

2008 AS Mathematics & Statistics

評卷參考*

Marking Scheme * 此部分只設英文版本。

AS Mathematics and Statistics

General Instructions To Markers

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.
2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept / technique had been used.
3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
4. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.
5. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.
6. In the marking scheme, marks are classified into the following three categories:
 - 'M' marks – awarded for applying correct methods
 - 'A' marks – awarded for the accuracy of the answers
 - Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates' work in awarding 'M' marks.) However, 'A' marks for the corresponding answers should NOT be awarded, unless otherwise specified.
7. In the marking scheme, steps which can be skipped are enclosed by dotted rectangles, whereas alternative answers are enclosed by solid rectangles.
8. Marks may be deducted for poor presentation (*pp*). The symbol $\textcircled{pp-1}$ should be used to denote 1 mark deducted for *pp*.
 - (a) At most deduct 1 mark for *pp* in each section.
 - (b) In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.
9. Marks may be deducted for numerical answers with inappropriate degree of accuracy (*a*). The symbol $\textcircled{a-1}$ should be used to denote 1 mark deducted for *a*.
 - (a) At most deduct 1 mark for *a* in each section.
 - (b) In any case, do not deduct any marks for *a* in those steps where candidates could not score any marks.
10. Marks entered in the Page Total Box should be the NET total scored on that page.

Solution		Marks	Remarks
1. (a) (i)	$\frac{1}{\sqrt{1+ax}} = 1 + \binom{-1}{2}(ax) + \frac{\binom{-1}{2}\binom{-1}{2}(-1)}{2!}(ax)^2 + \dots$	1M	For binomial expansion on $(1+ax)^{\frac{-1}{2}}$
	$= 1 - \frac{a}{2}x + \frac{3a^2}{8}x^2 + \dots$	1A	For any two terms correct. pp-1 for omitting '...'
	$\therefore \begin{cases} \frac{-a}{2} = \frac{3}{2} \\ \frac{3a^2}{8} = b \end{cases}$	1M	For both
	$\therefore a = -3 \text{ and } b = \frac{27}{8}$	1A	For both correct
	(ii) The expansion is valid for $ x < \frac{1}{3}$	1A	
(b)	$\therefore \frac{1}{\sqrt{1-3\left(\frac{1}{30}\right)}} = 1 + \frac{3}{2}\left(\frac{1}{30}\right) + \frac{27}{8}\left(\frac{1}{30}\right)^2 + \dots$		
	$\sqrt{\frac{10}{9}} \approx \frac{843}{800}$	1A	For RHS, accept 1.05375
	$\sqrt{10} \approx \frac{2529}{800}$	1A	Accept 3.16125 and 3.1613
		(7)	
2. (a)	$y^3 - uy = 1$		
	$3y^2 \frac{dy}{du} - \left(u \frac{dy}{du} + y\right) = 0$	1M	
	$\frac{dy}{du} = \frac{y}{3y^2 - u}$	1A	
	<u>Alternative Solution</u>		
	$u = y^2 - \frac{1}{y}$		
$\frac{du}{dy} = 2y + \frac{1}{y^2}$	1M		
$\frac{dy}{du} = \frac{y^2}{2y^3 + 1}$	1A		
(b)	$u = 2^{x^2}$		
	$\ln u = x^2 \ln 2$	1M	
	$\frac{1}{u} \frac{du}{dx} = 2x \ln 2$	1M	
	$\frac{du}{dx} = 2^{x^2} \cdot 2x \ln 2$	1A	
	<u>Alternative Solution</u>		
$u = 2^{x^2} = e^{x^2 \ln 2}$	1M		
$\frac{du}{dx} = e^{x^2 \ln 2} \cdot 2x \ln 2$	1M		
$= 2^{x^2} \cdot 2x \ln 2$	1A		

