## AS Mathematics and Statistics

### General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits *all the marks* allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

| | |
|---|---|
| 'M' marks | awarded for correct methods being used; |
| 'A' marks | awarded for the accuracy of the answers; |
| Marks without 'M' or 'A' | awarded for correctly completing a proof or arriving at an answer given in a question. |

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

6. Marks may be deducted for poor presentation (*pp*). The symbol (pp-1) should be used to denote 1 mark deducted for *pp*. At most deducted 1 mark from Section A and 1 mark from Section B for *pp*. In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.

7. Marks may be deducted for numerical answers with inappropriate degree of accuracy (*a*). The symbol (a-1) should be used to denote 1 mark deducted for *a*. At most deducted 1 mark from Section A and 1 mark from Section B for *a*. In any case, do not deduct any marks for *a* in those steps where candidates could not score any marks.

8. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles.



| Solution | Marks |
|---|---|
| 1. (a) $(1+ax)^n$ | |
| $=1+nax+\frac{n(n-1)}{2!}(ax)^2+\frac{n(n-1)(n-2)}{3!}(ax)^3+\cdots$ | 1M for any two terms correct |
| $=1+nax+\frac{n(n-1)}{2}a^2x^2+\frac{n(n-1)(n-2)}{6}a^3x^3+\cdots$ | 1A |
| So, we have $na=\frac{3}{2}$, $\frac{n(n-1)}{2}a^2=\frac{27}{8}$ and $\frac{n(n-1)(n-2)}{6}a^3=b$. | |
| Solving, we have $n=\frac{-1}{2}$, $a=-3$ and $b=\frac{135}{16}$. | 1A for any one + 1A for all |
| (b) The required range of values of $x$ is $\lvert -3x\rvert<1$. | 1M can be absorbed |
| Thus, the expansion is valid for $\frac{-1}{3}<x<\frac{1}{3}$. | 1A accept $\lvert x\rvert<\frac{1}{3}$ |
| | ----------(6) |
| 2. (a) $\int_0^8 te^{\frac{t}{5}}\,dt$ | |
| $\approx\frac{8-0}{2(4)}\left(0+8e^{\frac{8}{5}}+2(2e^{\frac{2}{5}}+4e^{\frac{4}{5}}+6e^{\frac{6}{5}})\right)$ | 1M for trapezoidal rule |
| $\approx 103.2372887$ | |
| $\approx 103.2373$ | 1A *a*-1 for r.t. 103.237 |
| (b) $\int_0^8\frac{dx}{dt}\,dt=\int_0^8\left(4te^{\frac{t}{5}}+\frac{200}{t+1}\right)dt$ | 1M for considering $\int_0^8\frac{dx}{dt}dt$ |
| $x(8)-x(0)=\int_0^8\left(4te^{\frac{t}{5}}+\frac{200}{t+1}\right)dt$ | 1A |
| $x(8)-x(0)=4\int_0^8 te^{\frac{t}{5}}dt+200\int_0^8\frac{dt}{t+1}$ | |
| $x(8)-100\approx 4(103.2372887)+200\int_0^8\frac{dt}{t+1}$ (by (a)) | 1M for using (a) |
| Note that $\int_0^8\frac{dt}{t+1}$ | |
| $=[\ln(t+1)]_0^8$ | 1A for $\int\frac{dt}{t+1}=\ln(t+1)+C$ |
| $=\ln 9$ | |
| So, we have $x(8)\approx 952.3940702\approx 950$ (correct to 2 significant figures). | 1A |
| Thus, the required number is 950. | |
| | ----------(7) |

