評卷參考 *
Marking Scheme

香港考試及評核局
Hong Kong Examinations and Assessment Authority

2006年香港高級程度會考
Hong Kong Advanced Level Examination 2006

數學及統計學　高級補充程度
Mathematics and Statistics　AS-Level

本文件專爲閱卷員而設，其內容不應視爲標準答案。考生以及沒有參與評卷工作的教師在詮釋本文件時應小心謹慎。

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

* 此部分只設英文版本

2006-AS-M & S-1

AS Mathematics and Statistics

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

6. Marks may be deducted for poor presentation (*pp*). The symbol *pp-1* should be used to denote 1 mark deducted for *pp*. At most deducted 1 mark from Section A and 1 mark from Section B for *pp*. In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.

7. Marks may be deducted for numerical answers with inappropriate degree of accuracy (*a*). The symbol *a-1* should be used to denote 1 mark deducted for *a*. At most deducted 1 mark from Section A and 1 mark from Section B for *a*. In any case, do not deduct any marks for *a* in those steps where candidates could not score any marks.

8. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles.

Solution	Marks
1. (a) (i) $\left(1+\frac{x}{a}\right)^{\frac{-1}{n}}$	
$=1-\frac{x}{na}+\frac{1}{2}\left(\frac{-1}{n}\right)\left(\frac{-1}{n}-1\right)\left(\frac{x}{a}\right)^2-\cdots$	1M for any two terms correct pp−1 for omitting ' $\cdots$ '
$=1-\frac{x}{na}+\left(\frac{1+n}{2n^2a^2}\right)x^2-\cdots$	
So, we have $\frac{-1}{na}=\frac{-1}{18}$ and $\frac{1+n}{2n^2a^2}=\frac{1}{24a}$.	
Solving, we have $a=9$ and $n=2$.	1A + 1A
(ii) The binomial expansion is valid for $\left\lvert\frac{x}{9}\right\rvert<1$.	
Thus, the range of values of x is $-9<x<9$.	1A accept $\lvert x\rvert<9$
(b) (i) By (a)(i), we have $\left(1+\frac{x}{9}\right)^{\frac{-1}{2}}=1-\frac{x}{18}+\frac{x^2}{216}-\cdots$.	
So, we have $(9+x)^{\frac{-1}{2}}=\frac{1}{3}-\frac{x}{54}+\frac{x^2}{648}-\cdots$.	1M for $\frac{1}{3}$(a)(i) pp−1 for omitting ' $\cdots$ '
(ii) By (a)(ii), the range of values of x is $-9<x<9$.	1M ----------(6)

Solution	Marks
2. (a) $S(9) = S(19)$	
$2(10^2)e^{-9\lambda} + 15 = 2(20^2)e^{-19\lambda} + 15$	
$e^{10\lambda} = 4$	
$\lambda = \dfrac{\ln 4}{10}$	1A
Thus, we have $\lambda = \dfrac{\ln 2}{5}$.	
(b) $S(t) = 2(t+1)^2 e^{-\lambda t} + 15$	
$\dfrac{dS(t)}{dt} = 2\left(2(t+1)e^{-\lambda t} - \lambda(t+1)^2 e^{-\lambda t}\right)$	1A
$= 2(t+1)(2-\lambda-\lambda t)e^{-\lambda t}$	
$\dfrac{dS(t)}{dt} = 0$ when $t = \dfrac{2-\lambda}{\lambda} = \dfrac{2-\frac{\ln 2}{5}}{\frac{\ln 2}{5}} = \dfrac{10-\ln 2}{\ln 2} = T \approx 13.42695041$	1M
$\dfrac{dS(t)}{dt}\begin{cases} >0 & \text{if} \quad 0 \le t < T \\ =0 & \text{if} \quad t = T \\ <0 & \text{if} \quad t > T \end{cases}$	1M for testing + 1A
Therefore, $S(t)$ attains its greatest value when $t = T$.	
The greatest value of $S(t)$	
$= 2\left(\dfrac{10-\ln 2}{\ln 2} + 1\right)^2 e^{\frac{\ln 2}{5} - 2} + 15$	
≈ 79.71368176	
< 90	
Thus, the temperature will not get higher than $90°C$.	1A f.t.