Solution	Marks	Remarks
(c) $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ $= \frac{y}{3y^2 - u} \cdot 2x \ln 2$ $= \frac{2x^2 \cdot 2xy \ln 2}{3y^2 - 2x^2}$	1M 1A (7)	OR $\frac{2^{x^2+1} xy \ln 2}{3y^2 - 2x^2}$
3. (a) $\frac{dx}{dt} = 5.3 \left(\frac{1}{t+2} - \frac{1}{t+5} \right) + 1.2e^{-0.1t}$ $x = \int \left[5.3 \left(\frac{1}{t+2} - \frac{1}{t+5} \right) + 1.2e^{-0.1t} \right] dt$ $= 5.3[\ln(t+2) - \ln(t+5)] - 12e^{-0.1t} + C \quad (\text{since } t \geq 0)$ When $t = 0$, $x = 0$. $\therefore 0 = 5.3(\ln 2 - \ln 5) - 12 + C$ $C = 5.3(\ln 5 - \ln 2) + 12$ ≈ 16.8563 i.e. $x = 5.3[\ln(t+2) - \ln(t+5)] - 12e^{-0.1t} + 16.8563$ (b) $\lim_{t \rightarrow \infty} \{ 5.3[\ln(t+2) - \ln(t+5)] - 12e^{-0.1t} + 16.8563 \}$ $= 5.3 \lim_{t \rightarrow \infty} \ln \frac{t+2}{t+5} - 12 \lim_{t \rightarrow \infty} e^{-0.1t} + 16.8563$ $= 16.8563$ i.e. the concentration of the drug after a long time = 16.8563 mg/L	1M 1A 1M 1A 1M (6)	OR $5.3[\ln t+2 - \ln t+5] - 12e^{-0.1t} + C$ OR $x = \dots + 5.3 \ln 2.5 + 12$
4. (a) $P(A \cap B) = P(A B) \cdot P(B)$ $= \frac{k}{6}$ (b) $\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\therefore \frac{9}{20} = \frac{1}{5} + k - \frac{k}{6}$ i.e. $k = \frac{3}{10}$ (c) $P(A' \cap B) = P(B) - P(A \cap B)$ $= \left(\frac{3}{10} \right) - \frac{1}{6} \left(\frac{3}{10} \right)$	1A 1M 1A 1M	
<div style="border: 1px solid black; padding: 5px; width: fit-content;"> Alternative Solution $P(A' \cap B) = P(A \cup B) - P(A)$ $= \frac{9}{20} - \frac{1}{5}$ $= \frac{1}{4}$ </div>	1M 1A	

Solution	Marks	Remarks
(d) $P(A' \cap B') = 1 - P(A \cup B)$ $= 1 - \frac{9}{20}$		
<u>Alternative Solution 1</u> $P(A' \cap B') = P(A') - P(A' \cap B)$ $= \left(1 - \frac{1}{5}\right) - \left(\frac{1}{4}\right)$		
<u>Alternative Solution 2</u> $P(A' \cap B') = P(A') + P(B') - P(A' \cup B')$ $= P(A') + P(B') - P((A \cap B)')$ $= \left(1 - \frac{1}{5}\right) + \left(1 - \frac{3}{10}\right) - \left(1 - \frac{1}{6} \cdot \frac{3}{10}\right)$ $= \frac{11}{20}$ $\neq 0$ Hence A' and B' are not mutually exclusive.	1M 1	For $P(A' \cap B') \neq 0$ Follow through
<u>Alternative Solution</u> $P(A') + P(B') = \left(1 - \frac{1}{5}\right) + \left(1 - \frac{3}{10}\right)$ $= \frac{3}{2}$ $\neq P(A' \cup B')$ since $P(A' \cup B') \leq 1$ Hence A' and B' are not mutually exclusive.	1M 1	For $P(A') + P(B') \neq P(A' \cup B')$ Follow through
(7)		
5. (a) Let $\$X$ be the amount of money spent by a randomly selected customer. $P(X > 30000) = 0.242$ $P\left(Z > \frac{30000 - \mu}{6000}\right) = 0.242$ $\therefore P\left(0 < Z \leq \frac{30000 - \mu}{6000}\right) = 0.258$ $\therefore \frac{30000 - \mu}{6000} = 0.7$ i.e. $\mu = 25800$ (b) The required probability $= \frac{P(16500 < X < 30000)}{P(X < 30000)}$ $= \frac{P\left(\frac{16500 - 25800}{6000} < Z < \frac{30000 - 25800}{6000}\right)}{1 - P(X \geq 30000)}$ $= \frac{P(-1.55 < Z < 0.7)}{1 - 0.242}$ $= \frac{0.4394 + 0.258}{0.758}$ ≈ 0.9201	1M 1A 1A 1A 1A 1A	For standardization For $P(16500 < X < 30000)$ For denominator
(6)		

