2011-ASL
M & S

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG ADVANCED LEVEL EXAMINATION 2011

MATHEMATICS AND STATISTICS AS-LEVEL

8.30 am – 11.30 am (3 hours)

This paper must be answered in English

1. This paper consists of Section A and Section B.

2. Answer **ALL** questions in Section A, **using the AL(E) answer book.**

3. Answer any **FOUR** questions in Section B, **using the AL(C) answer book.**

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

Not to be taken away before the end of the examination session

Section A (40 marks)
Answer **ALL** questions in this section.
Write your answers in the AL(E) answer book.

1. (a) Prove that $\dfrac{1}{1+\sqrt{1-x}}=\dfrac{1}{x}(1-\sqrt{1-x})$ for $x<1$ and $x\neq 0$.

   (b) Let $I=\displaystyle\int_{\frac{1}{4}}^{\frac{1}{2}}\frac{1}{1+\sqrt{1-x}}\,\mathrm{d}x$. By considering the expansion of $\dfrac{1}{1+\sqrt{1-x}}$ in ascending powers of x as far as the term in x^2 , estimate the value of I .

   (c) Let $J=\displaystyle\int_{-2}^{-1}\frac{1}{1+\sqrt{1-x}}\,\mathrm{d}x$. Can we use the same method in (b) to estimate the value of J ? Explain your answer.

(7 marks)

2. A company launches a promotion plan to raise revenue. The total amount of money X (in million dollars) invested in the plan can be modelled by

$$\frac{\mathrm{d}X}{\mathrm{d}t}=6\left(\frac{t}{0.2t^3+1}\right)^2 , \qquad t\geq 0 ,$$

where t is the number of months elapsed since the launch of the plan.
Initially, 4 million dollars are invested in the plan.

   (a) Using the substitution $u=0.2t^3+1$, or otherwise, express X in terms of t .

   (b) Find the number of months elapsed since the launch of the plan if a total amount of 13 million dollars are invested in the plan.

   (c) If the company has a budget of 14.5 million dollars only, can the plan be run for a long time? Explain your answer.

(7 marks)

3. Let $\mathrm{f}(x)=x+\dfrac{3}{x}+\ln(x^2)$, where $x\neq 0$. Find the coordinates of all the turning point(s) and point(s) of inflexion of $y=\mathrm{f}(x)$.

(7 marks)

4. Peter and Susan play a shooting game. Each of them will shoot a target twice. Each shot will score 1 point if it hits the target. The one who has a higher score is the winner. It is known that the probabilities of hitting the target in one shot for Peter and Susan are 0.55 and 0.75 respectively. Assume that all shots are independent.

   (a) Find the probability that Susan will be the winner.

   (b) Given that Peter scores at least 1 point, what is the probability that Susan is the winner?

(6 marks)

5.

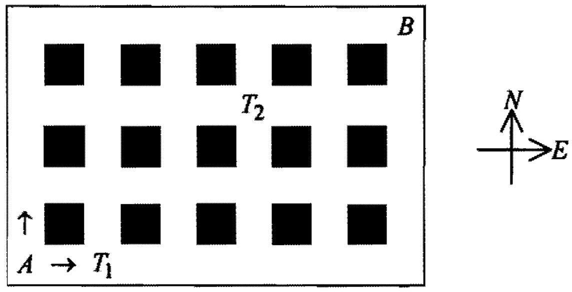


Figure 1

Figure 1 shows a board with routes blocked by shaded squares for an electronic toy car that goes from A to B. At each junction, the toy car will go either East or North as shown by the arrows at A. The toy car will choose randomly a route from A to B. There may be traps being set at some junctions. If the car reaches a trapped junction, it will stop and cannot reach B.

(a) If a trap is set at T_1, how many different routes are there for the toy car to go from A to B?

(b) If a trap is set at T_2, how many different routes are there for the toy car to go from A to B?

(c) If two traps are set at T_1 and T_2, find the probability that the toy car can reach B from A.

(6 marks)

6.

Tens	Units
2	6 7
3	0 0 a 2 9 9
4	b 3 3 3 6 8 8
5	6 9
6	5 9

Figure 2

The revision times (in minutes) of 19 students are represented by the stem-and-leaf diagram in Figure 2. It is known that the mean revision time is $(40+b)$ minutes.

(a) Find a and b.

(b) Find the standard deviation of the revision times for the students.

(c) The revision times of 2 more students are added. If both the range and the mean do not change after the inclusion of the 2 data, find the range of possible values of the standard deviation of the revision times for the 21 students.

(7 marks)

Section B (60 marks)
Answer any **FOUR** questions in this section. Each question carries 15 marks.
Write your answers in the AL(C) answer book.

