + \frac{2t^3 + t^2 + \dots}{2} + \frac{t^3 + \dots}{6} + \dots$ $= 1 + t + \frac{3t^2}{2} + \frac{7t^3}{6} + \dots$ <p>$V = \int_0^{\frac{1}{2}} \frac{1}{25} e^{t^2+t+2} dt$</p> $\approx \frac{e^2}{25} \int_0^{\frac{1}{2}} \left(1 + t + \frac{3t^2}{2} + \frac{7t^3}{6} \right) dt$ $= \frac{e^2}{25} \left[t + \frac{t^2}{2} + \frac{t^3}{2} + \frac{7t^4}{24} \right]_0^{\frac{1}{2}}$ $= \frac{271}{9600} e^2 \text{ hundred thousand m}^3$ <p>Since for $t > 0$, $e^{t^2+t} = 1 + t + \frac{3t^2}{2} + \frac{7t^3}{6} + \text{positive terms}$,</p> $e^{t^2+t} > 1 + t + \frac{3t^2}{2} + \frac{7t^3}{6}$ <p>Hence the estimation is an under-estimate.</p>	<p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>(6)</p>	<p>OR 0.2086 hund. th. m³</p> <p>OR 20858.6896 m³</p>
<p>(b) (i) Area $\approx \frac{0.2}{2} [0 + 2(3.8 + 4.2 + 4.3 + 4.1 + 3.4) + 0]$</p> $= 3.96 \text{ km}^2$ <p>Since the upper half of the curve is concave downwards and the lower half is concave upwards, the estimation is an under-estimate.</p> <p>(ii) Thickness $\approx \frac{20858.6896}{3.96 \times 1000^2} \text{ m}$</p> $\approx 0.0053 \text{ m}$ <p>Since both the numerator and denominator are under-estimates, we cannot determine whether the thickness is an over- or under-estimate.</p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>(5)</p>	 <p>OR $5.2673 \times 10^{-3} \text{ m}$</p> <p>OR 5.2673 mm</p>

Solution	Marks	Remarks								
<p>(c) $\frac{dW}{dt} = \frac{-(W+1)^{\frac{1}{3}}}{40}$</p> $\frac{dt}{dW} = -40(W+1)^{-\frac{1}{3}}$ $t = -40 \int (W+1)^{-\frac{1}{3}} dW$ $= -60(W+1)^{\frac{2}{3}} + C$ <p>When $t = 0$, $W = \frac{271}{9600}e^2$.</p> $\therefore 0 = -60\left(\frac{271}{9600}e^2 + 1\right)^{\frac{2}{3}} + C$ $C = 68.07743296$ <p>Hence $t = -60(W+1)^{\frac{2}{3}} + 68.07743296$</p> <p>When $W = 0$, $t \approx 8.0774$.</p> <p>Thus all the oil spread will be cleaned up after 8.0774 days.</p>	<p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>(4)</p>									
<p>9. (a) (i) $P = \frac{ke^{-\lambda t}}{t^2}$</p> $\ln P + 2 \ln t = \ln k - \lambda t$ <p>(ii) Intercept of vertical axis = 2.3 = $\ln k$</p> $\therefore k \approx 10 \text{ correct to the nearest integer}$ $\text{slope} = \frac{2.3 - 0}{0 - (-1.15)} = -\lambda$ $\therefore \lambda = -2$ $P = \frac{10e^{2t}}{t^2}$ $\frac{dP}{dt} = 10 \frac{t^2 2e^{2t} - e^{2t} 2t}{t^4}$ $= \frac{20e^{2t}(t-1)}{t^3}$ <table border="1" data-bbox="274 1657 880 1758"> <thead> <tr> <th>t</th> <th>$0 < t < 1$</th> <th>$t = 1$</th> <th>$t > 1$</th> </tr> </thead> <tbody> <tr> <td>$\frac{dP}{dt}$</td> <td>-ve</td> <td>0</td> <td>+ve</td> </tr> </tbody> </table> <p>Hence the minimum population size is attained when $t = 1$.</p> $P = \frac{10e^{2(1)}}{(1)^2}$ $= 74$ <p>Hence the minimum population size is 74 hundred thousands.</p>	t	$0 < t < 1$	$t = 1$	$t > 1$	$\frac{dP}{dt}$	-ve	0	+ve	<p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>(6)</p>	
t	$0 < t < 1$	$t = 1$	$t > 1$							
$\frac{dP}{dt}$	-ve	0	+ve							

