評卷參考 *
Marking Scheme

香港考試及評核局
Hong Kong Examinations and Assessment Authority

2006年香港高級程度會考
Hong Kong Advanced Level Examination 2006

數學及統計學 高級補充程度
Mathematics and Statistics   AS-Level

*此部分只設英文版本

This document was prepared for markers’ reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.
AS Mathematics and Statistics

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits all the marks allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

   'M' marks awarded for correct methods being used;
   'A' marks awarded for the accuracy of the answers;
   Marks without 'M' or 'A' awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates’ work. In general, marks for a certain step should be awarded if candidates’ solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

5. In marking candidates’ work, the benefit of doubt should be given in the candidates’ favour.

6. Marks may be deducted for poor presentation (pp). The symbol \( pp- \) should be used to denote 1 mark deducted for \( pp \). At most deducted 1 mark from Section A and 1 mark from Section B for \( pp \). In any case, do not deduct any marks for \( pp \) in those steps where candidates could not score any marks.

7. Marks may be deducted for numerical answers with inappropriate degree of accuracy (a). The symbol \( a- \) should be used to denote 1 mark deducted for \( a \). At most deducted 1 mark from Section A and 1 mark from Section B for \( a \). In any case, do not deduct any marks for \( a \) in those steps where candidates could not score any marks.

8. In the marking scheme, ‘r.t.’ stands for ‘accepting answers which can be rounded off to’ and ‘f.t.’ stands for ‘follow through’. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles.

2006-AS-M & S-2
1. (a) (i) \( \left( 1 + \frac{x}{a} \right)^{-\frac{1}{n}} \)

\[
= 1 - \frac{x}{na} + \frac{1}{2} \left( \frac{-1}{n} \right) \left( \frac{-1}{n} - 1 \right) \left( \frac{x}{a} \right)^2 - \ldots
\]

\[
= 1 - \frac{x}{na} + \frac{1 + n}{2n^2a^2} x^2 - \ldots
\]

So, we have \( \frac{-1}{2n} = \frac{-1}{18} \) and \( \frac{1 + n}{2n^2a^2} = \frac{1}{24a} \).

Solving, we have \( a = 9 \) and \( n = 2 \).

(ii) The binomial expansion is valid for \( \left| \frac{x}{9} \right| < 1 \).

Thus, the range of values of \( x \) is \( -9 < x < 9 \).

(b) (i) By (a)(i), we have \( \left( 1 + \frac{x}{9} \right)^{-\frac{1}{2}} = 1 - \frac{x}{18} + \frac{x^2}{216} - \ldots \).

So, we have \( (9 + x)^{\frac{1}{2}} = \frac{1}{3} - \frac{x}{54} + \frac{x^2}{648} - \ldots \).

(iii) By (a)(ii), the range of values of \( x \) is \( -9 < x < 9 \).
2. (a) \( S(9) = S(19) \)

\[
2(10^2)e^{-9\lambda} + 15 = 2(20^2)e^{-19\lambda} + 15
\]

\( e^{10\lambda} = 4 \)

\( \lambda = \frac{\ln 4}{10} \)

Thus, we have \( \lambda = \frac{\ln 2}{5} \).

(b) \( S(t) = 2(t+1)^2e^{-\lambda t} + 15 \)

\[
\frac{dS(t)}{dt} = 2 \left( 2(t+1)e^{-\lambda t} - \lambda(t+1)^2e^{-\lambda t} \right)
\]

\[
= 2(t+1)(2-\lambda - \lambda t)e^{-\lambda t}
\]

\[
\frac{dS(t)}{dt} = 0 \text{ when } t = \frac{2-\lambda}{\lambda} = \frac{2 - \frac{\ln 2}{5}}{\frac{\ln 2}{5}} = \frac{10 - \ln 2}{\ln 2} = T \approx 13.42695041
\]

\[
\frac{dS(t)}{dt} = \begin{cases} 
> 0 & \text{if } 0 \leq t < T \\
= 0 & \text{if } t = T \\
< 0 & \text{if } t > T 
\end{cases}
\]

Therefore, \( S(t) \) attains its greatest value when \( t = T \).