7. Let C be the curve $y=\dfrac{ax+b}{cx+d}$ for all $x\neq\dfrac{-d}{c}$, where a, b, c, d are constants, $a\neq 0$ and $c\neq 0$. It is given that C has a vertical asymptote $2x-1=0$ and a horizontal asymptote $y+2=0$, and C passes through the origin.

   Let D be the curve $y=\dfrac{cx+d}{ax+b}$ for all $x\neq\dfrac{-b}{a}$.

   (a) Show that $d=\dfrac{-c}{2}$.
   Hence find the equation of C.
   (4 marks)

   (b) Find the coordinates of all the intersecting points of curves C and D.
   (3 marks)

   (c) Sketch the curves C and D on the same diagram, indicating their asymptotes, intercepts and their points of intersection.
   (4 marks)

   (d) Find the exact value of the area of the region bounded by the curves C, D and the positive x-axis.
   (4 marks)

8. An oil tanker leaks out oil for half a day at the rate of

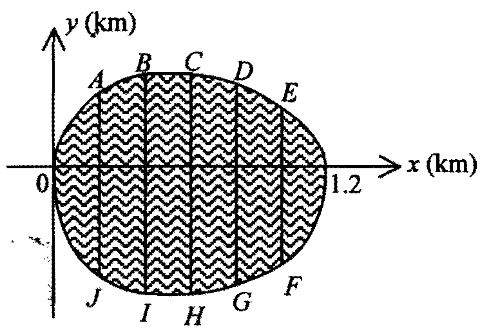
$$\frac{dV}{dt} = \frac{1}{25} e^{t^2+t+2}$$

where V is the volume of the oil (in hundred thousand m^3) leaked out and t $(0 \le t \le 0.5)$ is the number of days elapsed since the leakage begins.

(a) By finding a polynomial in t of degree 3 which approximates e^{t^2+t}, estimate the volume of the oil leaked out.
Is this an over-estimate or under-estimate? Explain your answer.

(6 marks)

(b) After half a day, the surface area of the ocean affected by the oil spread is as shown by the shaded region in Figure 3:



$A(0.2, 1.8)$	$F(1, -1.8)$
$B(0.4, 2)$	$G(0.8, -2)$
$C(0.6, 2.2)$	$H(0.6, -2.1)$
$D(0.8, 2.1)$	$I(0.4, -2.2)$
$E(1, 1.6)$	$J(0.2, -2)$

Figure 3

(i) Using the trapezoidal rule, estimate the surface area of the ocean affected.
Is this an over-estimate or under-estimate? Explain your answer.

(ii) Assuming that the thickness of the oil spread is uniform, estimate the thickness of the oil spread.
Is this an over-estimate or under-estimate? Explain your answer.

(5 marks)

(c) Subsequently, the oil company uses a new technology to clean up the oil spread. The rate of cleaning up the oil spread can be modelled by

$$\frac{dW}{dt} = \frac{-(W+1)^{\frac{1}{3}}}{40}$$

where W is the volume of the oil spread (in hundred thousand m^3) remained and t is the number of days elapsed since the beginning of the cleaning up.
How long will it take for all the oil spread to be cleaned up?

(4 marks)

9. A researcher studies the growth of the population size and the electricity consumption of a certain city. Suppose that the population size P (in hundred thousand) of the city can be modelled by

$$P=\frac{ke^{-\lambda t}}{t^2}\ , \qquad 0<t<6\ ,$$

where k and λ are constants and t is the time in years elapsed since the start of the research.

(a) (i) Express $\ln P+2\ln t$ as a linear function of t.

(ii) Given that the intercepts on the horizontal and vertical axes of the graph of the linear function in (a)(i) above are -1.15 and 2.3 respectively, find the values of k and λ correct to the nearest integer.
Hence find the minimum population size correct to the nearest hundred thousand.

(6 marks)

(b) The annual electricity consumption E (in thousand terajoules per year) of the city can be modelled by

$$\frac{dE}{dt}=hte^{ht}-1.2e^{ht}+4.214\ , \qquad t\geq 0\ ,$$

where h is a non-zero constant and t is the time in years elapsed since the start of the research. It is known that the population size and the rate of change of annual electricity consumption both attain minimum at the same time t_0, and when $t=0$, $E=1$.

(i) Find the value of h.

(ii) By considering $\frac{d}{dt}(te^{ht})$, find $\int te^{ht}\,dt$.
Hence find the annual electricity consumption of the city at t_0 correct to the nearest thousand terajoules per year.

(iii) A green campaign is launched to save the annual electricity consumption immediately after t_0. The new annual electricity consumption F (in thousand terajoules per year) of the city can then be modelled by

$$F=\frac{6}{1-5e^{rt}+3e^{2rt}}+2 \qquad ,\ t\geq t_0\ .$$

If the new annual electricity consumption is the same as the original annual electricity consumption at $t=t_0$, find the value of r.

