2007-ASL
M & S

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG ADVANCED LEVEL EXAMINATION 2007

MATHEMATICS AND STATISTICS AS-LEVEL

8.30 am – 11.30 am (3 hours)

This paper must be answered in English

1. This paper consists of Section A and Section B.
2. Answer **ALL** questions in Section A, **using the AL(E) answer book.**
3. Answer any **FOUR** questions in Section B, **using the AL(C) answer book.**
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

Not to be taken away before the end of the examination session

SECTION A (40 marks)
Answer **ALL** questions in this section.
Write your answers in the AL(E) answer book.

1. The binomial expansion of $\left(1+\frac{x}{a}\right)^r$ in ascending powers of x is $1+\frac{2r}{3}x-\frac{1}{18}x^2+\cdots$, where a is a non-zero constant and r is a rational number.

   (a) (i) Find the values of a and r.

   (ii) State the range of values of x for which the binomial expansion of $\left(1+\frac{x}{a}\right)^r$ is valid.

   (b) Using the results of (a),

   (i) write down the binomial expansion of $\left(1-\frac{x}{a}\right)^r$ in ascending powers of x as far as the term in x^2 ;

   (ii) state the range of values of x for which the binomial expansion of $\left(1-\frac{x}{a}\right)^r$ is valid.

   (6 marks)

2. A researcher models the rate of change of the number of certain bacteria under controlled conditions by

$$\frac{dN}{dt}=\frac{800\,t}{(2t^2+50)^2},$$

   where N is the number in millions of bacteria and t (≥ 0) is the number of days elapsed since the start of the research. It is given that $N=4$ when $t=0$.

   (a) Using the substitution $u=2t^2+50$, or otherwise, express N in terms of t.

   (b) When will the number of bacteria be 6 million after the start of the research?

   (7 marks)

3. Let $y=\frac{1-e^{4x}}{1+e^{8x}}$.

   (a) Find the value of $\frac{dy}{dx}$ when $x=0$.

   (b) Let $(z^2+1)e^{3z}=e^{\alpha+\beta x}$, where α and β are constants.

   (i) Express $\ln(z^2+1)+3z$ as a linear function of x.

   (ii) It is given that the graph of the linear function obtained in (b)(i) passes through the origin and the slope of the graph is 2. Find the values of α and β.

   (iii) Using the values of α and β obtained in (b)(ii), find the value of $\frac{dy}{dz}$ when $z=0$.

   (7 marks)

4. Albert conducted a survey on the time spent (in hours) on watching television by 16 students. The data recorded are 3.7, 1.2, 2.1, 5.1, 2.1, 4.7, 1.9, 2.4, 2.4, 2.9, 3.6, 2.3, 3.9, 2.2, 1.8 and k, where k is the missing datum.

(a) Albert assumes that the range of these data is 5.3 hours.

(i) Find the value of k.

(ii) Construct a stem and leaf diagram for these data.

(iii) Find the mean and the median of these data.

(b) Albert finds that the assumption in (a) is incorrect and he can only assume that the range of these data is greater than 5.3 hours. Describe the change in the mean and the change in the median of these data due to the revision of Albert's assumption.

(7 marks)

5. Let A and B be two events with $P(A)=a$ and $P(B)=b$, where $0<a<1$ and $0<b<1$. Suppose that $P(A'|B)=0.6$, $P(B|A')=0.3$ and $P(B'|A)=0.7$, where A' and B' are complementary events of A and B respectively.

(a) By considering $P(A'\cap B)$, prove that $a+2b=1$.

(b) Using the fact that $A\cup B'$ is the complementary event of $A'\cap B$, or otherwise, find the values of a and b.

(c) Are A and B independent events? Explain your answer.

(7 marks)

6. David has 10 shirts and 3 bags:

1 blue shirt,
4 yellow shirts,
5 white shirts,
1 yellow bag and
2 white bags.