The greatest value of \( S(t) \)

\[
= 2 \left( \frac{10 - \ln 2}{\ln 2} + 1 \right) \left( \frac{\ln 2}{5} \right)^2 + 15
\]

\( \approx 79.71368176 \)

\( < 90 \)

Thus, the temperature will not get higher than 90°C.
\[ S(t) = 2(t+1)^2 e^{-\lambda t} + 15 \]
\[
\frac{dS(t)}{dt} = 2\left(2(t+1)e^{-\lambda t} - \lambda (t+1)^2 e^{-\lambda t}\right)
= 2(t+1)(2 - \lambda - \lambda t)e^{-\lambda t}
\]
\[
\frac{d^2S(t)}{dt^2} = 2\left(2e^{-\lambda t} - 2\lambda (t+1) e^{-\lambda t} - 2\lambda (t+1) e^{-\lambda t} + \lambda^2 (t+1)^2 e^{-\lambda t}\right)
= 2\left(\lambda^2 - 4\lambda + 2\right) + (2\lambda^2 - 4\lambda + \lambda^2 + \lambda^2) e^{-\lambda t}
\]
\[
\frac{dS(t)}{dr} = 0 \text{ when } r = \frac{2 - \lambda}{\lambda} = \frac{2 - \frac{\ln 2}{5}}{\frac{\ln 2}{5}} = \frac{10 - \ln 2}{\ln 2} = T \approx 13.42698041
\]
\[
\frac{d^2S(t)}{dr^2}\bigg|_{r=T} = -4 e^{-\lambda T} < 0
\]

Note that there is only one local maximum.
So, \( S(t) \) attains its greatest value when \( t = T \).

The greatest value of \( S(t) \)
\[
= 2\left(10 - \ln 2 + 1\right)^2 e^{\frac{\ln 2}{5} - 2} + 15
\]
\[
\approx 79.71368176
\]
\[
< 90
\]
Thus, the temperature will not get higher than 90°C.

1A ft.

---------(6)
3. (a) The total amount

\[ \int_{0}^{11} f(t) \, dt \]

\[ \approx \frac{11 - 1}{10} \left( f(1) + f(11) + 2(f(3) + f(5) + f(7) + f(9)) \right) \]

\[ \approx 22.57906572 \]

\[ \approx 22.5791 \text{ litres} \]

(b) \[ f(t) = \frac{500}{(t + 2)^2 e^t} \]

\[ \frac{df(t)}{dt} = -500 \left( \frac{2(t + 2)e^t + (t + 2)^2 e'}{(t + 2)^3 e^t} \right) \]

\[ = -500 \left( \frac{t + 4}{(t + 2)^3 e^t} \right) \]

\[ \frac{d^2 f(t)}{dt^2} = -500 \left( \frac{(t + 2)^3 e^t - (t + 4)(t + 5)(t + 2)^2 e' + (t + 2)^3 e'}{(t + 2)^6 e^{2t}} \right) \]

\[ = -500 \left( \frac{t^2 + 8t + 18}{(t + 2)^5 e^t} \right) \]

\[ f(t) = 500(t + 2)^{-2} e^{-t} \]

\[ \frac{df(t)}{dt} = 500(-2)(t + 2)^{-3} e^{-t} + 500(t + 2)^{-2}(-1)e^{-t} \]

\[ = -1000(t + 2)^{-3} e^{-t} - 500(t + 2)^{-2} e^{-t} \]

\[ \frac{d^2 f(t)}{dt^2} = 3000(t + 2)^{-4} e^{-t} + 1000(t + 2)^{-3} e^{-t'} + 1000(t + 2)^{-3} e^{-t} + 500(t + 2)^{-2} e^{-t} \]

\[ = 3000(t + 2)^{-4} e^{-t} + 2000(t + 2)^{-3} e^{-t} + 500(t + 2)^{-2} e^{-t} \]

(c) Note that \[ \frac{d^2 f(t)}{dt^2} > 0 \] for all \( 1 \leq t \leq 11 \).

