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HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG ADVANCED LEVEL EXAMINATION 2006

MATHEMATICS AND STATISTICS AS-LEVEL

8.30 am – 11.30 am (3 hours)

This paper must be answered in English

1. This paper consists of Section A and Section B.
2. Answer ALL questions in Section A, using the AL(E) answer book.
3. Answer any FOUR questions in Section B, using the AL(C) answer book.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

SECTION A (40 marks)
Answer ALL questions in this section.
Write your answers in the AL(E) answer book.

1. (a) The coefficients of x and x^2 in the binomial expansion of $\left(1+\frac{x}{a}\right)^{\frac{-1}{n}}$ are $\frac{-1}{18}$ and $\frac{1}{24a}$ respectively, where a is a non-zero constant and n is a positive integer.

(i) Find the values of a and n.

(ii) State the range of values of x for which the binomial expansion of $\left(1+\frac{x}{a}\right)^{\frac{-1}{n}}$ is valid.

(b) Using the results of (a), or otherwise,

(i) write down the binomial expansion of $(9+x)^{\frac{-1}{2}}$ in ascending powers of x as far as the term in x^2 ;

(ii) state the range of values of x for which the binomial expansion of $(9+x)^{\frac{-1}{2}}$ is valid.

(6 marks)

2. After adding a chemical into a bottle of solution, the temperature $S(t)$ of the surface of the bottle can be modelled by

$$S(t)=2(t+1)^2e^{-\lambda t}+15 ,$$

where $S(t)$ is measured in °C, $t\ (\geq 0)$ is the time measured in seconds after the chemical has been added and λ is a positive constant. It is given that $S(9)=S(19)$.

(a) Find the exact value of λ.

(b) Will the temperature of the surface of the bottle get higher than 90°C ? Explain your answer.

(6 marks)

3. The rate of change of the amount of water in litres flowing into a tank can be modelled by

$$f(t)=\frac{500}{(t+2)^2e^t} ,$$

where $t\ (\geq 0)$ is the time measured in minutes.

(a) Using the trapezoidal rule with 5 sub-intervals, estimate the total amount of water flowing into the tank from $t=1$ to $t=11$.

(b) Find $\frac{d^2f(t)}{dt^2}$.

(c) Determine whether the estimate in (a) is an over-estimate or under-estimate.

(7 marks)

4. The stem-and-leaf diagram below shows the distribution of the numbers of books read by 24 students of a school in the first term:

Stem (tens)	Leaf (units)
0	3 4 6 7
1	1 2 2 3 5 6 7 8 8 9
2	1 3 4 5 5 7 8 9
3	0 0

(a) Find the median and the interquartile range of the numbers of books read.

(b) The librarian of the school ran a reading award scheme in the second term. The following table shows some statistics of the distribution of the numbers of books read by these 24 students in the second term:

Minimum	Lower quartile	Median	Upper quartile	Maximum
8	26	35	41	46

(i) Draw two box-and-whisker diagrams of the same scale to compare the numbers of books read by these students in the first term and in the second term.

(ii) The librarian claims that not less than 50% of these students read at least 5 more books in the second term than that in the first term. Do you agree? Explain your answer.

(7 marks)

Go on to the next page

5. A and B are two events. Suppose that $P(A \cap B) = 0.2$ and $P(A \mid B') = 0.5$, where B' is the complementary event of B. Let $P(B) = b$, where $b < 1$.

(a) Express $P(A \cap B')$ and $P(A)$ in terms of b.

(b) If A and B are independent events, find the value(s) of b.

(7 marks)

6. A teacher, Susan, built up a performance assessment scheme in which her 100 students were awarded merit points according to their test scores. She also used a normal distribution and a Poisson distribution to model the test scores and the merit points of the students respectively (see the following table).

Test score (X)	Merit points	Observed number of students	Expected number of students *	
			Normal distribution	Poisson distribution
$50 \le X < 150$	0	20	14.65	24.66
$150 \le X < 250$	1	41	44.00	34.52
$250 \le X < 350$	2	28	33.45	24.17
$350 \le X < 450$	3	9	6.38	11.28
$X \ge 450$	4	2	0.30	3.95
		Sum = 100	Sum = 98.78	Sum = 98.58

* Correct to 2 decimal places.