Solution		Marks	Remarks
6.	<p>(a) Since the mean of the 17 student marks is 74, $k = 74$</p> <p>(b) The required probability $= \frac{C_1^3 \cdot C_2^{15}}{C_3^{18}}$ $= \frac{105}{272}$</p> <p>(c) (i) The standard deviation of the 18 student marks is 9.32737905. The corresponding interval is $(74 - 2 \times 9.32737905, 74 + 2 \times 9.32737905)$ $\approx (55.34524189, 92.65475811)$ Thus, 55 is an outlier.</p> <p>(ii) There is no change in the median, which is 74 in both cases. The standard deviation decreases as the extreme datum (the outlier) is removed.</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>(7)</p>	<p>For denominator</p> <p>OR 0.3860</p> <p>Accept 1 d.p. or above</p> <p>Follow through</p> <p>Awarded only when (i) is correct</p>
7.	<p>(a) $\because \lim_{x \rightarrow \infty} \frac{3x-2}{x+2} = \lim_{x \rightarrow \infty} \frac{3 - \frac{2}{x}}{1 + \frac{2}{x}} = 3$</p> <p>$\therefore$ the equation of the horizontal asymptote to C is $y = 3$.</p> <p>$\because \lim_{x \rightarrow -2^-} \frac{3x-2}{x+2} = +\infty$ and $\lim_{x \rightarrow -2^+} \frac{3x-2}{x+2} = -\infty$</p> <p>$\therefore$ the equation of the vertical asymptote to C is $x = -2$.</p>	<p>1A</p> <p>1A</p> <p>(2)</p>	
(b)		<p>1A</p> <p>1A</p> <p>1A</p> <p>(3)</p>	<p>For intercepts</p> <p>For asymptotes</p> <p>For shape</p> <p>(pp-1) for origin and labels of axes were all missing</p>

Solution	Marks	Remarks
<p>(c) (i) $f(x) = \frac{3x-2}{x+2}$ $f'(x) = \frac{(x+2)(3) - (3x-2)(1)}{(x+2)^2}$ $= \frac{8}{(x+2)^2}$ So the equation of L_1 is $y - k = f'(h)(x - h)$ $y - \frac{3h-2}{h+2} = \frac{8}{(h+2)^2}(x - h)$ $(h+2)^2 y - (3h-2)(h+2) = 8x - 8h$ $8x - (h+2)^2 y + 3h^2 - 4h - 4 = 0$</p>	<p>1A 1M 1</p>	<p>For numerator Follow through</p>
<p>(ii) (1) If L_1 passes through the origin, then $8(0) - (h+2)^2(0) + 3h^2 - 4h - 4 = 0$ $h = 2$ or $\frac{-2}{3}$ (rejected since P lies in the first quadrant) $\therefore k = \frac{3(2)-2}{(2)+2} = 1$ Slope of $L_2 = \frac{[(2)+2]^2}{-8} = -2$ Hence the equation of L_2 is $y - 1 = -2(x - 2)$ i.e. $2x + y - 5 = 0$</p>	<p>1A 1M 1A</p>	<p>(pp-1) if $\frac{-2}{3}$ was not rejected</p>
<p>(2) The x-intercept of L_2 is $\frac{5}{2}$ So the required area $= \int_{\frac{2}{3}}^2 \frac{3x-2}{x+2} dx + \frac{1}{2} \left(\frac{5}{2} - 2 \right) (1)$ $= \int_{\frac{2}{3}}^2 \left(3 - \frac{8}{x+2} \right) dx + \frac{1}{2} \left(\frac{1}{2} \right) (1)$ $= [3x - 8 \ln x+2]_{\frac{2}{3}}^2 + \frac{1}{4}$ $= \frac{17}{4} + 8 \ln \frac{2}{3}$</p>	<p>1A+1A 1M 1A</p>	<p>OR $\dots + \int_{\frac{2}{3}}^{\frac{5}{2}} (5 - 2x) dx$ For $3 - \frac{8}{x+2}$ OR 1.0063</p>
(10)		