Solution	Marks
$S(t)=2(t+1)^2e^{-\lambda t}+15$	
$\dfrac{dS(t)}{dt}=2\left(2(t+1)e^{-\lambda t}-\lambda(t+1)^2e^{-\lambda t}\right)$	1A
$\quad =2(t+1)(2-\lambda-\lambda t)e^{-\lambda t}$	
$\dfrac{d^2S(t)}{dt^2}=2\left(2e^{-\lambda t}-2\lambda(t+1)e^{-\lambda t}-2\lambda(t+1)e^{-\lambda t}+\lambda^2(t+1)^2e^{-\lambda t}\right)$	
$\quad =2\left((\lambda^2-4\lambda+2)+(2\lambda^2-4\lambda)t+\lambda^2t^2\right)e^{-\lambda t}$	
$\dfrac{dS(t)}{dt}=0$ when $t=\dfrac{2-\lambda}{\lambda}=\dfrac{2-\frac{\ln 2}{5}}{\frac{\ln 2}{5}}=\dfrac{10-\ln 2}{\ln 2}=T\approx 13.42695041$	1M
$\left.\dfrac{d^2S(t)}{dt^2}\right\|_{t=T}=-4e^{-\lambda T}<0$	1M for testing + 1A
Note that there is only one local maximum.	
So, $S(t)$ attains its greatest value when $t=T$.	
The greatest value of $S(t)$	
$=2\left(\dfrac{10-\ln 2}{\ln 2}+1\right)^2e^{\frac{\ln 2}{5}-2}+15$	
≈ 79.71368176	
<90	
Thus, the temperature will not get higher than $90°C$.	1A f.t.
	----------(6)

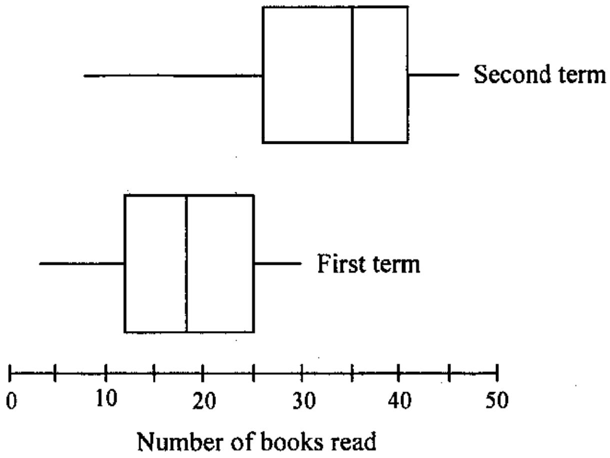
Solution	Marks
3. (a) The total amount	
$= \int_1^{11} f(t)\,dt$	1A can be absorbed
$\approx \frac{11-1}{10}\left(f(1)+f(11)+2\left(f(3)+f(5)+f(7)+f(9)\right)\right)$	1M for trapezoidal rule
≈ 22.57906572	
≈ 22.5791 litres	1A $a-1$ for r.t. 22.579
(b) $f(t)=\frac{500}{(t+2)^2 e^t}$	
$\frac{df(t)}{dt}=\frac{-500\left(2(t+2)e^t+(t+2)^2 e^t\right)}{(t+2)^4 e^{2t}}$	1M for quotient rule
$=\frac{-500(t+4)}{(t+2)^3 e^t}$	
$\frac{d^2f(t)}{dt^2}=-500\left(\frac{(t+2)^3 e^t-(t+4)\left(3(t+2)^2 e^t+(t+2)^3 e^t\right)}{(t+2)^6 e^{2t}}\right)$	
$=-500\left(\frac{t+2-(t+4)(t+5)}{(t+2)^4 e^t}\right)$	
$=500\left(\frac{t^2+8t+18}{(t+2)^4 e^t}\right)$	1A or equivalent
$f(t)=500(t+2)^{-2}e^{-t}$	
$\frac{df(t)}{dt}=500(-2)(t+2)^{-3}e^{-t}+500(t+2)^{-2}(-1)e^{-t}$	1M for product rule
$=-1000(t+2)^{-3}e^{-t}-500(t+2)^{-2}e^{-t}$	
$\frac{d^2f(t)}{dt^2}=3000(t+2)^{-4}e^{-t}+1000(t+2)^{-3}e^{-t}+1000(t+2)^{-3}e^{-t}+500(t+2)^{-2}e^{-t}$	
$=3000(t+2)^{-4}e^{-t}+2000(t+2)^{-3}e^{-t}+500(t+2)^{-2}e^{-t}$	1A or equivalent
(c) Note that $\frac{d^2f(t)}{dt^2}>0$ for all $1\le t\le 11$.	1M for considering the sign of $\frac{d^2f(t)}{dt^2}$
So, $f(t)$ is concave upward on $[1,11]$.	
Thus, the estimate in (a) is an over-estimate.	1A f.t.
	----------(7)