Solution	Marks	Remarks
<p>(b) (i) $\frac{dE}{dt} = hte^{ht} - 1.2e^{ht} + 4.214$</p> $\frac{d^2E}{dt^2} = he^{ht} + h^2te^{ht} - 1.2he^{ht}$ $= he^{ht}(ht - 0.2)$ <p>When $t = 1$, $\frac{dE}{dt}$ is minimum and hence $\frac{d^2E}{dt^2} = 0$.</p> <p>Thus, $h = 0.2$.</p>	1A	
<p>(ii) $\frac{d}{dt}(te^{0.2t}) = 0.2te^{0.2t} + e^{0.2t}$</p> $\therefore te^{0.2t} = 5 \frac{d}{dt}(te^{0.2t}) - 5e^{0.2t}$ $\int te^{0.2t} dt = 5te^{0.2t} - 5 \int e^{0.2t} dt$ $= 5te^{0.2t} - 25e^{0.2t} + C_1$ $\frac{dE}{dt} = 0.2te^{0.2t} - 1.2e^{0.2t} + 4.214$ $E = 0.2 \int te^{0.2t} dt - 1.2 \int e^{0.2t} dt + \int 4.214 dt$ $= te^{0.2t} - 5e^{0.2t} - 6e^{0.2t} + 4.214t + C$ $= te^{0.2t} - 11e^{0.2t} + 4.214t + C$ <p>When $t = 0$, $E = 1$.</p> <p>Hence $1 = 0 - 11 + 0 + C$ which gives $C = 12$.</p> <p>i.e. $E = te^{0.2t} - 11e^{0.2t} + 4.214t + 12$</p> <p>When $t = 1$, $E = e^{0.2} - 11e^{0.2} + 4.214 + 12$</p> ≈ 4 <p>Thus the annual electricity consumption is 4 thousand terajoules per year.</p>	1A 1A 1M 1A	
<p>(iii) $F = \frac{6}{1 - 5e^r + 3e^{2r}} + 2$</p> $\frac{6}{1 - 5e^r + 3e^{2r}} + 2 \approx 4$ $3e^{2r} - 5e^r - 2 \approx 0$ $e^r \approx 2 \text{ or } \frac{-1}{3} \text{ (rejected)}$ $r \approx \ln 2$	1M 1A	OR 0.6931
(9)		

Solution	Marks	Remarks
<p>10. (a) $P(160 \leq Y < K) = 78.88\%$</p> $P\left(\frac{160-165}{4} \leq Z < \frac{K-165}{4}\right) = 0.7888$ $0.3944 + P\left(0 \leq Z < \frac{K-165}{4}\right) = 0.7888$ $\frac{K-165}{4} = 1.25$ $K = 170$	1M	
	1A	
	(2)	
<p>(b) $P(\text{score } 30) = P(170 \leq Y < 174)$</p> $= P(1.25 \leq Z < 2.25)$ $= 0.4878 - 0.3944$ $= 0.0934$	1M	
	1A	
	(2)	
<p>(c) $P(\text{6th game is the 3rd Bingo}) = C_2^5 (0.2112)^3 (0.7888)^3$</p> ≈ 0.0462	1M	
	1A	
	(2)	
<p>(d) The number of "Bingo" in n games $\sim B(n, 0.7888)$.</p> $\therefore n(0.7888)(0.2112) \leq 2.3$ $n \leq 13.80597302$ <p>Thus the largest value of n is 13.</p>	1M	
	1A	
	(2)	
<p>(e) (i) $P(\text{score } 20) = P(-2.75 \leq Z < -1.25) = 0.1026$</p> $\therefore P(\text{win a prize})$ $= P(\text{total score in 4 games} \geq 160)$ $= (0.7888)^4 + C_1^4 (0.7888)^3 (0.0934 + 0.1026) + C_2^4 (0.7888)^2 (0.0934)^2$ ≈ 0.804490478 ≈ 0.8045	1A	
	1M	
	1A	
<p>(ii) $P(\text{win a prize and average score in the first 2 games} \geq 40)$</p> $= P(\text{total score in 4 games} \geq 160 \text{ and total score in first 2 games} \geq 80)$ $= (0.7888)^4 + C_1^4 (0.7888)^3 (0.0934) + C_1^2 (0.7888)^3 (0.1026)$ $+ (C_2^4 - 1)(0.7888)^2 (0.0934)^2$	1M	
<p><u>Alternative Solution</u></p> $= P(\text{total score in 4 games} \geq 160) - P(\text{total score in 4 games} \geq 160 \text{ and total score in first 2 games} < 80)$ $\approx 0.804490478 - (0.7888)^2 (0.0934)^2 - C_1^2 (0.7888)^3 (0.1026)$	1M	
	1A	
<p>(iii) $P(\text{average score in the first 2 games} < 40 \mid \text{win a prize})$</p> $\approx \frac{0.804490478 - 0.698351364}{0.804490478}$ ≈ 0.1319	1M	
	1A	
	(7)	