So, \( f(t) \) is concave upward on \( [1, 11] \).

Thus, the estimate in (a) is an over-estimate.

1A can be absorbed

1M for trapezoidal rule

1A \( a-1 \) for r.t. 22.579

1M for quotient rule

1A or equivalent

1M for product rule

1A or equivalent

1M for considering the sign of \( \frac{d^2 f(t)}{dt^2} \)

1A f.t.

\[ \text{------(7)} \]
4. (a) The median
    \[= 18\]
    The interquartile range
    \[= 25 - 12\]
    \[= 13\]

(b) (i) 

[Box-and-whisker diagrams showing number of books read]

(ii) Note that the median (35) of the numbers of books read in the second term is greater than the maximum (30) of the numbers of books read in the first term and the difference between 35 and 30 is 5. So, not less than 50% of these students read at least 5 more books in the second terms than that in the first term. Thus, the claim is correct.
<table>
<thead>
<tr>
<th>Solution</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 5. (a) \[ P(A | B') = \frac{P(A \cap B')}{P(B')} \]  
\[ 0.5 = \frac{P(A \cap B')}{1 - b} \]  
\[ P(A \cap B') = 0.5(1 - b) \]  
\[ P(A) = P(A \cap B) + P(A \cap B') \]  
\[ = 0.2 + 0.5(1 - b) \]  
\[ = 0.7 - 0.5b \]  
(b) \[ P(A \cap B) = P(A)P(B) \]  
\[ 0.2 = (0.7 - 0.5b)b \]  
\[ 5b^2 - 7b + 2 = 0 \]  
\[ b = 0.4 \text{ or } b = 1 \text{ (rejected)} \]  
Thus, we have \( b = 0.4 \).  
Since \( A \) and \( B \) are independent events, we have \( P(A \cap B) = P(A)P(B) \).  
So, we have \( P(A | B')P(B') = P(A \cap B') = P(A) - P(A)P(B) = P(A)P(B') \).  
Since \( P(B') \neq 0 \), we have \( P(A | B') = P(A) \).  
By (a), we have \( 0.5 = 0.7 - 0.5b \).  
Therefore, we have \( 0.5b = 0.2 \).  
Thus, we have \( b = 0.4 \).  
1M for using (a) + 1M for using independence  
1A |
| 6. (a) For the normal distribution, the expected numbers of the students with test scores less than 50 are omitted.  
For the Poisson distribution, the expected numbers of the students with merit points greater than 4 are omitted.  
1A do not accept rounding errors  
1A do not accept rounding errors  
(b) The difference between the sum of the observed numbers of students and the sum of the expected numbers of students fitted by the normal distribution is \( 100 - 98.78 = 1.22 \).  
Let \( SE_1 \) be the sum of errors for model fitted by the normal distribution. Then,  
\[ SE_1 = |20 - 14.65| + |41 - 44.00| + |28 - 33.45| + |9 - 6.38| + |2 - 0.30| + 0 - 1.22 | \]  
\[ = 19.34 \]  
The difference between the sum of the observed numbers of students and the sum of the expected numbers of students fitted by the Poisson distribution is \( 100 - 98.58 = 1.42 \).  
Let \( SE_2 \) be the sum of errors for model fitted by the Poisson distribution. Then,  
\[ SE_2 = |20 - 24.66| + |41 - 34.52| + |28 - 24.17| + |9 - 11.28| + |2 - 3.95| + 0 - 1.42 | \]  
\[ = 20.62 \]  
Since \( SE_1 < SE_2 \), the normal distribution fits the observed data better.  
1A can be absorbed  
1A + 1M (1A for the first 5 terms)  
1M for the last term  
1A  
---------(7)
7. (a) \( \therefore \) the \( y \)-intercept of \( C_1 \) is \( \frac{-3}{2} \).