(a) In the above table, why is the sum of the expected numbers of students under each distribution less than 100 ?

(b) The absolute values of the differences between the observed numbers of students and the expected numbers of students are regarded as errors. The distribution with a smaller sum of errors will fit the observed data better. Which distribution, normal or Poisson, fits the observed data better? Explain your answer.

(7 marks)

SECTION B (60 marks)
Answer any FOUR questions in this section. Each question carries 15 marks.
Write your answers in the AL(C) answer book.

7. Define $f(x) = \dfrac{a+bx}{4-x}$ for all $x \neq 4$. Let $g(x) = f(-x)$ for all $x \neq -4$.

Let C_1 and C_2 be the curves $y = f(x)$ and $y = g(x)$ respectively.

It is given that the y-intercept of C_1 is $\dfrac{-3}{2}$ while the x-intercept of C_2 is -2.

(a) Find the values of a and b.

(2 marks)

(b) (i) Find the equations of the vertical asymptote(s) and the horizontal asymptote(s) to C_1.

(ii) Sketch C_1 and indicate its asymptote(s) and its intercept(s).

(5 marks)

(c) On the diagram sketched in (b)(ii), sketch C_2 and indicate its asymptote(s), its intercept(s) and the point(s) of intersection of the two curves.

(4 marks)

(d) Find the area enclosed by C_1, C_2 and the straight line $y = 9$.

(4 marks)

Go on to the next page

8. An airline manager, Christine, notices that the *weekly number of passengers* of the airline is declining, so she starts a promotion plan to boost the *weekly number of passengers*. She models the rate of change of the *weekly number of passengers* by

$$\frac{dx}{dt}=\frac{30t-90}{t^2-6t+11} \qquad (t \ge 0) \ ,$$

where x is the *weekly number of passengers* recorded at the end of a week in thousands of passengers and t is the number of weeks elapsed since the start of the plan.

Christine finds that at the start of the plan (i.e. $t=0$), the *weekly number of passengers* is 40 thousand.

(a) Let $v=t^2-6t+11$, find $\frac{dv}{dt}$.
Hence, or otherwise, express x in terms of t .
(4 marks)

(b) How many weeks after the start of the plan will the *weekly number of passengers* be the same as at the start of the plan?
(2 marks)

(c) Find the least *weekly number of passengers* after the start of the plan. Give your answer correct to the nearest thousand.
(3 marks)

(d) The week when the *weekly number of passengers* drops to the least is called the *Recovery Week*.

(i) Find the change in the *weekly number of passengers* from the *Recovery Week* to its following week. Give your answer correct to the nearest thousand.

(ii) Prove that $(t+1)^2-6(t+1)+11<3(t^2-6t+11)$ for all t .

(iii) Christine's assistant claims that after the *Recovery Week*, the change in the *weekly number of passengers* from a certain week to its following week will be greater than 25 thousand. Do you agree? Explain your answer.
(6 marks)

9. After upgrading the production line of a cloth factory, two engineers, John and Mary, model the rate of change of the amount of cloth production in thousand metres respectively by

$$f(t)=25t^2(t+10)^{\frac{-1}{3}} \quad \text{and} \quad g(t)=28+ke^{ht^2} \ ,$$

where h and k are positive constants and $t\ (\ge 0)$ is the time measured in months since the upgrading of the production line.