He randomly chooses 3 shirts from the 10 shirts and randomly puts the chosen shirts into 3 bags so that each bag contains 1 shirt.

(a) Find the probability that the yellow bag contains the blue shirt and each of the two white bags contains 1 yellow shirt.

(b) Find the probability that each of these three bags contains 1 shirt of a colour different from the bag.

(6 marks)

Go on to the next page

SECTION B (60 marks)
Answer any **FOUR** questions in this section. Each question carries 15 marks.
Write your answers in the AL(C) answer book.

7. Define $f(x)=\dfrac{8x-40}{x+4}$ for all $x\neq -4$. Let $g(x)=\dfrac{(x+4)^2(x-5)}{8}$.

Let C_1 and C_2 be the curves $y=f(x)$ and $y=g(x)$ respectively.

(a) Sketch C_1 and indicate its asymptote(s) and its intercept(s).

(3 marks)

(b) (i) Find the coordinates of the relative extreme point(s) and the point(s) of inflexion of C_2.

(ii) On the diagram sketched in (a), sketch C_2 and indicate its relative extreme point(s), its intercept(s), its point(s) of inflexion and the point(s) of intersection of the two curves.

(8 marks)

(c) Find the area enclosed by C_1 and C_2.

(4 marks)

8. A financial analyst, Mary, models the rates of change of profit (in billion dollars) made by companies A and B respectively by

$$f(t)=\ln(e^t+2)+3 \quad \text{and} \quad g(t)=\frac{8e^t}{40-t^2},$$

where t is the time measured in months.
Assume that the two models are valid for $0 \le t \le 6$.

(a) (i) Using the trapezoidal rule with 6 sub-intervals, estimate the total profit made by company A from $t=0$ to $t=6$.

(ii) Find $\frac{d^2f(t)}{dt^2}$ and hence determine whether the estimate in (a)(i) is an over-estimate or an under-estimate.

(7 marks)

(b) (i) Expand $\frac{1}{40-t^2}$ in ascending powers of t as far as the term in t^4 .

(ii) Using the result of (b)(i), find the expansion of $\frac{8e^t}{40-t^2}$ in ascending powers of t as far as the term in t^4 .

(iii) Using the result of (b)(ii), estimate the total profit made by company B from $t=0$ to $t=6$.

(6 marks)

(c) Mary claims that the total profit made by company A from $t=0$ to $t=6$ is less than that of company B . Do you agree? Explain your answer.

(2 marks)

Go on to the next page

9. In a certain year, the amount of water (in million cubic metres) stored in a reservoir can be modelled by

$$A(t)=(-t^2+5t+a)e^{kt}+7 \qquad (0 \le t \le 12),$$

where a and k are constants and t is the time measured in months from the start of the year.

The amount of water stored in the reservoir is the greatest when $t=2$. It is found that $A(0)=3$.

(a) Find the value of a ;

Hence find the amount of water stored in the reservoir when $t=1$.

(2 marks)

(b) Find the value of k .

(3 marks)

(c) In that year, the period during which the amount of water stored in the reservoir is 7 million cubic metres or more is termed *adequate*.

(i) How long does the *adequate* period last?

(ii) Find the least amount of water stored in the reservoir, within that year, after the *adequate* period has ended.

(iii) Find $\frac{d^2A(t)}{dt^2}$.

(iv) Describe the behaviour of $A(t)$ and $\frac{dA(t)}{dt}$, within that year, after the *adequate* period has ended for 6 months.

(10 marks)

10. The manager, Teresa, of a superstore launches a promotion plan to increase the sales volume. The number of customers shopping at the superstore in a minute can be modelled by a Poisson distribution with a mean of 2.4 customers per minute. The expenses of customers in the superstore are assumed to be independent and follow a normal distribution with a mean of \$ 375 and a standard deviation of \$ 125. A customer who spends more than \$ 300 but less than \$ 600 in the superstore can enter lucky draw X in which the probability of winning a gift is 0.25. A customer who spends \$ 600 or more in the superstore can enter lucky draw Y in which the probability of winning a gift is 0.8. Assume that each customer enters at most one lucky draw for each visit.