\( \therefore a = \frac{-3}{2} \)

Thus, we have \( a = -6 \).

\( \therefore \) the \( x \)-intercept of \( C_2 \) is \(-2\).

\( \therefore \frac{a - (-2)b}{4 + (-2)} = 0 \)

Thus, we have \( b = 3 \).

(b) (i) \( \therefore \) \( \lim_{x \to 4^-} \frac{3x - 6}{4 - x} = -\infty \) and \( \lim_{x \to 4^+} \frac{3x - 6}{4 - x} = -\infty \)

\( \therefore \) the equation of the vertical asymptote to \( C_1 \) is \( x = 4 \).

\( \therefore \) \( \lim_{x \to 2^\pm} \frac{3x - 6}{4 - x} = \lim_{x \to 2^\pm} \frac{3 - 6}{x - 1} = -3 \)

\( \therefore \) the equation of the horizontal asymptote to \( C_1 \) is \( y = -3 \).

(ii)

\( (c) \) The equation of the vertical asymptote to \( C_2 \) is \( x = -4 \).

The equation of the horizontal asymptote to \( C_2 \) is \( y = -3 \).

The \( x \)-intercept of \( C_2 \) is \(-2 \).

The \( y \)-intercept of \( C_2 \) is \( \frac{-3}{2} \).

The coordinates of the point of intersection of the two curves are \((0, -1.5)\).
(d) The required area

\[
\int_{-\infty}^{0} (9 - g(x)) \, dx + \int_{0}^{\frac{7}{2}} (9 - f(x)) \, dx
\]

\[
= \int_{0}^{\frac{7}{2}} (9 - f(x)) \, dx
\]

\[
= \int_{0}^{\frac{7}{2}} \left( 9 - \frac{3x - 6}{4 - x} \right) \, dx
\]

\[
= \int_{0}^{\frac{7}{2}} \left( 12 - \frac{6}{4 - x} \right) \, dx
\]

\[
= 12 \left[ 2x + \ln|4 - x| \right]_{0}^{\frac{7}{2}}
\]

\[
= 84 + 12 \ln \frac{1}{2} - 12 \ln 4
\]

\[
= 84 - 12 \ln 8 - 84 - 36 \ln 2
\]

\[
\approx 59.0467015
\]

\[
\approx 59.046
\]

1A for all the asymptotes of \( C_2 \)
1M for the shape and position of \( C_2 \)
1A for all the intercepts of \( C_2 \)
1A for the intersection point

\[\text{---------}(4)\]

1M for division
1A for correct integration
1A

\[\alpha - 1 \text{ for } r.t. 59.047\]

\[\text{---------}(4)\]

2006-AS-M & S-10
8. (a) \( \frac{dv}{dt} = 2t - 6 \)

\[ x = \int \frac{30t - 90}{t^2 - 6t + 11} \, dt \]

\[ = 15 \int \frac{dv}{v} + C \]

\[ = 15 \ln |v| + C \quad (\because t^2 - 6t + 11 = (t - 3)^2 + 2 > 0) \]

Using the condition that \( x = 40 \) when \( t = 0 \), we have \( C = 40 - 15 \ln 11 \).

Thus, we have \( x = 15 \ln (t^2 - 6t + 11) + 40 - 15 \ln 11 \).

(b) \( 15 \ln (t^2 - 6t + 11) + 40 - 15 \ln 11 = 40 \)

\[ 15 \ln (t^2 - 6t + 11) = 15 \ln 11 \]

\[ t^2 - 6t + 11 = 11 \]

\[ t(t - 6) = 0 \]

\( t = 6 \) or \( t = 0 \) (rejected)

Therefore, we have \( t = 6 \).