	Solution	Marks	Remarks
8.	(a)		
	(i)		
	$N'(t) = \frac{20}{1 + he^{-kt}} \quad (t \geq 0)$		
	$\ln \left[\frac{20}{N'(t)} - 1 \right] = -kt + \ln h$	1A	
	(ii)		
	$\ln h = 1.5$	1M	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> Either One </div>
	$h = e^{1.5}$		
	$\approx 4.4817 \text{ (correct to 4 d.p.)}$	1A	
	$-k = \frac{1.5 - 0}{0 - 7.6}$		
	$k = \frac{15}{76}$		
	$\approx 0.1974 \text{ (correct to 4 d.p.)}$	1A	
		(4)	
	(b)		
	(i)		
	$v = 4.5 + e^{0.2t}$		
	$\frac{dv}{dt} = 0.2e^{0.2t}$		1A
	$N(t) = \int \frac{20}{1 + 4.5e^{-0.2t}} dt$		
	$= \int \frac{100}{e^{0.2t} + 4.5} (0.2e^{0.2t}) dt$		
	$= \int \frac{100}{v} dv$		1M
	$= 100 \ln v + C$		
	$= 100 \ln(4.5 + e^{0.2t}) + C \quad (\because 4.5 + e^{0.2t} > 0)$		1A
	Since $N(0) = 50$, so $C = 50 - 100 \ln 5.5$		1M
	i.e. $N(t) = 100 \ln \frac{4.5 + e^{0.2t}}{5.5} + 50$		1A
	(ii)		
	(1)		
	$M(20) = N(20)$		
	$21 \left[(20) + \frac{4.5}{0.2} e^{-0.2(20)} \right] + b = 100 \ln \frac{4.5 + e^{0.2(20)}}{5.5} + 50$		
	$b \approx -141.2090$		1A
	(2)		
	Consider $M'(t) - N'(t)$		
	$= 21(1 - 4.5e^{-0.2t}) - \frac{20}{1 + 4.5e^{-0.2t}}$		1A
	$= \frac{1 - 425.25e^{-0.4t}}{1 + 4.5e^{-0.2t}}$		1A
	$\therefore M'(t) - N'(t) > 0 \text{ when } e^{-0.4t} < \frac{1}{425.25}$		1M
	i.e. $t > \frac{\ln 425.25}{0.4} \approx 15.1317$		1A
	Since $M(20) = N(20)$ and $M(t) - N(t)$ is increasing when $t > 20$,		
	so $M(t) > N(t)$ for $t > 20$.		
	Hence the biologist's claim is correct.		1A
		(11)	Follow through

Solution	Marks	Remarks
<p>9. (a) (i) $\int_0^{10} \frac{1}{40} \sqrt{1+t^2} dt$ $\approx \frac{10}{2(4)} \cdot \frac{1}{40} \left(\sqrt{1+0^2} + 2\sqrt{1+2.5^2} + 2\sqrt{1+5^2} + 2\sqrt{1+7.5^2} + \sqrt{1+10^2} \right)$ ≈ 1.305182044 ≈ 1.3052 So the increase of temperature is about 1.3052°C.</p> <p>(ii) $\frac{d}{dt} \left(\frac{1}{40} \sqrt{1+t^2} \right) = \frac{t}{40\sqrt{1+t^2}}$ $\frac{d^2}{dt^2} \left(\frac{1}{40} \sqrt{1+t^2} \right) = \frac{1}{40(1+t^2)^{\frac{3}{2}}}$ > 0 Hence it is an over-estimate.</p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p>	<p>Follow through</p>
(4)		
<p>(b) (i) $100(\ln x_0)^2 - 630 \ln x_0 + 1960 = 968$ $50(\ln x_0)^2 - 315 \ln x_0 + 496 = 0$ $\ln x_0 = \frac{31}{10}$ or $\frac{16}{5}$ $x_0 \approx 22.1980$ or 24.5325</p> <p>(ii) $W'(x) = \frac{200 \ln x}{x} - \frac{630}{x}$ $\therefore W'(x) < 0$ when $200 \ln x - 630 < 0$ ($\because x \geq 22$) $\therefore \ln x < 3.15$ i.e. $22 \leq x < e^{3.15} \approx 23.3361$</p>	<p>1A</p> <p>1A</p> <p>1A</p>	<p>Accept $x < 23.3361$</p>
<p>(iii) $\frac{dW}{dt} = \frac{dW}{dx} \cdot \frac{dx}{dt}$ $= \frac{200 \ln x - 630}{x} \cdot \frac{\sqrt{1+t^2}}{40}$ When $t = 0$, $x = 22$. $\therefore \left. \frac{dW}{dt} \right _{t=0} = \frac{200 \ln 22 - 630}{22} \cdot \frac{\sqrt{1+0}}{40}$ ≈ -0.0134 Hence the rate of change of electricity consumption at $t = 0$ is -0.0134 units per year.</p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>Accept $x < 23.3361$</p>
<p>(iv) The electricity consumption at $t = 10$ is approximately $W(22 + 1.305182044)$ $= 100(\ln 23.305182044)^2 - 630 \ln 23.305182044 + 1960$ ≈ 967.7502 units Since the estimate in (a)(i) is an over-estimate, the actual temperature when $t = 10$ is $x < 23.305182044$. Moreover, $W(x)$ is decreasing for $22 \leq x < 23.3361$ by (b)(ii). Therefore the actual electricity consumption is larger than this estimate.</p>	<p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p>Follow through</p>
(11)		