(a) Using the substitution $u=t+10$, or otherwise, find the total amount of cloth production from $t=0$ to $t=3$ under John's model.
(5 marks)

(b) Express $\ln\left(g(t)-28\right)$ as a linear function of t^2 .
(1 mark)

(c) Given that the slope and the intercept on the vertical axis of the graph of the linear function in (b) are measured to be 0.3 and 1.0 respectively, estimate the values of h and k correct to 1 decimal place.
(2 marks)

(d) Using the estimated values of h and k obtained in (c) correct to 1 decimal place,

(i) expand $g(t)$ in ascending powers of t as far as t^6 , and hence estimate the total amount of cloth production from $t=0$ to $t=3$ under Mary's model;

(ii) determine whether the estimate in (d)(i) is an over-estimate or an under-estimate;

(iii) determine whether the total amount of cloth production from $t=0$ to $t=3$ under Mary's model is greater than that under John's model.
(7 marks)

Go on to the next page

10. A researcher models the number of cars entering a roundabout in five-second time intervals (FSTIs) by a Poisson distribution with a mean of 4.7 cars per FSTI, and the speed of a car entering the roundabout by a normal distribution with a mean of 42.8 km/h and a standard deviation of 12 km/h . A car is *speeding* if the speed of the car is over 50 km/h .

(a) Find the probability that fewer than 6 cars enter the roundabout in a certain FSTI.

(3 marks)

(b) Find the probability that a car entering the roundabout is *speeding*.

(2 marks)

(c) Find the probability that the 6th car entering the roundabout is the 1st *speeding* car.

(3 marks)

(d) The roundabout is *hazardous* in a certain FSTI if at least 4 cars enter the roundabout in that FSTI and more than 2 of them are *speeding*.

(i) If exactly 4 cars enter the roundabout in a certain FSTI, find the probability that the roundabout is *hazardous* in that FSTI.

(ii) Given that fewer than 6 cars enter the roundabout in a certain FSTI, find the probability that the roundabout is *hazardous* in that FSTI.

(7 marks)

11. A manufacturer of brand E grape juice starts a marketing campaign by issuing points which can be exchanged for gifts. The number of points is shown on the back of the lid of each can of brand E grape juice. The probabilities for a customer to get a can of brand E grape juice with a 2-point lid and 5-point lid are 0.8 and 0.2 respectively. A total of 15 points or more can be exchanged for a packet of potato chips while a total of 20 points or more can be exchanged for a radio.

(a) Find the probability that a customer can exchange for a packet of potato chips in buying 5 cans of brand E grape juice.

(3 marks)

(b) A customer, Peter, buys 7 cans of brand E grape juice.

(i) Find the probability that only when the 7th can of brand E grape juice has been opened, Peter gets a 5-point lid.

(ii) Find the probability that only when the 7th can of brand E grape juice has been opened, Peter can exchange for a radio.

(iii) Given that Peter can exchange for a radio only when the 7th can of brand E grape juice has been opened, find the probability that the 7th can of brand E grape juice has a 5-point lid.

(iv) Given that Peter cannot get a packet of potato chips after opening 5 cans of brand E grape juice, find the probability that he can exchange for a radio only when the 7th can of brand E grape juice has been opened.

(12 marks)

Go on to the next page

12. There are many plants in a greenhouse and all of them are of the same species. Assume that the numbers of infected leaves on the plants in the greenhouse are independent and the number of infected leaves on each plant follows a Poisson distribution with a mean of 2.6 . A plant with at least 4 infected leaves is classified as *unhealthy*.

(a) Find the probability that a certain plant in the greenhouse is *unhealthy*.

(3 marks)

(b) A researcher, Teresa, inspects the plants one by one in the greenhouse. She finds that the Mth inspected plant is the first *unhealthy* plant.

(i) Find the probability that M is less than 5 .

(ii) Given that M is less than 5 , find the probability that M is greater than 2 .

(iii) If Teresa inspects m plants in the greenhouse, find the least value of m so that the probability of finding an *unhealthy* plant is greater than 0.95 .

(9 marks)

(c) It is given that there are 150 plants in the greenhouse. The number of *unhealthy* plants in the greenhouse is recorded every Friday. Let N be the number of *unhealthy* plants recorded on a Friday. Find the mean and the variance of N.

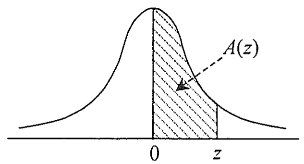
(3 marks)

END OF PAPER

Table: Area under the Standard Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note: An entry in the table is the proportion of the area under the entire curve which is between $z = 0$ and a positive value of z. Areas for negative values of z are obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \, dx$$