| Solution | Marks |
|---|---|
| (a) $\ln w=\frac{3}{2}\ln(x-1)-\frac{1}{2}\ln(x+2)-\frac{1}{2}\ln(2x+1)$ | 1A |
| Differentiate both sides w.r.t. $x$, we have | |
| $\frac{1}{w}\frac{dw}{dx}=\frac{3}{2(x-1)}-\frac{1}{2(x+2)}-\frac{1}{2x+1}$ | 1M |
| $\frac{dw}{dx}=w\left(\frac{3}{2(x-1)}-\frac{1}{2(x+2)}-\frac{1}{2x+1}\right)$ | 1A |
| $\boxed{\frac{dw}{dx}=\sqrt{\frac{(x-1)^3}{(x+2)(2x+1)}}\left(\frac{3}{2(x-1)}-\frac{1}{2(x+2)}-\frac{1}{2x+1}\right)}$ | |
| $\boxed{\frac{dw}{dx}=\frac{w(2x^2+14x+11)}{2(x-1)(x+2)(2x+1)}}$ | |
| (b) $w=2^y$ | |
| $\ln w=y\ln 2$ | 1M for taking log on both sides and can be absorbed |
| $y=\frac{\ln w}{\ln 2}$ | |
| $\frac{dy}{dw}=\frac{1}{w\ln 2}$ | 1A |
| $\frac{dy}{dx}=\frac{dy}{dw}\frac{dw}{dx}$ | |
| $\frac{dy}{dx}=\left(\frac{1}{w\ln 2}\right)\left[w\left(\frac{3}{2(x-1)}-\frac{1}{2(x+2)}-\frac{1}{2x-1}\right)\right]$ | 1M for Chain Rule |
| $\frac{dy}{dx}=\frac{1}{\ln 2}\left(\frac{3}{2(x-1)}-\frac{1}{2(x+2)}-\frac{1}{2x+1}\right)$ | 1A accept $\frac{(2x^2+14x+11)}{2(x-1)(x+2)(2x+1)\ln 2}$ |
| $w=2^y$ | 1M for using log on both sides and can be absorbed |
| $\ln w=y\ln 2$ | |
| Differentiate both sides w.r.t. $y$, we have | |
| $\frac{1}{w}\frac{dw}{dy}=\ln 2$ | |
| $\frac{dw}{dy}=w\ln 2$ | |
| $\frac{dy}{dw}=\frac{1}{w\ln 2}$ | 1A |
| $\frac{dy}{dx}=\frac{dy}{dw}\frac{dw}{dx}$ | |
| $\frac{dy}{dx}=\left(\frac{1}{w\ln 2}\right)\left[w\left(\frac{3}{2(x-1)}-\frac{1}{2(x+2)}-\frac{1}{2x+1}\right)\right]$ | 1M for Chain Rule |
| $\frac{dy}{dx}=\frac{1}{\ln 2}\left(\frac{3}{2(x-1)}-\frac{1}{2(x+2)}-\frac{1}{2x+1}\right)$ | 1A accept $\frac{(2x^2+14x+11)}{2(x-1)(x+2)(2x+1)\ln 2}$ |
| | ---------(7) |