Solution	Marks	Remarks
10. (a) The required probability $= 1 - \left(\frac{3.9^0 e^{-3.9}}{0!} + \frac{3.9^1 e^{-3.9}}{1!} + \frac{3.9^2 e^{-3.9}}{2!} + \frac{3.9^3 e^{-3.9}}{3!} \right)$ ≈ 0.546753239 ≈ 0.5468	1M+1M 1A (3)	1M for cases correct 1M for Poisson probability
(b) The required probability $= 1 - P(\text{no busy counters are found after the 4th counter is checked})$ $\approx 1 - (1 - 0.546753239)^4$ ≈ 0.9578	1M 1A	
<u>Alternative Solution 1</u> The required probability $\approx C_1^4 (0.546753239)(1 - 0.546753239)^3 + C_2^4 (0.546753239)^2 (1 - 0.546753239)^2$ $+ C_3^4 (0.546753239)^3 (1 - 0.546753239) + (0.546753239)^4$ ≈ 0.9578	1M 1A	1M for Binomial probability
<u>Alternative Solution 2</u> The required probability $\approx (0.546753239) + (1 - 0.546753239)(0.546753239)$ $+ (1 - 0.546753239)^2 (0.546753239) + (1 - 0.546753239)^3 (0.546753239)$ ≈ 0.9578	1M 1A (2)	1M for Geometric probability
(c) The required probability $\approx (0.546753239)^{10} + C_9^{10} (0.546753239)^9 (1 - 0.546753239)$ $+ C_8^{10} (0.546753239)^8 (1 - 0.546753239)^2$ ≈ 0.096004444 ≈ 0.0960	1M+1M 1A (3)	1M for cases correct 1M for Binomial probability
(d) The required probability $\approx C_8^{10} (0.546753239)^8 (1 - 0.546753239)^2 \times \frac{2}{10}$ $+ C_9^{10} (0.546753239)^9 (1 - 0.546753239) \times \frac{1}{10} + 0$ ≈ 0.0167	1M+1A 1A (3)	1M for form correct
(e) The required probability $(0.546753239)^{10} \times [(0.546753239)^5 + C_4^5 (0.546753239)^4 (1 - 0.546753239)]$ $\approx \frac{+ C_9^{10} (0.546753239)^9 (1 - 0.546753239) \times (0.546753239)^5}{0.096004444}$	1M+1M+1A	1M for denominator using (c) 1M for numerator form correct 1A for numerator correct
<u>Alternative Solution</u> The required probability $\approx \frac{(0.546753239)^{15} + C_{14}^{15} (0.546753239)^{14} (1 - 0.546753239)}{0.096004444}$ ≈ 0.0163	1M+1M+1A 1A (4)	1M for denominator using (c) 1M for numerator form correct 1A for numerator correct

