Section A (40 marks)
Answer **ALL** questions in this section.
Write your answers in the AL(E) answer book.

1. (a) Expand $\frac{1}{\sqrt{1-x}}$ in ascending powers of x as far as the term in x^3 .

   (b) By putting $x=\frac{1}{5}$ in the expansion obtained in (a), find an approximate value of $\sqrt{5}$.

   (c) Josephine claims that she can approximate $\sqrt{5}$ by putting $x=-4$ in the expansion obtained in (a). Do you agree? Why?

(6 marks)

2. A scientist models the proportion, P , of the initial population of an endangered species of animal still surviving by

$$\frac{dP}{dt}=\frac{-0.09}{\sqrt{3t+1}} \quad (0 \le t \le T) ,$$

where t is time in months since the beginning of his study, and T is the number of months elapsed for the population size to decrease to 0 . It is given that when $t=0$, $P=1$.

   (a) Find the proportion of the endangered species surviving after t months from the beginning of the study.

   (b) What is the proportion of the endangered species dying off within the first 5 months of the study?

   (c) Determine the value of T .

(6 marks)

3. Let C be the curve $x=y^4-y$.

   (a) Find $\frac{dy}{dx}$.

   (b) Find the equation of the tangent to C if the slope of the tangent is $\frac{1}{3}$.

(7 marks)

4. Let A and B be two events. Suppose $P(A \cup B)=\frac{5}{12}$, $P(A)=a$, $P(B)=\frac{1}{4}$ and $P(A \mid B')=k$.

   (a) Find $P(A \cap B)$ in terms of a .

   (b) Find the value of k .

   (c) If A and B are independent, find the value of a .

(8 marks)

5. It is known that 36% of the customers of a certain supermarket will bring their own shopping bags. There are 3 cashiers and each cashier has 5 customers in queue.

   (a) Find the probability that among all the customers in queue, at least 4 of them have brought their own shopping bags.

   (b) If exactly 4 customers in queue have brought their own shopping bags, what is the probability that each cashier will have at least 1 customer who has brought his/her own shopping bag?

(6 marks)

6. There are 18 managers in a company and they can be classified into two groups A and B according to their educational background. The following table summarizes some of the statistics of their monthly salaries (in thousand dollars).

Group	Number of Managers	Highest Monthly Salary	Standard Deviation
A	11	42.5	2.5
B	7	50.1	2.8

   (a) It is known that the means of the monthly salaries of the two groups are the same. Find the standard deviation of the monthly salaries of all the 18 managers.

   (b) In a set of n observations ranked in ascending order, let h and k be the nearest integers to $\frac{3}{4}(n+1)$ and $\frac{1}{4}(n+1)$ respectively. Define the h^{th} and k^{th} observations to be the upper and lower quartiles of the set of observations respectively.

      (i) If the upper quartiles of the monthly salaries in both group A and group B are \$40 000 , determine whether the monthly salary of the manager with the second highest salary in group A is lower than that in group B .

      (ii) If the manager with the highest monthly salary in group B resigns, determine whether the inter-quartile range of the monthly salaries of the remaining managers in group B will be increased, unchanged or decreased.

(7 marks)

Go on to the next page

Section B (60 marks)
Answer any **FOUR** questions in this section. Each question carries 15 marks.
Write your answers in the AL(C) answer book.

7. Let C_1 be the curve $y = \dfrac{2x-1}{hx-1}$, where h is a non-zero constant and $x \neq \dfrac{1}{h}$.
It is given that C_1 has a vertical asymptote $x = 1$ and a horizontal asymptote $y = k$, where k is a constant.

(a) (i) Find the values of h and k .

(ii) Sketch the graph of C_1 . Indicate its asymptotes and intercepts.

(5 marks)

(b) Let C_2 be the curve $y = -x^2 + px + q$, where p and q are constants. Let A be the point where C_1 cuts the x-axis. It is given that C_2 intersects C_1 at A , and the tangents to both curves at A are perpendicular to each other.

(i) Find the values of p and q .

(ii) On the diagram sketched in (a)(ii), sketch the graph of C_2 .