Solution	Marks

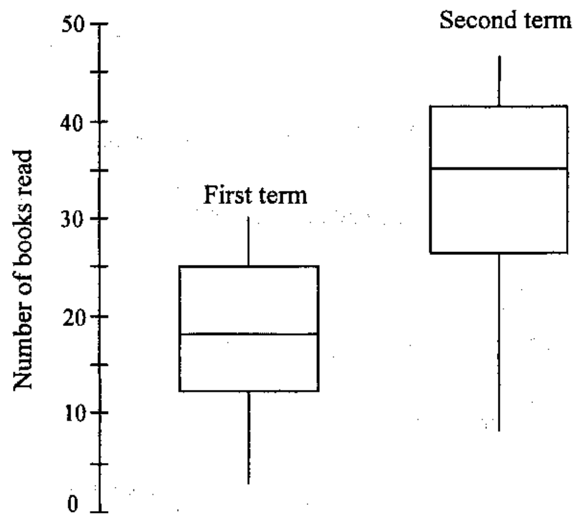
4. (a) The median
$= 18$ — 1A

The interquartile range
$= 25 - 12$
$= 13$ — 1A

(b) (i)



1A + 1A for correct box-and-whisker diagrams
1A for correct scale and same scale
pp−1 for incomplete specifications



1A + 1A for correct box-and-whisker diagrams
1A for correct scale and same scale
pp−1 for incomplete specifications

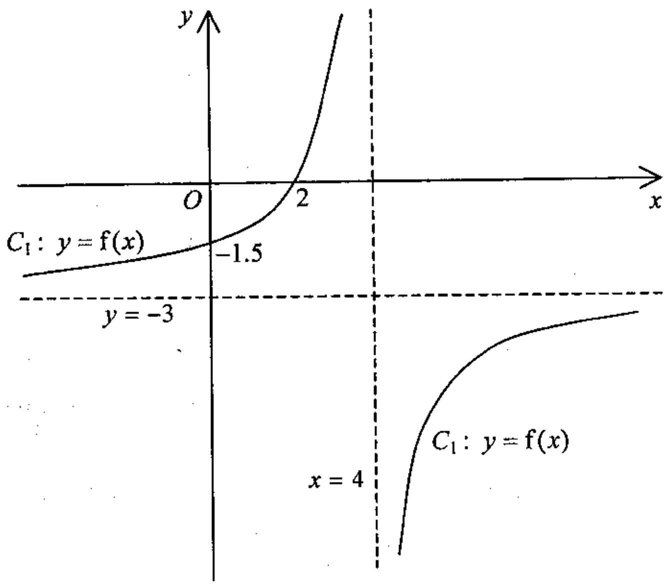
(ii) Note that the median (35) of the numbers of books read in the second term is greater than the maximum (30) of the numbers of books read in the first term and the difference between 35 and 30 is 5. — 1M for using the median in the 2nd term and the maximum in the 1st term

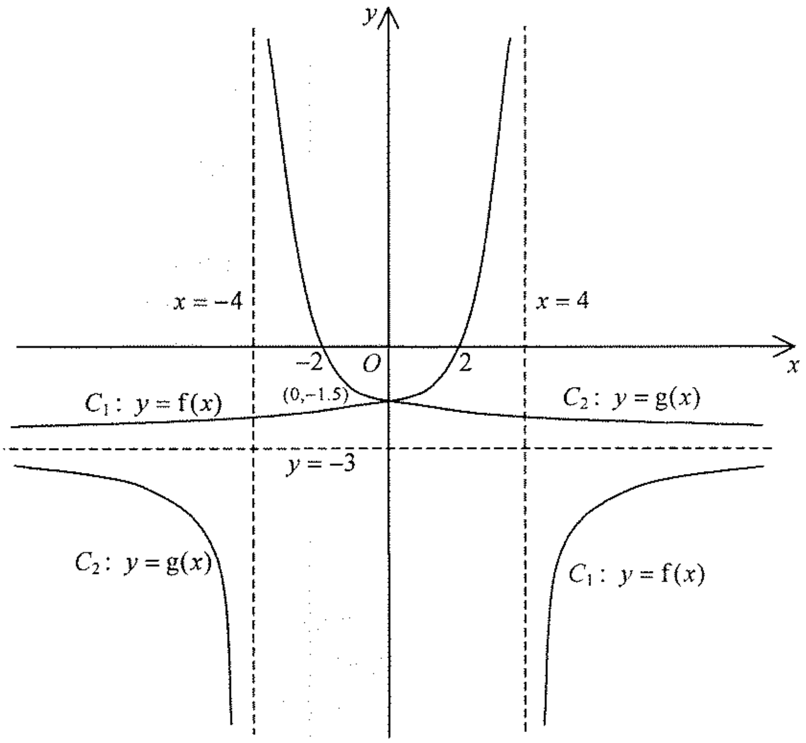
So, not less than 50% of these students read at least 5 more books in the second terms than that in the first term. — 1A f.t.