Solution	Marks	Remarks
11. (a) The required probability $= \frac{1.8^0 e^{-1.8}}{0!} + \frac{1.8^1 e^{-1.8}}{1!} + \frac{1.8^2 e^{-1.8}}{2!} + \frac{1.8^3 e^{-1.8}}{3!} + \frac{1.8^4 e^{-1.8}}{4!}$ ≈ 0.963593339 ≈ 0.9636	1M+1M 1A	1M for $P(X \leq 4)$ 1M for Poisson probability with correct λ
	(3)	
(b) $p_0 = P\left(Z > \frac{4.6-3}{0.8}\right) = P(Z > 2) = 0.5 - 0.4772 = 0.0228$ $p_2 = P\left(Z < \frac{2-3}{0.8}\right) = P(Z < -1.25) = 0.5 - 0.3944 = 0.1056$ $p_1 = 1 - p_0 - p_2 = 1 - 0.0228 - 0.1056 = 0.8716$	1M 1A 1A	For standardization For any one correct For all correct
	(3)	
(c) (i) The required probability $= C_1^3 p_2 p_1^2 + C_1^3 p_2^2 p_0$ $= 3(0.1056)(0.8716)^2 + 3(0.1056)^2(0.0228)$ ≈ 0.241431455 ≈ 0.2414	1M 1A	1M for form correct
(ii) The required probability $= C_2^4 p_2^2 p_0^2 + \frac{4!}{1!2!1!} p_2 p_1^2 p_0 + p_1^4$ $= 6(0.1056)^2(0.0228)^2 + 12(0.1056)(0.8716)^2(0.0228) + (0.8716)^4$ ≈ 0.599107436 ≈ 0.5991	1M 1A 1A	1M for form correct
	(5)	
(d) The required probability $\approx \frac{\frac{1.8^2 e^{-1.8}}{2!} (0.1056)^2 + \frac{1.8^3 e^{-1.8}}{3!} (0.241431455) + \frac{1.8^4 e^{-1.8}}{4!} (0.599107436)}{0.963593339}$ ≈ 0.0883	1M+1M+1A 1A	1M for any 2 cases 1M for denominator using (a) 1A for all correct
	(4)	

	Solution	Marks	Remarks
12. (a)	<p>The required probability</p> $= 1 - [(1 - 0.01)^{40} + C_1^{40}(1 - 0.01)^{39}(0.01) + C_2^{40}(1 - 0.01)^{38}(0.01)^2]$ ≈ 0.007497363 ≈ 0.0075	<p>1M+1M</p> <p>1A</p> <p>(3)</p>	<p>1M for cases correct</p> <p>1M for Binomial probability</p> <p>(Can be awarded in (d)(i))</p>
(b)	<p>The required probability</p> $\approx (1 - 0.007497363)^4(0.007497363)$ ≈ 0.0073	<p>1M</p> <p>1A</p> <p>(2)</p>	
(c)	$C_1^k(0.007497363)(1 - 0.007497363)^{k-1} + C_2^k(0.007497363)^2(1 - 0.007497363)^{k-2}$ $+ \dots + C_{k-1}^k(0.007497363)^{k-1}(1 - 0.007497363) + (0.007497363)^k > 0.05$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p><u>Alternative Solution</u></p> $0.007497363 + (1 - 0.007497363)(0.007497363) + (1 - 0.007497363)^2(0.007497363)$ $+ \dots + (1 - 0.007497363)^{k-1}(0.007497363) > 0.05$ </div>		
	$1 - (1 - 0.007497363)^k > 0.05$ $0.992502636^k < 0.95$ $k \ln 0.992502636 < \ln 0.95$ $k > \frac{\ln 0.95}{\ln 0.992502636} \approx 6.815832223$ <p>Hence the least value of k is 7.</p>	<p>1M</p> <p>1M</p> <p>1A</p> <p>(3)</p>	
(d) (i)	<p>The required probability</p> $= 1 - [(1 - 0.015)^{40} + C_1^{40}(1 - 0.015)^{39}(0.015) + C_2^{40}(1 - 0.015)^{38}(0.015)^2]$ ≈ 0.022069897 ≈ 0.0221	<p>1A</p>	
(ii)	<p>The required probability</p> $= [C_0^8(1 - 0.007497363)^8(0.007497363)^0][C_2^{12}(1 - 0.022069897)^{10}(0.022069897)^2]$ $+ [C_1^8(1 - 0.007497363)^7(0.007497363)][C_1^{12}(1 - 0.022069897)^{11}(0.022069897)]$ $+ [C_2^8(1 - 0.007497363)^6(0.007497363)^2][C_0^{12}(1 - 0.022069897)^{12}(0.022069897)^0]$ ≈ 0.037154780 ≈ 0.0372	<p>1M+1M</p> <p>1A</p>	<p>1M for any 1 case correct</p> <p>1M for all cases correct</p>
(iii)	<p>The required probability</p> $\approx \frac{[C_0^8(1 - 0.007497363)^8(0.007497363)^0][C_2^{12}(1 - 0.022069897)^{10}(0.022069897)^2]}{0.037154780}$ ≈ 0.6517	<p>1M+1M</p> <p>1A</p> <p>(7)</p>	<p>1M for form correct</p> <p>1M for denominator using (ii)</p>