(iii) Find the area of the region bounded by the curves C_1 , C_2 and the y-axis.

(10 marks)

8. A researcher models the population size R , in hundreds, of a certain species of fish in a lake by

$$R = kt^{1.2}e^{\frac{\lambda t}{20}} \quad (0 \leq t \leq 30) ,$$

where t is the number of months elapsed since the beginning of the study and k and λ are constants.

(a) (i) Express $\ln R - 1.2 \ln t$ as a linear function of t .

(ii) It is given that the graph of $\ln R - 1.2 \ln t$ against t has intercept 2.89 on the vertical axis and slope -0.05 . Find the values of k and λ correct to the nearest integer.

(iii) Using the approximate integral values of k and λ obtained in (a)(ii), find the maximum population size of the species of fish correct to the nearest hundreds.
When will this take place?

(7 marks)

(b) In order to stimulate the growth of this species of fish, more food is added immediately when the population size of the fish attains 240 hundreds. The population size of the species of fish can then be modelled by

$$Q = L - 20(6e^{-t} + t^3) \quad (0 \leq t \leq 2) ,$$

where Q is the population size (in hundreds) of the species of fish, t is the number of months elapsed since more food has been added and L is a constant.

(i) Find the value of L .

(ii) Expand e^{-t} in ascending powers of t as far as the term in t^3 .
Hence, find a quadratic polynomial which approximates Q .

(iii) Using the result obtained in (b)(ii), check whether the species of fish will reach a population size of 300 hundreds.

(iv) Do you think that the conclusion in (b)(iii) is still valid if terms up to and including t^7 in the expansion of e^{-t} in (b)(ii) are used? Explain your answer briefly.

(8 marks)

9. A shop owner wants to launch two promotion plans A and B to raise the revenue. Let R and Q (in million dollars) be the respective cumulative weekly revenues of the shop after the launching of the promotion plans A and B. It is known that R and Q can be modelled by

$$\frac{dR}{dt}=\begin{cases}\ln(2t+1) & \text{when } 0\le t\le 6\,,\\ 0 & \text{when } t>6\,;\end{cases}$$

and

$$\frac{dQ}{dt}=\begin{cases}45t(1-t)+\dfrac{1.58}{t+1} & \text{when } 0\le t\le 1\,,\\ \dfrac{30e^{-t}}{(3+2e^{-t})^2} & \text{when } t>1\,;\end{cases}$$

respectively, where t is the number of weeks elapsed since the launching of a promotion plan.

(a) Suppose plan A is adopted.

(i) Using the trapezoidal rule with 6 sub-intervals, estimate the total amount of revenue in the first 6 weeks since the start of the plan.

(ii) Is the estimate in (a)(i) an over-estimate or under-estimate? Explain your answer briefly.

(4 marks)

(b) Suppose plan B is adopted.

(i) Find the total amount of revenue in the first week since the start of the plan.

(ii) Using the substitution $u=3+2e^{-t}$, or otherwise, find the total amount of revenue in the first n weeks, where $n>1$, since the start of the plan. Express your answer in terms of n.

(6 marks)

(c) Which of the plans will produce more revenue in the long run? Explain your answer briefly.

(5 marks)

10.

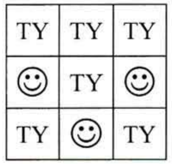


Figure 1

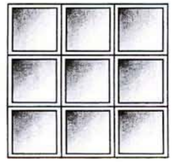


Figure 2

A soft-drink company proposes a promotion programme by attaching a scratch card to each can of soft drink. Every card has nine squares, with 3 or 4 randomly selected squares each containing a smiley face and in each of the rest a 'TY' denoting 'Thank You'. An example is shown in Figure 1. All squares are covered by metallic films (see Figure 2).

(a) A customer is asked to rub off the metallic films on 3 squares of a scratch card. If 3 smiley faces are found, the customer will win a prize. Find the probability that the customer can win a prize if the card has

(i) 3 smiley faces,

(ii) 4 smiley faces.

(2 marks)

(b) If the company wants to set the probability of winning a prize to be at most $\frac{1}{60}$, what should be the largest value of the proportion (p) of the cards with 4 smiley faces?