Thus, the claim is correct.

----------(7)

Solution	Marks
5. (a) $\mathrm{P}(A \mid B') = \dfrac{\mathrm{P}(A \cap B')}{\mathrm{P}(B')}$	
$0.5 = \dfrac{\mathrm{P}(A \cap B')}{1-b}$	1M
$\mathrm{P}(A \cap B') = 0.5(1-b)$	1A accept $0.5 - 0.5b$
$\mathrm{P}(A)$	
$= \mathrm{P}(A \cap B) + \mathrm{P}(A \cap B')$	
$= 0.2 + 0.5(1-b)$	1M
$= 0.7 - 0.5b$	1A
(b) $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$	
$0.2 = (0.7 - 0.5b)\,b$	1M for using (a) + 1M for using independence
$5b^2 - 7b + 2 = 0$	
$b = 0.4$ or $b = 1$ (rejected)	1A
Thus, we have $b = 0.4$.	
Since A and B are independent events, we have $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$.	
So, we have $\mathrm{P}(A \mid B')\mathrm{P}(B') = \mathrm{P}(A \cap B') = \mathrm{P}(A) - \mathrm{P}(A)\mathrm{P}(B) = \mathrm{P}(A)\mathrm{P}(B')$.	
Since $\mathrm{P}(B') \neq 0$, we have $\mathrm{P}(A \mid B') = \mathrm{P}(A)$.	
By (a), we have $0.5 = 0.7 - 0.5b$.	1M for using (a) + 1M for using independence
Therefore, we have $0.5b = 0.2$.	
Thus, we have $b = 0.4$.	1A
	-----------(7)
6. (a) For the normal distribution, the expected numbers of the students with test scores less than 50 are omitted.	1A do not accept rounding errors
For the Poisson distribution, the expected numbers of the students with merit points greater than 4 are omitted.	1A do not accept rounding errors
(b) The difference between the sum of the observed numbers of students and the sum of the expected numbers of students fitted by the normal distribution is $100 - 98.78 = 1.22$	1A can be absorbed
Let SE_1 be the sum of errors for model fitted by the normal distribution. Then,	
SE_1	
$= \lvert 20-14.65\rvert + \lvert 41-44.00\rvert + \lvert 28-33.45\rvert + \lvert 9-6.38\rvert + \lvert 2-0.30\rvert + \lvert 0-1.22\rvert$	1A + 1M (1A for the first 5 terms 1M for the last term)
$= 19.34$	1A
The difference between the sum of the observed numbers of students and the sum of the expected numbers of students fitted by the Poisson distribution is $100 - 98.58 = 1.42$	either one
Let SE_2 be the sum of errors for model fitted by the Poisson distribution. Then,	
SE_2	
$= \lvert 20-24.66\rvert + \lvert 41-34.52\rvert + \lvert 28-24.17\rvert + \lvert 9-11.28\rvert + \lvert 2-3.95\rvert + \lvert 0-1.42\rvert$	either one
$= 20.62$	both
Since $\mathrm{SE}_1 < \mathrm{SE}_2$, the normal distribution fits the observed data better.	1M
	----------(7)

Solution	Marks
7. (a) $\because$ the y-intercept of C_1 is $\frac{-3}{2}$.	
$\therefore \frac{a}{4}=\frac{-3}{2}$	
Thus, we have $a=-6$.	1A
$\because$ the x-intercept of C_2 is -2 .	
$\therefore \frac{a-(-2)b}{4+(-2)}=0$	
Thus, we have $b=3$.	1A ----------(2)
(b) (i) $\because \lim_{x\to 4^-}\frac{3x-6}{4-x}=+\infty$ and $\lim_{x\to 4^+}\frac{3x-6}{4-x}=-\infty$	
$\therefore$ the equation of the vertical asymptote to C_1 is $x=4$.	1A
$\because \lim_{x\to\pm\infty}\frac{3x-6}{4-x}=\lim_{x\to\pm\infty}\frac{3-\frac{6}{x}}{\frac{4}{x}-1}=-3$	
$\therefore$ the equation of the horizontal asymptote to C_1 is $y=-3$.	1A
(ii) 	1A for all the asymptotes of C_1 1A for all the intercepts of C_1 1A for the shape and position of C_1 ----------(5)
(c) The equation of the vertical asymptote to C_2 is $x=-4$. The equation of the horizontal asymptote to C_2 is $y=-3$. The x-intercept of C_2 is -2 . The y-intercept of C_2 is $\frac{-3}{2}$. The coordinates of the point of intersection of the two curves are $(0,-1.5)$.	