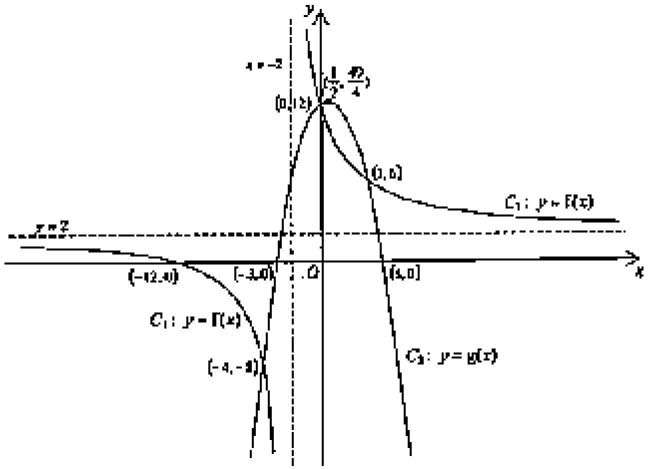


| Solution | Marks |
|---|---|
| 4. (a) Increase in the mean | |
| $=\frac{(10)(3)}{32}$ | |
| $=\frac{15}{16}$ cm | 1A (accept 160cm → $\frac{2575}{16}$ cm) |
| = 0.9375 cm | a−1 for r.t. 0.938 cm |
| (b) Change in the median = 0 | 1A (accept no change) |
| (c) Change in the mode = 0 | 1A (accept no change) |
| (d) Case 1 : The three incorrect records are 145 cm , 146 cm and 146 cm . Change in the range = 0 | 1A (accept no change) |
| Case 2 : The three incorrect records are 145 cm , 145 cm and 146 cm . Decrease in the range = 1 cm | 1A ( accept 29cm → 28cm ) |
| (e) Decrease in the interquartile range | |
| = 155 − 154 | |
| = 1 cm | 1A ( accept 14 cm → 13 cm ) |
| | ---------(6) |
| 5. (a) $P(A\cap B')$ | |
| $=P(A\mid B')P(B')$ | 1M can be absorbed (either one) |
| $=(\frac{5}{12})(1-\frac{2}{5})$ | |
| $=\frac{1}{4}$ | 1A |
| (b) $P(A'\cap B)$ | |
| $=P(B\mid A')P(A')$ | |
| $=(\frac{8}{15})(1-a)$ | 1A or equivalent |
| (c) $P(A'\cup B)$ | |
| $=1-P(A\cap B')$ | 1M accept $P(A)+P(A'\cap B)+P(A)+P(A\cap B')$ |
| $=1-\frac{1}{4}$ ( by (a) ) | |
| $=\frac{3}{4}$ | |
| Note that $P(A'\cup B)=P(A')+P(B)-P(A'\cap B)$ . | |
| Hence, we have $\frac{3}{4}=(1-a)+\frac{2}{5}-(\frac{8}{15})(1-a)$ ( by (b) ) . | 1M for using (b) |
| Thus, we have $a=\frac{1}{4}$ . | 1A |
| (d) $\because$ $P(A)=P(A\cap B)+P(A\cap B')$ , $P(A)=\frac{1}{4}$ ( by (c) ) | |
| and $P(A\cap B')=\frac{1}{4}$ ( by (a) ) | |
| $\therefore$ $P(A\cap B)=0$ | |
| Thus, $A$ and $B$ are mutually exclusive. | 1A must show reasons |
| | ---------(7) |

| Solution | Marks |
|---|---|
| 6. (a) The required probability | |
| $=C_2^4(\frac{5}{12})(\frac{4}{11})(\frac{7}{10})(\frac{6}{9})$ | 1M for $C_2^4$ + 1A for $(\frac{5}{12})(\frac{4}{11})(\frac{7}{10})(\frac{6}{9})$ |
| $=\frac{14}{33}$ | 1A |
| $\approx 0.424242424$ | |
| $\approx 0.4242$ | a−1 for r.t. 0.424 |
| The required probability | |
| $=\frac{C_2^5 C_2^7}{C_4^{12}}$ | 1M for numerator + 1A for denominator |
| $=\frac{14}{33}$ | 1A |
| $\approx 0.424242424$ | |
| $\approx 0.4242$ | a−1 for r.t. 0.424 |
| (b) (i) The required probability | |
| $=(\frac{2}{4})(\frac{1}{3})$ | |
| $=\frac{1}{6}$ | 1A |
| $\approx 0.166666667$ | |
| $\approx 0.1667$ | a−1 for r.t. 0.167 |
| The required probability | |
| $=\frac{C_2^2}{C_2^4}$ | |
| $=\frac{1}{6}$ | 1A |
| $\approx 0.166666667$ | |
| $\approx 0.1667$ | a−1 for r.t. 0.167 |
| (ii) The required probability | |
| $=(\frac{14}{33})(\frac{1}{6})+C_1^4(\frac{5}{12})(\frac{4}{11})(\frac{3}{10})(\frac{7}{9})(\frac{3}{4})(\frac{2}{3})$ | 1M for the 2 cases + 1M for either case correct |
| $=\frac{14}{99}$ | 1A |
| $\approx 0.141414141$ | |
| $\approx 0.1414$ | a−1 for r.t. 0.141 |
| The required probability | |
| $=(\frac{14}{33})(\frac{1}{6})+(\frac{C_3^5 C_1^7}{C_4^{12}})(\frac{C_2^3}{C_2^4})$ | 1M for the 2 cases + 1M for either case correct |
| $=\frac{14}{99}$ | 1A |
| [redacted] | |
| $\approx 0.1414$ | a−1 for r.t. 0.141 |
| | ---------(7) |