(a) Find the probability that there are more than 2 customers shopping at the superstore in a certain minute.

(3 marks)

(b) Find the probability that a randomly selected customer shopping at the superstore can enter lucky draw X.

(2 marks)

(c) Find the probability that a randomly selected customer shopping at the superstore wins a gift.

(2 marks)

(d) Find the probability that there are exactly 3 customers shopping at the superstore in a certain minute and each of them wins a gift.

(2 marks)

(e) Given that there are more than 2 customers shopping at the superstore in a certain minute, find the probability that there are fewer than 5 customers shopping at the superstore in this minute and each of them wins a gift.

(3 marks)

(f) If Teresa wants to revise the least expense of a customer for entering lucky draw Y so that 33% of the customers shopping at the superstore could enter lucky draw Y, what should be the revised least expense?

(3 marks)

11. A factory produces brand D coffee beans which are packed into boxes of 30 cans each. The net weight of each can of coffee beans follows a normal distribution with a mean of 300 g and a standard deviation of 7.5 g. A can of coffee beans with net weight less than 283.5 g or more than 316.5 g is classified as *exceptional*.

(a) Find the probability that a randomly selected can of brand D coffee beans is *exceptional*.

(2 marks)

(b) The manager of the factory randomly selects a box of brand D coffee beans and inspects every can in the box one by one.

(i) Find the probability that the 12th inspected can is the 1st *exceptional* can of coffee beans in the box.

(ii) Find the probability that there is exactly 1 *exceptional* can of coffee beans in the box.

(iii) Find the probability that there is at most 1 *exceptional* can of coffee beans in the box.

(8 marks)

(c) The shopkeeper of a coffee shop buys one box of brand D coffee beans. The shopkeeper regards a can of coffee beans as *unacceptable* if the net weight of the can is less than 283.5 g.

(i) Find the probability that in the box there is exactly 1 *exceptional* can of coffee beans which is *unacceptable*.

(ii) Given that in the box there is at most 1 *exceptional* can of coffee beans, find the probability that there is exactly 1 *unacceptable* can of coffee beans in the box.

(5 marks)

12. In game A, two players take turns to draw a ball randomly, with replacement, from a bag containing 4 green balls and 1 red ball. The first player who draws the red ball wins the game. Christine and Donald play the game until one of them wins. Christine draws a ball first.

(a) Find the probability that Donald wins game A before his 4th draw.
(2 marks)

(b) Find the probability that Donald wins game A.
(3 marks)

(c) Given that Donald wins game A, find the probability that Donald does not win game A before his 4th draw.
(3 marks)

(d) After game A, Christine and Donald play game B. In game B, there are box X and box Y. Box X contains 2 cards which are numbered 4 and 8 respectively while box Y contains 7 cards which are numbered $1, 2, \ldots, 7$ respectively. A player randomly draws one card from each box without replacement. If the number drawn from box X is greater than that from box Y, then the player wins game B. Christine and Donald take turns to draw cards until one of them wins game B. Donald draws cards first.

(i) Find the probability that Donald wins game B in his 1st draw.

(ii) Find the probability that Christine wins game B.

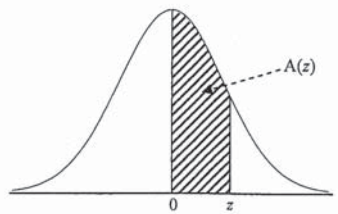
(iii) Given that Christine and Donald win one game each, find the probability that Donald wins game A.
(7 marks)

END OF PAPER

Table: Area under the Standard Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the proportion of the area under the entire curve which is between $z = 0$ and a positive value of z. Areas for negative values of z are obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$