Solution	Marks
	1A for all the asymptotes of C_2 1M for the shape and position of C_2 1A for all the intercepts of C_2 1A for the intersection point ----------(4)
(d) The required area	
$= \int_{\frac{-7}{2}}^{0} (9-\mathrm{g}(x))\mathrm{d}x + \int_{0}^{\frac{7}{2}} (9-\mathrm{f}(x))\mathrm{d}x$	1M accept $\int_{-1.5}^{9} \frac{4y+6}{y+3}\mathrm{d}y - \int_{-1.5}^{9} \frac{-4y-6}{y+3}\mathrm{d}y$
$=2\int_{0}^{\frac{7}{2}} (9-\mathrm{f}(x))\mathrm{d}x$	
$=2\int_{0}^{\frac{7}{2}} \left(9-\frac{3x-6}{4-x}\right)\mathrm{d}x$	
$=2\int_{0}^{\frac{7}{2}} \left(12-\frac{6}{4-x}\right)\mathrm{d}x$	1M for division
$=12\Big[2x+\ln\lvert 4-x\rvert\Big]_{0}^{\frac{7}{2}}$	1A for correct integration
$=84+12\ln\frac{1}{2}-12\ln 4$	
$=84-12\ln 8$	1A
$=84-36\ln 2$	
≈ 59.0467015	
≈ 59.0467	a−1 for r.t. 59.047 ----------(4)

Solution	Marks
8. (a) $\dfrac{dv}{dt} = 2t - 6$	1A
x $= \displaystyle\int \frac{30t-90}{t^2-6t+11}\, dt$	
$= 15\displaystyle\int \frac{dv}{v}$	1A
$= 15\ln\lvert v\rvert + C$	
$= 15\ln(t^2-6t+11)+C \qquad (\because\ t^2-6t+11=(t-3)^2+2>0)$	
Using the condition that $x = 40$ when $t = 0$, we have $C = 40 - 15\ln 11$.	1M for finding C
Thus, we have $x = 15\ln(t^2-6t+11)+40-15\ln 11$.	1A --------(4)
(b) $15\ln(t^2-6t+11)+40-15\ln 11 = 40$ $15\ln(t^2-6t+11) = 15\ln 11$	
$t^2-6t+11 = 11$ $t(t-6)=0$ $t=6$ or $t=0$ (rejected)	1M
Therefore, we have $t=6$. Thus, 6 weeks after the start of the plan, the *weekly number of passengers* will be the same as at the start of the plan.	1A --------(2)
(c) $\dfrac{dx}{dt} = \dfrac{30(t-3)}{(t-3)^2+2}$	
$\begin{cases} <0 & \text{if } 0 \le t < 3 \\ =0 & \text{if } t = 3 \\ >0 & \text{if } t > 3 \end{cases}$	1M for testing + 1A
So, x attains its least value when $t=3$.	
The least *weekly number of passengers* $= 15\ln 2 + 40 - 15\ln 11$ $= 40 - 15\ln\dfrac{11}{2}$ ≈ 14.42877862 ≈ 14 thousand	1A

Solution	Marks
$\dfrac{d^2x}{dt^2} = \dfrac{-30(t^2-6t+7)}{(t^2-6t+11)^2}$	
Note that $\dfrac{dx}{dt} = 0$ when $t = 3$.	
$\left.\dfrac{d^2x}{dt^2}\right\|_{t=3} = 15 > 0$	1M for testing + 1A
Note that there is only one local minimum.	
So, x attains its least value when $t = 3$.	
The least *weekly number of passengers*	
$= 15\ln 2 + 40 - 15\ln 11$	
$= 40 - 15\ln\dfrac{11}{2}$	
≈ 14.42877862	
≈ 14 thousand	1A
	--------(3)
(d) By (c), note that the end of the *Recovery Week* corresponds to $t = 3$.	
(i) The required change	
$= x(4) - x(3)$	
$= 15\ln(4^2 - 24 + 11) - 15\ln(3^2 - 18 + 11)$	1M
$= 15(\ln 3 - \ln 2)$	
$= 15\ln\dfrac{3}{2}$	
≈ 6.081976622	
≈ 6 thousand	1A
The required change	
$= \displaystyle\int_3^4 \frac{30t-90}{t^2-6t+11}\,dt$	
$= 15\Big[\ln(t^2-6t+11)\Big]_3^4$	
$= 15\ln(4^2 - 24 + 11) - 15\ln(3^2 - 18 + 11)$	1M
$= 15(\ln 3 - \ln 2)$	
$= 15\ln\dfrac{3}{2}$	
≈ 6.081976622	
≈ 6 thousand	1A
(ii) $(t+1)^2 - 6(t+1) + 11 - 3(t^2 - 6t + 11)$	
$= -2t^2 + 14t - 27$	
$= -2\left(t - \dfrac{7}{2}\right)^2 - \dfrac{5}{2}$	1M accept using discriminant < 0
< 0	1
Thus, we have $(t+1)^2 - 6(t+1) + 11 < 3(t^2 - 6t + 11)$ for all t .	