| Solution | Marks |
|---|---|
| 7. (a) ∵ $C_1$ and $C_2$ have a common $y$-intercept and $f(3)=g(3)$ . | |
| ∴ $f(0)=g(0)$ and $f(3)=g(3)$ . | |
| Therefore, we have $b=\frac{a}{2}$ and $\frac{6+a}{5}=b-6$ . | 1M for setting up simultaneous equations |
| Solving, we have $a=24$ and $b=12$ . | 1A + 1A<br>---------(3) |
| (b) ∵ $\lim_{x\to\infty}\frac{2x+24}{x+2}=\lim_{x\to\infty}\frac{2+\frac{24}{x}}{1+\frac{2}{x}}=2$ | |
| ∴ the equation of the horizontal asymptote to $C_1$ is $y=2$ . | 1A |
| ∵ $\lim_{x\to-2^-}f(x)=\lim_{x\to-2^-}\frac{2x+24}{x+2}=-\infty$ and $\lim_{x\to-2^+}f(x)=\lim_{x\to-2^+}\frac{2x+24}{x+2}=+\infty$ | |
| ∴ the equation of the vertical asymptote to $C_1$ is $x=-2$ . | 1A<br>---------(2) |
| (c) $C_1: y=f(x)$ , where $f(x)=\frac{2x+24}{x+2}$ . | |
| $C_2: y=g(x)$ , where $g(x)=-x^2+x+12$ . | |
| Note that $f(x)=g(x)$ | |
| $\Leftrightarrow \frac{2x+24}{x+2}=-x^2+x+12$ | |
| $\Leftrightarrow 2x+24=-x^3-x^2+14x+24$ | |
| $\Leftrightarrow x^3+x^2-12x=0$ | |
| $\Leftrightarrow x(x^2+x-12)=0$ | |
| $\Leftrightarrow x(x-3)(x+4)=0$ | |
| $\Leftrightarrow x=0$ , $x=3$ or $x=-4$ | |
| So, all the points of intersection are $(0,12)$ , $(3,6)$ and $(-4,-8)$ . | |
| Also, the $y$-intercepts of $C_1$ and $C_2$ are 12 . | |
| When $f(x)=0$ , we have $x=-12$ . | |
| When $g(x)=0$ , we have $x=-3$ or $x=4$ . | |
| So, the $x$-intercept of $C_1$ is $-12$ . | |
| Also, the $x$-intercepts of $C_2$ are $-3$ and $4$ . | |
| ∵ $g(x)=-x^2+x+12=-(x-\frac{1}{2})^2+\frac{49}{4}$ | |
| ∴ the maximum point of $C_2$ is $(\frac{1}{2},\frac{49}{4})$ . | |

| Solution | Marks |
|---|---|
|   | 1A for all the asymptotes<br>1A for the shape of $C_1$<br>1A for the shape of $C_2$<br>1A for the intercepts of $C_1$<br>1A for the intercepts of $C_2$<br>1A for the points of intersection<br>———(6) |
| (d) The required area | |
| $= \int_0^3 (g(x) - f(x))\,dx$ | 1A accept $\int_3^0 (f(x) - g(x))\,dx$ |
| $= \int_0^3 (-x^2 + x + 12 - \frac{2x+24}{x+2})\,dx$ | |
| $= \int_0^3 (-x^2 + x + 10 - \frac{20}{x+2})\,dx$ | 1M for division |
| $= \left[\frac{-x^3}{3} + \frac{x^2}{2} + 10x - 20\ln(x+2)\right]_0^3$ | 1A for correct integration |
| $= \frac{51}{2} - 20\ln\left(\frac{5}{2}\right)$ | 1A |
| [redacted] | |
| $\approx 7.1742$ | a−1 for r.t. 7.174<br>———(4) |