(3 marks)

(c) The company then produces the scratch card according to the proportion p found in (b). The company changes the rule of the game that customers will be asked to rub off the metallic films on 4 squares now and the prizes will be given as follows:

Gold Prize — exactly 4 smiley faces are found on 1 card
Silver Prize — exactly 3 smiley faces are found on 1 card
Bronze Prize — exactly 2 smiley faces are found on each of 2 cards

Find the probability of winning

(i) a Gold Prize with 1 card,

(ii) a Silver Prize with 1 card,

(iii) one or two prizes with 2 cards.

(10 marks)

11. Suppose the width of the tongues of normal new born babies can be modelled by a normal distribution with mean μ cm and standard deviation 0.4 cm. It is known that 24.2% of the normal babies will have their tongue widths less than 2.22 cm. If babies have inherited a certain genetic disease A, their tongues will be wider. It is known that 5% of new born babies have inherited disease A and the width of their tongues can be modelled by a normal distribution with mean $(\mu + 0.3)$ cm and standard deviation 0.2 cm. A diagnostic test is proposed such that if the width of the tongue of a baby is wider than $(\mu + 0.5)$ cm, he/she is diagnosed to have inherited disease A.

(a) Find the value of μ.

(1 mark)

(b) (i) What is the probability that a normal baby is diagnosed as having inherited disease A?

(ii) What is the probability of a wrong diagnosis?

(iii) Given that a baby is diagnosed as NOT having inherited disease A, what is the probability that the baby has actually inherited the disease?

(8 marks)

(c) A group of 20 babies are going to take the test one by one.

(i) Given that exactly 4 babies are diagnosed wrongly among the 20 babies, what is the probability that exactly 3 babies are diagnosed wrongly in the first 8 tests?

(ii) Given that at most 4 babies are diagnosed wrongly among the 20 babies, what is the probability that the 8^{th} baby to take the test is the 3^{rd} baby who is diagnosed wrongly?

(6 marks)

12.

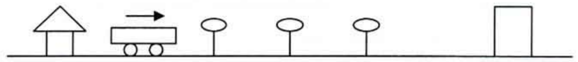


Figure 3

During peak hours, a 16-seat shuttle bus of a shopping mall will leave the bus terminus whenever there are exactly 12 passengers on the bus. The bus has three stops between the terminus and the shopping mall and the passengers are only allowed to get off at the shopping mall (see Figure 3). It is assumed that the numbers of passengers queuing up at different stops during peak hours are independent and follow the same Poisson distribution and any vacant seat will be filled by the next passenger in the queue. A survey is conducted to record the number of passengers waiting at a stop during peak hours and the results are shown in the following table. Two Poisson distributions are proposed to model the data.

Number of passengers waiting	Observed frequency f_o	* Expected frequency f_e [Po (2)]	* Expected frequency f_e [Po (λ)]
0	28	27.07	b
1	50	54.13	41.04
2	52	54.13	c
3	40	36.09	d
4	24	18.04	26.72
5	6	a	e
Total	200		

* Correct to 2 decimal places

(a) (i) Find λ correct to 1 decimal place.

(ii) Write down the values of a, b, c, d and e correct to 2 decimal places.

(iii) Suppose the absolute values of the difference between observed and expected frequencies are defined as errors. The distribution with a smaller sum of errors is regarded as fitting the data better. Which distribution fits the observed data better?

(7 marks)

(b) A bus has left the terminus. Adopting the distribution in (a)(iii) which fits the observed data better, find the probability that

(i) the bus is full when it leaves the first stop,

(ii) there are at least 2 passengers queuing at the second stop and the second passenger is able to get on the bus,

(iii) at each of the three stops some passenger(s) will get on the bus.

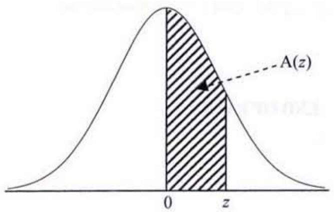
(8 marks)

END OF PAPER

Table: Area under the Standard Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the proportion of the area under the entire curve which is between $z = 0$ and a positive value of z. Areas for negative values of z are obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$