Solution	Marks
Note that $t^2-6t+11=(t-3)^2+2>0$. $\dfrac{(t+1)^2-6(t+1)+11}{t^2-6t+11}-3$ $=\dfrac{-2t^2+14t-27}{t^2-6t+11}$ $=\dfrac{-2\left(t-\frac{7}{2}\right)^2-\frac{5}{2}}{(t-3)^2+2}$ <0 Thus, we have $(t+1)^2-6(t+1)+11<3(t^2-6t+11)$ for all t.	 1M accept using discriminant <0 1
Let $\mathrm{f}(t)=(t+1)^2-6(t+1)+11-3(t^2-6t+11)$ for all $t\geq 0$. $\dfrac{\mathrm{d}\,\mathrm{f}(t)}{\mathrm{d}t}=-4t+14$ $\dfrac{\mathrm{d}\,\mathrm{f}(t)}{\mathrm{d}t}\begin{cases}>0 & \text{if } 0\leq t<\frac{7}{2}\\ =0 & \text{if } t=\frac{7}{2}\\ <0 & \text{if } t>\frac{7}{2}\end{cases}$ So, $\mathrm{f}(t)$ attains its greatest value when $t=\dfrac{7}{2}$. The greatest value of $\mathrm{f}(t)$ $=\dfrac{-5}{2}$ <0 Thus, we have $(t+1)^2-6(t+1)+11<3(t^2-6t+11)$ for all t.	 1M for testing 1
(iii) $x(t+1)-x(t)$ $=15\ln\left((t+1)^2-6(t+1)+11\right)-15\ln(t^2-6t+11)$ $=15\ln\left(\dfrac{(t+1)^2-6(t+1)+11}{t^2-6t+11}\right)$ $<15\ln 3$ (by (d)(ii) and $t^2-6t+11>0$) <25 Thus, the claim is incorrect.	 1M for using (d)(ii) and taking ln 1A f.t.
By (d)(ii), we have $(t+1)^2-6(t+1)+11<3(t^2-6t+11)$ Note that $(t+1)^2-6(t+1)+11>0$ and $3(t^2-6t+11)>0$. $\ln\left((t+1)^2-6(t+1)+11\right)<\ln 3+\ln(t^2-6t+11)$ $15\ln\left((t+1)^2-6(t+1)+11\right)-15\ln(t^2-6t+11)<15\ln 3$ $x(t+1)-x(t)<25$ Thus, the claim is incorrect.	 1M for using (d)(ii) and taking ln 1A f.t.
	--------(6)