| Solution | Marks |
|---|---|
| 8. (a) $r(t) = \alpha t e^{-\beta t}$ | |
| $\frac{r(t)}{t} = \alpha e^{-\beta t}$ | |
| $\ln\frac{r(t)}{t} = \ln\alpha - \beta t$ | 1A<br>———(1) |
| (b) $\because \ln\alpha = 2.3$ | |
| $\therefore \alpha \approx 10$ ( correct to 1 significant figure ) | 1A |
| Also, we have $\beta \approx 0.5$ ( correct to 1 significant figure ). | 1A |
| $r(t) = 10te^{-0.5t}$ | |
| $\frac{dr(t)}{dt} = 10t(-0.5e^{-0.5t}) + 10e^{-0.5t}$ | |
| $= 10e^{-0.5t} - 5te^{-0.5t}$ | 1A |
| $= (10 - 5t)e^{-0.5t}$ | |
| $\frac{dr(t)}{dt} \begin{cases} > 0 & \text{if } 0 \le t < 2 \\ = 0 & \text{if } t = 2 \\ < 0 & \text{if } t > 2 \end{cases}$ | 1M for testing + 1A |
| So, $r(t)$ attains its greatest value when $t = 2$. | |
| Hence, greatest value of $r(t)$ is $(10)(2)e^{-0.5(2)} \approx 7.357588823$. | |
| Thus, the greatest rate of change is 7 ppm per hour. | 1A |
| $\frac{dr(t)}{dt} = 10t(-0.5e^{-0.5t}) + 10e^{-0.5t}$ | |
| $= 10e^{-0.5t} - 5te^{-0.5t}$ | 1A |
| $= (10 - 5t)e^{-0.5t}$ | |
| $\frac{d^2r(t)}{dt^2} = -5e^{-0.5t} + (-5 + 2.5t)e^{-0.5t} = (2.5t - 10)e^{-0.5t}$ | |
| $\frac{dr(t)}{dt} = 0$ when $t = 2$ only and $\left.\frac{d^2r(t)}{dt^2}\right\|_{t=2} = -5e^{-1} < 0$ | 1M for testing + 1A |
| So, $r(t)$ attains its greatest value when $t = 2$. | |
| Hence, greatest value of $r(t)$ is $(10)(2)e^{-0.5(2)} \approx 7.357588823$. | |
| Thus, the greatest rate of change is 7 ppm per hour. | 1A<br>———(6) |
| (c) (i) $\frac{d}{dt}\left((t + \frac{1}{\beta})e^{-\beta t}\right)$ | |
| $= \frac{d}{dt}\left((t+2)e^{-0.5t}\right)$ | |
| $= e^{-0.5t} - 0.5(t+2)e^{-0.5t}$ | 1M for product rule or chain rule |
| $= -0.5te^{-0.5t}$ | 1M accept $-\beta te^{-\beta t}$ |

| Solution | Marks |
| --- | --- |
| The required amount | |
| $= \int_0^T r(t)\,dt$ | |
| $= \int_0^T 10te^{-0.5t}\,dt$ | 1M |
| $= \left[-20(t+2)e^{-0.5t}\right]_0^T$ | 1M + 1A |
| $= \left(40-20(T+2)e^{-0.5T}\right)$ ppm | 1A |
| Note that <br> $\int r(t)\,dt$ <br> $= \int 10te^{-0.5t}\,dt$ <br> $= -20(t+2)e^{-0.5t}+C$ | 1M + 1A |
| Let $A(t)$ ppm be the amount of soot reduced when the petrol additive has been used for $t$ hours. <br> Then, we have $A(t) = -20(t+2)e^{-0.5t}+C$ . | 1M |
| Since $A(0)=0$ , we have $C=40$ . <br> So, we have $A(t)=\left(40-20(t+2)e^{-0.5t}\right)$ . <br> Note that $A(0)=0$ . <br> Thus, the required amount $= A(T) = \left(40-20(T+2)e^{-0.5T}\right)$ ppm | 1A |
| Note that <br> $\int r(t)\,dt$ <br> $= \int 10te^{-0.5t}\,dt$ <br> $= -20(t+2)e^{-0.5t}+C$ | 1M + 1A |
| Let $A(t)$ ppm be the amount of soot reduced when the petrol additive has been used for $t$ hours. <br> Then, we have $A(t) = -20(t+2)e^{-0.5t}+C$ . | 1M |
| The required amount <br> $= A(T)-A(0)$ <br> $= \left(-20(T+2)e^{-0.5T}+C\right)-\left(-40+C\right)$ <br> $= \left(40-20(T+2)e^{-0.5T}\right)$ ppm | 1A |
| (ii) The required amount <br> $= \lim_{T\to\infty}\left(40-20(T+2)e^{-0.5T}\right)$ <br> $= 40-20\lim_{T\to\infty} Te^{-0.5T} - 40\lim_{T\to\infty} e^{-0.5T}$ | |
| $= 40-20(0)-40(0)$ | 1M for $\lim_{T\to\infty} e^{-0.5T}=0$ and can be absorbed |
| $= 40$ ppm | 1A <br> ----------(8) |