Thus, 6 weeks after the start of the plan, the weekly number of passengers will be the same as at the start of the plan.

(c) \( \frac{dx}{dt} = \frac{30(t - 3)}{(t - 3)^2 + 2} \)

\[ = 0 \quad \text{if} \quad t = 3 \]

\[ > 0 \quad \text{if} \quad t > 3 \]

\[ < 0 \quad \text{if} \quad 0 \leq t < 3 \]

So, \( x \) attains its least value when \( t = 3 \).

The least weekly number of passengers

\[ = 15 \ln 2 + 40 - 15 \ln 11 \]

\[ = 40 - 15 \ln \frac{11}{2} \]

\[ \approx 14,428,778.62 \]

\[ \approx 14 \text{ thousand} \]
\[
\frac{d^2x}{dt^2} = \frac{-30(t^2 - 6t + 7)}{(t^2 - 6t + 11)^2}
\]

Note that \(\frac{dx}{dt} = 0\) when \(t = 3\).

\[
\frac{d^2x}{dt^2} \bigg|_{t=3} = 15 > 0
\]

Note that there is only one local minimum.

So, \(x\) attains its least value when \(t = 3\).

The least weekly number of passengers

\[
= 15 \ln 2 + 40 - 15 \ln 11
\]

\[
= 40 - 15 \ln \frac{11}{2}
\]

\[
\approx 14.42877862
\]

\(
\approx 14\text{ thousand}
\)

1M for testing + 1A

(d) By (c), note that the end of the Recovery Week corresponds to \(t = 3\).

(i) The required change

\[
x(4) - x(3)
\]

\[
= 15 \ln(4^2 - 24 + 11) - 15 \ln(3^2 - 18 + 11)
\]

\[
= 15(\ln 3 - \ln 2)
\]

\[
= 15 \ln \frac{3}{2}
\]

\[
\approx 6.081976622
\]

\(
\approx 6\text{ thousand}
\)

1M

1A

The required change

\[
= \int_{3}^{4} \frac{30t - 90}{t^2 - 6t + 11} \, dt
\]

\[
= 15 \left[ \ln(t^2 - 6t + 11) \right]_{3}^{4}
\]

\[
= 15 \ln(4^2 - 24 + 11) - 15 \ln(3^2 - 18 + 11)
\]

\[
= 15(\ln 3 - \ln 2)
\]

\[
= 15 \ln \frac{3}{2}
\]

\[
\approx 6.081976622
\]

\(
\approx 6\text{ thousand}
\)

1M

1A

(ii) \((t + 1)^2 - 6(t + 1) + 11 - 3(t^2 - 6t + 11) \)

\[
= -2t^2 + 14t - 27
\]

\[
= -2 \left( \frac{t - \frac{7}{2}}{2} \right)^2 - \frac{5}{2}
\]

\(< 0
\]

Thus, we have \((t + 1)^2 - 6(t + 1) + 11 < 3(t^2 - 6t + 11)\) for all \(t\).

1M accept using discriminant < 0

1
<table>
<thead>
<tr>
<th>Solution</th>
<th>Marks</th>
</tr>
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<tbody>
<tr>
<td>Note that $t^2 - 6t + 11 = (t - 3)^2 + 2 &gt; 0$.</td>
<td></td>
</tr>
<tr>
<td>$(t+1)^2 - 6(t+1) + 11 - 3$</td>
<td></td>
</tr>
<tr>
<td>$= \frac{-2t^2 + 14t - 27}{t^2 - 6t + 11}$</td>
<td></td>
</tr>
<tr>
<td>$= \frac{-2\left(t - \frac{7}{2}\right)^2 - \frac{5}{2}}{(t - 3)^2 + 2}$</td>
<td></td>
</tr>
<tr>
<td>$&lt; 0$</td>
<td>1M accept using discriminant $&lt; 0$</td>
</tr>
<tr>
<td>Thus, we have $(t + 1)^2 - 6(t + 1) + 11 &lt; 3(t^2 - 6t + 11)$ for all $t$</td>
<td>1</td>
</tr>
</tbody>
</table>