Solution	Marks
9. (a) Let $u=t+10$. Then, we have $\frac{du}{dt}=1$.	1A can be absorbed
The total amount	
$=\int_0^3 f(t)\,dt$	
$=\int_0^3 25t^2(t+10)^{\frac{-1}{3}}\,dt$	
$=\int_{10}^{13} 25(u-10)^2 u^{\frac{-1}{3}}\,du$	1A
$=25\int_{10}^{13}(u^{\frac{5}{3}}-20u^{\frac{2}{3}}+100u^{\frac{-1}{3}})\,du$	1M
$=25\left[\frac{3}{8}u^{\frac{8}{3}}-12u^{\frac{5}{3}}+150u^{\frac{2}{3}}\right]_{10}^{13}$	1M
≈ 97.65521668	
≈ 97.6552 thousand metres	1A $a-1$ for r.t. 97.655 --------(5)
(b) $\ln\left(g(t)-28\right)=\ln k+ht^2$	1A --------(1)
(c) $h\approx 0.3$ (correct to 1 decimal place)	1A
$\ln k\approx 1.0$	
$k\approx 2.718281828$	
$k\approx 2.7$ (correct to 1 decimal place)	1A --------(2)
(d) (i) $g(t)\approx 28+2.7e^{0.3t^2}$	
$=28+2.7\left(1+0.3t^2+\frac{(0.3t^2)^2}{2!}+\frac{(0.3t^2)^3}{3!}+\cdots\right)$	1M
$=30.7+0.81t^2+0.1215t^4+0.01215t^6+\cdots$	1A pp−1 for omitting ‘ $\cdots$ ’
The total amount	
$=\int_0^3 g(t)\,dt$	
$\approx\int_0^3(30.7+0.81t^2+0.1215t^4+0.01215t^6)\,dt$	1M
$=\left[30.7t+\frac{0.81t^3}{3}+\frac{0.1215t^5}{5}+\frac{0.01215t^7}{7}\right]_0^3$	
≈ 109.0909071	
≈ 109.0909 thousand metres	1A $a-1$ for r.t. 109.091
(ii) $e^{0.3t^2}=1+0.3t^2+\frac{(0.3t^2)^2}{2!}+\frac{(0.3t^2)^3}{3!}+r(t)$ and $r(t)>0$ for all $t>0$.	
Thus, the estimate in (d)(i) is an under-estimate.	1A f.t.
(iii) Note that the estimate in (d)(i) is greater than the total amount of cloth production under John's model and that the estimate in (d)(i) is an under-estimate.	1M for using (a), (d)(i) and (d)(ii)
Thus, the total amount of cloth production under Mary's model is greater than that under John's model.	1A f.t. --------(7)

Solution	Marks
10. (a) The required probability	
$=\frac{4.7^0 e^{-4.7}}{0!}+\frac{4.7^1 e^{-4.7}}{1!}+\frac{4.7^2 e^{-4.7}}{2!}+\frac{4.7^3 e^{-4.7}}{3!}+\frac{4.7^4 e^{-4.7}}{4!}+\frac{4.7^5 e^{-4.7}}{5!}$	1M for the 6 cases + 1M for Poisson probability
≈ 0.668438485	
≈ 0.6684	1A a–1 for r.t. 0.668 ----------(3)
(b) Let X km/h be the speed of a car entering the roundabout.	
Then, $X \sim \mathrm{N}(42.8\,,12^2)$.	
The required probability	
$=\mathrm{P}(X>50)$	
$=\mathrm{P}(Z>\frac{50-42.8}{12})$	1M (accept $\mathrm{P}(Z\geq\frac{50-42.8}{12})$)
$=\mathrm{P}(Z>0.6)$	
$=0.2743$	1A a–1 for r.t. 0.274 ----------(2)
(c) The required probability	
$=(1-0.2743)^5(0.2743)$	1M for $(1-p)^5 p$ + 1M for $p=$ (b)
≈ 0.055209196	
≈ 0.0552	1A a–1 for r.t. 0.055 ----------(3)
(d) (i) The required probability	
$=C_1^4(0.2743)^3(1-0.2743)+(0.2743)^4$	1M for the 2 cases + 1M for binomial probability
≈ 0.065570471	
≈ 0.0656	1A a–1 for r.t. 0.066
(ii) The required probability	
$\approx \dfrac{0.065570471\left(\frac{(4.7)^4 e^{-4.7}}{4!}\right)+\left((0.2743)^5+C_1^5(0.2743)^4(1-0.2743)+C_2^5(0.2743)^3(1-0.2743)^2\right)\left(\frac{(4.7)^5 e^{-4.7}}{5!}\right)}{0.668438485}$	1M + 1M for numerator + 1M for denominator using (a)
≈ 0.052151265	
≈ 0.0522	1A a–1 for r.t. 0.052 ----------(7)