Solution	Marks	Remarks
<p>11. (a) (i) P(lift is full at G/F)</p> $= 1 - e^{-4} \left(1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!} + \frac{4^5}{5!} \right)$ ≈ 0.214869613 ≈ 0.2149 <p>(ii) P(4 persons gets into the lift and it stops at each floor)</p> $= \frac{e^{-4} 4^4}{4!} \cdot 4! \left(\frac{1}{4} \right)^4$ ≈ 0.0183 <p>(iii) P(lift stops at each floor)</p> $= \frac{e^{-4} 4^4}{4!} \cdot \frac{4!}{4^4} + \frac{e^{-4} 4^5}{5!} \cdot \frac{C_2^5 \cdot 4!}{4^5} + 0.214869613 \cdot \frac{C_3^6 \cdot 4! + C_2^6 C_2^4 \cdot \frac{4!}{2}}{4^6}$ ≈ 0.1368	<p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1M+1M</p> <p>1A</p> <p>(7)</p>	<p>1M for using (i) and (ii)</p> <p>1M for correct cases</p>
<p>(b) (i) P(3 persons from different floor waits for the lift)</p> $= C_3^4 (e^{-3} 3)^3 (e^{-3})$ ≈ 0.0007 <p>(ii) P(2 persons waits for the lift)</p> $= C_1^4 \left(\frac{e^{-3} 3^2}{2!} \right) (e^{-3})^3 + C_2^4 (e^{-3} 3)^2 (e^{-3})^2$ ≈ 0.0004 <p>(iii) Let the number of persons waiting above 62/F be X. P(3 persons get into the lift at the 62/F 3 persons wait at the 62/F)</p> $= P(X=0) + P(X=1) + P(X=2) + P(X=3)$ $= (e^{-3})^2 + C_1^2 (e^{-3} 3)(e^{-3}) + \left[C_1^2 \left(\frac{e^{-3} 3^2}{2!} \right) (e^{-3}) + (e^{-3} 3)(e^{-3} 3) \right]$ $+ \left[C_1^2 \left(\frac{e^{-3} 3^3}{3!} \right) (e^{-3}) + C_1^2 \left(\frac{e^{-3} 3^2}{2!} \right) (e^{-3} 3) \right]$ ≈ 0.1512	<p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1M+1M+1M</p> <p>1A</p> <p>(8)</p>	<p>OR $\frac{e^{-12} 12^2}{2!}$</p> <p>1M for 4 cases</p> <p>1M for the case $X=2$</p> <p>1M for the case $X=3$</p> <p>OR $\sum_{k=0}^3 \frac{e^{-6} 6^k}{k!}$</p>
<p>12. (a) $P(Y < 100) = 0.121$</p> $P\left(\frac{100 - \mu}{\sigma} \leq Z < 0 \right) = 0.379$ $\frac{100 - \mu}{\sigma} = -1.17 \text{ ----- (1)}$ <p>$P(Y \geq 200) = 0.0918$</p> $P\left(0 < Z < \frac{200 - \mu}{\sigma} \right) = 0.4082$ $\frac{200 - \mu}{\sigma} = 1.33 \text{ ----- (2)}$ <p>Solving (1) and (2), we get $\mu = 146.8$ and $\sigma = 40$</p>	<p>1A</p> <p>1A</p> <p>1A</p> <p>(3)</p>	

Solution	Marks	Remarks
(b) $P(\text{High level})$ $= P(150 \leq Y < 200)$ $= P(0.08 \leq Z < 1.33)$ $\approx 0.4082 - 0.0319$ $= 0.3763$	1A	For conditional probability
	(1)	
(c) $P(\text{High} \mid \text{rainfall exceeds 100 mm})$ $= \frac{0.3763}{1 - 0.121}$ ≈ 0.428100113 ≈ 0.4281	1M	
	1A	
	(2)	
(d) (i) $P(\text{Severe} \mid \text{rainfall exceeds 100 mm})$ $= \frac{0.0918}{1 - 0.121}$ ≈ 0.10443686 $P(\text{Medium} \mid \text{rainfall exceeds 100 mm})$ $= \frac{1 - 0.121 - 0.0918 - 0.3763}{1 - 0.121}$ ≈ 0.467463026 $P(\text{job will NOT be postponed} \mid \text{rainfall exceeds 100 mm})$ $= (0.467463026)e^{-1} + (0.428100113)e^{-3} + (0.10443686)e^{-6}$ ≈ 0.193542759 ≈ 0.1935	1A	
	1A	
(ii) $P(\text{job will be postponed for 1 day} \mid \text{rainfall exceeds 100 mm})$ $= (0.467463026) \cdot e^{-1} + (0.428100113) \cdot e^{-3} + (0.10443686) \cdot e^{-6}$ ≈ 0.237464824 ≈ 0.2375	1M	
	1A	
(iii) $P(\text{job will be postponed for 2 days} \mid \text{rainfall exceeds 100 mm})$ $= (0.467463026) \cdot \frac{e^{-1} 1^2}{2!} + (0.428100113) \cdot \frac{e^{-3} 3^2}{2!} + (0.10443686) \cdot \frac{e^{-6} 6^2}{2!}$ ≈ 0.186557057 $P(\text{High level} \mid \text{job will be postponed for at least 3 days})$ $= \frac{0.428100113 \left(1 - e^{-3} - e^{-3} 3 - \frac{e^{-3} 3^2}{2!} \right)}{1 - 0.193542759 - 0.237464824 - 0.186557057}$ ≈ 0.6457	1A	
	1M	
	1A	
	(9)	