Let $f(t) = (t + 1)^2 - 6(t + 1) + 11 - 3(t^2 - 6t + 11)$ for all $t \geq 0$.

\[
\frac{df(t)}{dt} = -4t + 14 \\
\begin{cases} 
> 0 & \text{if } 0 \leq t < \frac{7}{2} \\
= 0 & \text{if } t = \frac{7}{2} \\
< 0 & \text{if } t > \frac{7}{2}
\end{cases}
\]

1M for testing

So, $f(t)$ attains its greatest value when $t = \frac{7}{2}$.

The greatest value of $f(t)$

$= -\frac{5}{2}$

$< 0$

Thus, we have $(t + 1)^2 - 6(t + 1) + 11 < 3(t^2 - 6t + 11)$ for all $t$.

(iii) \[x(t+1) - x(t)\]

\[= 15 \ln \left( \frac{(t+1)^2 - 6(t+1) + 11}{t^2 - 6t + 11} \right)\]

\[< 15 \ln 3 \quad (\text{by (d)(ii)) and } t^2 - 6t + 11 > 0)\]

$< 25$

Thus, the claim is incorrect.

By (d)(ii), we have $(t + 1)^2 - 6(t + 1) + 11 < 3(t^2 - 6t + 11)$

Note that $(t + 1)^2 - 6(t + 1) + 11 > 0$ and $3(t^2 - 6t + 11) > 0$.

\[\ln [(t+1)^2 - 6(t+1) + 11] < \ln 3 + \ln (t^2 - 6t + 11)\]

\[15 \ln [(t+1)^2 - 6(t+1) + 11] - 15 \ln (t^2 - 6t + 11) < 15 \ln 3\]

\[x(t+1) - x(t) < 25\]

Thus, the claim is incorrect.

1A f.t.

1M for using (d)(ii) and taking $\ln$

1A f.t.

--- (6)
9. (a) Let \( u = t + 10 \). Then, we have \( \frac{du}{dt} = 1 \).

The total amount
\[
= \int_0^3 f(t) \, dt \\
= \int_0^3 25t^2(t+10)^{-3} \, dt \\
= \int_{t=10}^{t=13} 25(u-10)^2 u^{-3} \, du \\
= 25 \left[ \frac{8}{8} - \frac{3}{8} - \frac{3}{12} + \frac{150}{150} \right] \int_{10}^{13} \\
= 97.65521668 \\
= 97.6552 \text{ thousand metres}
\]

(b) \( \ln(g(t) - 28) = \ln k + h t^2 \)

(c) \( h \approx 0.3 \) (correct to 1 decimal place)
\( \ln k = 1.0 \)
\( k \approx 2.718281828 \)
\( k \approx 2.7 \) (correct to 1 decimal place)

(d) (i) \( g(t) = 28 + 2.7e^{0.3t^2} \)
\[= 28 + 2.7 \left( 1 + 0.3t^2 + \frac{(0.3t^2)^2}{2!} + \frac{(0.3t^2)^3}{3!} + \cdots \right) \]
\[= 30.7 + 0.81t^2 + 0.1215t^4 + 0.01215t^6 + \cdots \]

The total amount
\[= \int_0^3 g(t) \, dt \]
\[\approx \int_0^3 (30.7 + 0.81t^2 + 0.1215t^4 + 0.01215t^6) \, dt \]
\[= \left[ 30.7t + \frac{0.81t^3}{3} + \frac{0.1215t^5}{5} + \frac{0.01215t^7}{7} \right]_0^3 \]
\[\approx 109.9090971 \]
\[\approx 109.909 \text{ thousand metres} \]

(ii) \( e^{0.3t^2} = 1 + 0.3t^2 + \frac{(0.3t^2)^2}{2!} + \frac{(0.3t^2)^3}{3!} + r(t) \) and \( r(t) > 0 \) for all \( t > 0 \).