Solution	Marks
11. (a) The required probability $=1-\left((0.8)^5+C_1^5(0.8)^4(0.2)\right)$ $=\frac{821}{3125}$ $=0.26272$ ≈ 0.2627	1M for cases correct + 1M for binomial probability 1A $a-1$ for r.t. 0.263
The required probability $=(0.2)^5+C_1^5(0.2)^4(0.8)+C_2^5(0.2)^3(0.8)^2+C_3^5(0.2)^2(0.8)^3$ $=\frac{821}{3125}$ $=0.26272$ ≈ 0.2627	1M for the 4 cases + 1M for binomial probability 1A $a-1$ for r.t. 0.263
	----------(3)
(b) (i) The required probability $=(0.8)^6(0.2)$ $=\frac{4096}{78125}$ $=0.0524288$ ≈ 0.0524	1M for $p^6(1-p)$, where $0<p<1$ 1A $a-1$ for r.t. 0.052
(ii) The required probability $=\left(C_2^6(0.8)^4(0.2)^2\right)(0.8)+\left(C_2^6(0.8)^4(0.2)^2\right)(0.2)+\left(C_1^6(0.8)^5(0.2)\right)(0.2)$ $=\frac{25344}{78125}$ $=0.3244032$ ≈ 0.3244	1M for the 3 cases + 1M for binomial probability 1A $a-1$ for r.t. 0.324
The required probability $=C_2^6(0.8)^4(0.2)^2+\left(C_1^6(0.8)^5(0.2)\right)(0.2)$ $=\frac{25344}{78125}$ $=0.3244032$ ≈ 0.3244	1M for the 2 cases + 1M for binomial probability 1A $a-1$ for r.t. 0.324
The required probability $=C_2^7(0.8)^5(0.2)^2+\left(C_2^6(0.8)^4(0.2)^2\right)(0.2)$ $=\frac{25344}{78125}$ $=0.3244032$ ≈ 0.3244	1M for the 2 cases + 1M for binomial probability 1A $a-1$ for r.t. 0.324
(iii) The required probability $=\frac{\left(C_2^6(0.8)^4(0.2)^2\right)(0.2)+\left(C_1^6(0.8)^5(0.2)\right)(0.2)}{0.3244032}$ $=\frac{13}{33}$ ≈ 0.3939393939 ≈ 0.3939	1A for numerator 1M for denominator using (b)(ii) 1A $a-1$ for r.t. 0.394

Solution	Marks
The required probability	
$=1-\dfrac{\left(C_2^6(0.8)^4(0.2)^2\right)(0.8)}{0.3244032}$	1A for numerator 1M for denominator using (b)(ii)
$=\dfrac{13}{33}$	1A
≈ 0.3939393939	
≈ 0.3939	a–1 for r.t. 0.394
(iv) The required probability	
$=\dfrac{(0.8)^5(0.2)^2+C_1^5(0.8)^4(0.2)\left((0.2)^2+C_1^2(0.8)(0.2)\right)}{1-0.26272}$	1M (one term) + 1A for numerator 1M for denominator using (a)
$=\dfrac{49}{225}$	1A
≈ 0.2177777777	
≈ 0.2178	a–1 for r.t. 0.218 ----------(12)

Solution	Marks
12. (a) The required probability	
$= 1 - \left(\frac{2.6^0 e^{-2.6}}{0!} + \frac{2.6^1 e^{-2.6}}{1!} + \frac{2.6^2 e^{-2.6}}{2!} + \frac{2.6^3 e^{-2.6}}{3!} \right)$	1M for cases correct + 1M for Poisson probability
≈ 0.263998355	
≈ 0.2640	1A a–1 for r.t. 0.264 ----------(3)
Let p be the probability described in (a).	
(b) (i) The required probability	
$= p + (1-p)p + (1-p)^2 p + (1-p)^3 p$	1M for the 4 cases + 1M for geometric probability
$= 1 - (1-p)^4$	
$\approx 1 - (1 - 0.263998355)^4$	
0.70656282	
≈ 0.7066	1A a–1 for r.t. 0.707
(ii) The required probability	
$\approx \frac{(1-0.263998355)^2(0.263998355) + (1-0.263998355)^3(0.263998355)}{0.70656282}$	1M for numerator using (a) 1M for denominator using (b)(i)
≈ 0.351364771	
≈ 0.3514	1A (accept 0.3513) a–1 for r.t. 0.351
(iii) The integer m satisfies $\mathrm{P}(M \le m) > 0.95$.	
$p + (1-p)p + (1-p)^2 p + \cdots + (1-p)^{m-1} p > 0.95$	1M withhold 1M for bearing an equality sign
$1 - (1-p)^m > 0.95$	
$(1-p)^m < 0.05$	
$(1 - 0.263998355)^m < 0.05$	
$m \ln(0.736001645) < \ln(0.05)$	1M for using log or trial and error
$m > 9.773273146$	
Thus, the least value of m is 10 .	1A ----------(9)
(c) Note that $N \sim \mathrm{B}(150, p)$.	
The mean of N	
$= 150p$	
$\approx (150)(0.263998355)$	1M (either one)
≈ 39.59975325	
≈ 39.5998	1A (accept 39.6) a–1 for r.t. 39.600
The variance of N	
$= 150p(1-p)$	
$\approx (150)(0.263998355)(1 - 0.263998355)$	
≈ 29.14548353	
≈ 29.1455	1A (accept 29.1456) a–1 for r.t. 29.145 ----------(3)