(9 marks)

10. In a scoring game, a player will roll a ball at a starting point along a long horizontal track. When the ball comes to rest, let Y cm be the distance of the ball having travelled. The scoring system is shown in the following table.

Range of Y	$154 \leq Y < 160$	$160 \leq Y < K$	$K \leq Y < 174$	Otherwise
Score	20	50	30	0

It is known that Y can be modelled by a normal distribution with mean 165 and variance 16 . It is also known that 78.88% of the players score 50 in a game. A game in which the player scores 50 is called "Bingo". Assume that the games are independent.

(a) Find the value of K.

(2 marks)

(b) Find the probability that a player will score 30 in a game.

(2 marks)

(c) Find the probability that the 6^{th} game is the 3^{rd} "Bingo".

(2 marks)

(d) If the variance of the number of "Bingo" in n games is at most 2.3 , determine the largest value of n.

(2 marks)

(e) A player will win a prize if his average score in 4 games is at least 40 .

(i) Find the probability that a player will win the prize.

(ii) Find the probability that he wins the prize and his average score in the first 2 games is at least 40.

(iii) Given that a player wins the prize, find the probability that his average score in the first 2 games is less than 40.

(7 marks)

11.

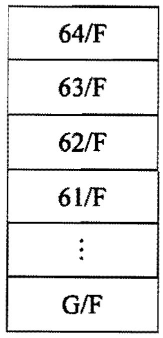


Figure 4

In a multi-storey office building as shown in Figure 4, there is a lift with maximum capacity of 6 persons that only serves G/F, 61/F – 64/F and operates on demand. The lift is said to be full when there are 6 persons in the lift. People waiting for the lift will enter the lift until it is full.

(a) In the morning, the lift only allows passengers from G/F to enter and travel to upper floors. The number of persons waiting at G/F can be modelled by a Poisson distribution with a mean of 4 persons. The probability that a person goes to each floor (61/F – 64/F) is the same.

(i) Find the probability that the lift is full at G/F.

(ii) Find the probability that there are exactly 4 persons getting into the lift at G/F and they will get off the lift at different floors.

(iii) Find the probability that at least 1 person will get off the lift at each floor (61/F – 64/F) in a single trip.

(7 marks)

(b) In the evening, the lift only takes passengers from upper floors (61/F – 64/F) to G/F and passengers are only allowed to exit the lift at G/F. The number of persons waiting at each floor (61/F – 64/F) can be modelled by a Poisson distribution with a mean of 3 persons.

(i) Find the probability that there are exactly 3 persons waiting for the lift and they are all from different floors.

(ii) Find the probability that there are exactly 2 persons waiting for the lift.

(iii) If there are exactly 3 persons waiting at 62/F, find the probability that all of them can get into the lift.

(8 marks)

12. A construction company proposes to use the daily rainfall precipitation to determine the effect of rainfall on a construction project. The following table shows the classification system.

Daily Rainfall Precipitation (Y mm)	$Y < 100$	$100 \leq Y < 150$	$150 \leq Y < 200$	$Y \geq 200$
Effect Level of the Day	Low	Medium	High	Severe

Assume that the daily rainfall precipitation recorded follows a normal distribution with mean μ mm and standard deviation σ mm . From past record, 12.10% of the days are classified as Low and 9.18% of the days are classified as Severe.

(a) Find the values of μ and σ .

(3 marks)

(b) Find the probability that a day is classified as High.

(1 mark)

(c) It is given that, in a certain rainy day, the rainfall precipitation exceeds 100 mm . Find the probability that the day is classified as High.

(2 marks)

(d) In a construction site, the numbers of days that a project is postponed under the precipitation levels Medium, High and Severe of a rainy day follow Poisson distributions with means 1 , 3 and 6 respectively. The project will not be postponed if a day is classified as Low. Given that during the construction of the project, there is exactly 1 rainy day with precipitation exceeding 100 mm .

(i) Find the probability that the project will NOT be postponed.

(ii) Find the probability that the project will be postponed for exactly 1 day.

(iii) Given that the project is postponed for at least 3 days, find the probability that the rainy day is classified as High.

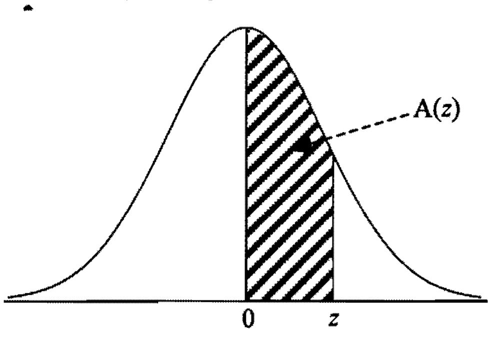
(9 marks)

END OF PAPER

Table: Area under the Standard Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the proportion of the area under the entire curve which is between $z = 0$ and a positive value of z. Areas for negative values of z are obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$