Thus, the estimate in (d)(i) is an under-estimate.

(iii) Note that the estimate in (d)(i) is greater than the total amount of cloth production under John’s model and that the estimate in (d)(i) is an under-estimate.

Thus, the total amount of cloth production under Mary’s model is greater than that under John’s model.
10. (a) The required probability
\[
= \frac{4.7^0e^{-4.7}}{0!} + \frac{4.7^1e^{-4.7}}{1!} + \frac{4.7^2e^{-4.7}}{2!} + \frac{4.7^3e^{-4.7}}{3!} + \frac{4.7^4e^{-4.7}}{4!} + \frac{4.7^5e^{-4.7}}{5!}
\]
\[
\approx 0.668438485
\]
\[
= 0.6684
\]

(b) Let \( X \) km/h be the speed of a car entering the roundabout. 
Then, \( X \sim N(42.8, 12^2) \).

The required probability
\[
= P(X > 50)
\]
\[
= P(Z > \frac{50 - 42.8}{12})
\]
\[
= P(Z > 0.6)
\]
\[
= 0.2743
\]

(c) The required probability
\[
= (1 - 0.2743)^5(0.2743)
\]
\[
\approx 0.055209196
\]
\[
= 0.0552
\]

(d) (i) The required probability
\[
= C_5^1(0.2743)^5(1 - 0.2743) + (0.2743)^6
\]
\[
\approx 0.065570471
\]
\[
= 0.0656
\]

(ii) The required probability
\[
= 0.065570471 \left( \frac{(4.7)^4e^{-4.7}}{4!} \right)
\]
\[
+ \left( (0.2743)^7 + C_7^5(0.2743)^5(1 - 0.2743) + C_7^2(0.2743)^2(1 - 0.2743)^3 \right) \left( \frac{(4.7)^5e^{-4.7}}{5!} \right)
\]
\[
\approx 0.052151265
\]
\[
= 0.0522
\]

Marks

1M for the 6 cases + 1M for Poisson probability

1A \( a=1 \) for r.t. 0.668

--------- (3)

1M ( accept \( P(Z \geq \frac{50 - 42.8}{12}) \) )

1A \( a=1 \) for r.t. 0.274

--------- (2)

1M for \((1 - p)^5 \) \( p \) + 1M for \( p = (b) \)

1A \( a=1 \) for r.t. 0.055

--------- (3)

1M for the 2 cases + 1M for binomial probability

1A \( a=1 \) for r.t. 0.066

1M + 1M for numerator +
1M for denominator using (a)

1A \( a=1 \) for r.t. 0.052

--------- (7)
11. (a) The required probability
\[ 1 - \left( 0.8 \right)^5 + C_1^5 (0.8)^4(0.2) \]
\[= \frac{821}{3125} \approx 0.2627 \]
\[ \approx 0.2627 \]

The required probability
\[ = (0.2)^5 + C_1^5 (0.2)^4(0.8) + C_2^5 (0.2)^3(0.8)^2 + C_3^5 (0.2)^2(0.8)^3 \]
\[= \frac{821}{3125} \approx 0.2627 \]
\[ \approx 0.2627 \]

(b) (i) The required probability
\[ = (0.8)^6 (0.2) \]
\[= \frac{4096}{78125} \approx 0.0524 \]
\[ \approx 0.0524 \]

(ii) The required probability
\[ = \left( C_2^6 (0.8)^4 (0.2)^2 \right) \left( 0.8 \right) + \left( C_3^6 (0.8)^3 (0.2)^3 \right) \left( 0.2 \right) + \left( C_4^6 (0.8)^2 (0.2)^4 \right) \left( 0.2 \right) \]
\[= \frac{25344}{78125} \approx 0.3244 \]
\[ \approx 0.3244 \]

The required probability
\[ = C_2^6 (0.8)^4 (0.2)^2 + \left( C_3^6 (0.8)^3 (0.2)^3 \right) \left( 0.2 \right) \]
\[= \frac{25344}{78125} \approx 0.3244 \]
\[ \approx 0.3244 \]

The required probability
\[ = C_2^6 (0.8)^5 (0.2)^2 + \left( C_3^6 (0.8)^4 (0.2)^3 \right) \left( 0.2 \right) \]
\[= \frac{25344}{78125} \approx 0.3244 \]
\[ \approx 0.3244 \]

(iii) The required probability
\[ = \left( C_2^6 (0.8)^4 (0.2)^2 \right) \left( 0.2 \right) + \left( C_3^6 (0.8)^3 (0.2)^3 \right) \left( 0.2 \right) \]
\[= \frac{13}{33} \approx 0.3939 \]
\[ \approx 0.3939 \]
The required probability
\[ = 1 - \left( C_5^5 (0.8)^4 (0.2)^1 \right)(0.8) \]
\[ = \frac{13}{33} \approx 0.3939393939 \approx 0.3939 \]

Marks
1A for numerator
1M for denominator using (b)(ii)
1A

\[ \alpha - 1 \text{ for r.t. 0.394} \]

(iv) The required probability
\[ = \frac{(0.8)^5 (0.2)^2 + C_3^5 (0.8)^4 (0.2)^1 (0.2)^2 + C_5^5 (0.8)^2 (0.2)^3)}{1 - 0.26272} \]
\[ = \frac{49}{225} \approx 0.2177777778 \approx 0.2178 \]

Marks
1M (one term) + 1A for numerator
1M for denominator using (a)
1A

\[ \alpha - 1 \text{ for r.t. 0.218} \]

----------(12)
12. (a) The required probability
\[ P = 1 - \left( \frac{2.6^0 e^{-2.6}}{0!} + \frac{2.6^1 e^{-2.6}}{1!} + \frac{2.6^2 e^{-2.6}}{2!} + \frac{2.6^3 e^{-2.6}}{3!} \right) \]
\[ \approx 0.263998355 \]
\[ \approx 0.2640 \]

Let \( p \) be the probability described in (a).

(b) (i) The required probability
\[ P = p + (1 - p)p + (1 - p)^2 p + (1 - p)^3 p \]
\[ = 1 - (1 - p)^4 \]
\[ = 1 - (1 - 0.263998355)^4 \]
\[ \approx 0.70656282 \]
\[ = 0.7066 \]

(ii) The required probability
\[ P = \frac{(1 - 0.263998355)^2 (0.263998355) + (1 - 0.263998355)^3 (0.263998355)}{0.70656282} \]
\[ \approx 0.351364771 \]
\[ \approx 0.3514 \]

(iii) The integer \( m \) satisfies \( P(M \leq m) > 0.95 \).
\[ p + (1 - p)p + (1 - p)^2 p + \cdots + (1 - p)^{m-1} p > 0.95 \]
\[ 1 \cdot (1 - p)^m > 0.95 \]
\[ (1 - p)^m < 0.05 \]
\[ (1 - 0.263998355)^m < 0.05 \]
\[ m \ln(0.736001645) < 0.05 \]
\[ m > 9.773273146 \]

Thus, the least value of \( m \) is 10.

(c) Note that \( N \sim B(150, p) \).

The mean of \( N \)
\[ = 150p \]
\[ = 150(0.263998355) \]
\[ \approx 39.59975325 \]
\[ \approx 39.5998 \]

The variance of \( N \)
\[ = 150 p (1 - p) \]
\[ = 150(0.263998355)(1 - 0.263998355) \]
\[ \approx 29.14548353 \]
\[ \approx 29.1455 \]