| Solution | Marks |
| --- | --- |
| 9. (a) (i) Let $v = 2+3te^{-0.02t}$ . Then, we have <br> $\frac{dv}{dt} = 3e^{-0.02t} - \frac{3t}{50}e^{-0.02t}$ <br> $= \frac{3}{50}(50-t)e^{-0.02t}$ | 1M for product rule or chain rule + 1A |
| (ii) When $t=0$ , $\frac{dN}{dt}=100$ . So, we have $100 = \frac{50k}{2}$ . <br> Thus, we have $k=4$ . | 1 |
| $N = \int \frac{4(50-t)}{2e^{0.02t}+3t}\,dt$ <br> $= \frac{200}{3}\int \frac{dv}{v}$ | 1M for using (a)(i) |
| $= \frac{200}{3}\ln v + C$ <br> $= \frac{200}{3}\ln(2+3te^{-0.02t}) + C$ | 1A |
| Note that when $t=0$ , $N=10$ . So, we have $C = 10-\frac{200}{3}\ln 2$ . | 1M for finding $C$ |
| Thus, we have <br> $N = \frac{200}{3}\ln(2+3te^{-0.02t}) + 10 - \frac{200}{3}\ln 2$ <br> $= \frac{200}{3}\ln(1+\frac{3te^{-0.02t}}{2}) + 10$ | 1A <br> ----------(7) |
| (b) $\frac{dN}{dt} = \frac{4(50-t)}{2e^{0.02t}+3t}$ <br> $\begin{cases} >0 & \text{if } 0 \le t < 50 \\ =0 & \text{if } t=50 \\ <0 & \text{if } t>50 \end{cases}$ | 1M for testing + 1A |
| So, $N$ attains its greatest value when $t=50$ . | |
| Note that $N(50) = \frac{200}{3}\ln(1+\frac{150}{2}e^{-1}) + 10 \approx 233.5393678 < 500$ . | 1M for comparing $N(50)$ and 500 |
| Thus, the claim is not correct. | 1A f.t. |

| Solution | Marks |
|---|---|
| $\frac{dN}{dt}=\frac{4(50-t)}{2e^{0.02t}+3t}$ | |
| $\frac{d^2N}{dt^2}=4\left(\frac{(2e^{0.02t}+3t)(-1)-(50-t)(\frac{2}{50}e^{0.02t}+3)}{(2e^{0.02t}+3t)^2}\right)$ | |
| $=\frac{4}{25}\left(\frac{te^{0.02t}-100e^{0.02t}-3750}{(2e^{0.02t}+3t)^2}\right)$ | |
| $\frac{dN}{dt}=0$ when $t=50$ . Also, when $t=50$ , | |
| $\frac{d^2N}{dt^2}=\frac{4}{25}\left(\frac{50e-100e-3750}{(2e+150)^2}\right)$ | |
| $=\frac{-2}{e+75}$ | |
| $<0$ | 1M for testing + 1A |
| So, $N$ attains its greatest value when $t=50$ . | |
| Note that $N(50)=\frac{200}{3}\ln(1+\frac{150}{2}e^{-1})+10\approx 233.3393678<300$ . | 1M for comparing $N(50)$ and 300 |
| Thus, the claim is not correct. | 1A f.t. |
| | ---(4) |
| (c) (i) $\frac{d^2N}{dt^2}=\frac{d}{dt}\left(\frac{4(50-t)}{2e^{0.02t}+3t}\right)$ | |
| $=4\left(\frac{(2e^{0.02t}+3t)(-1)-(50-t)(\frac{2}{50}e^{0.02t}+3)}{(2e^{0.02t}+3t)^2}\right)$ | |
| $=\frac{4}{25}\left(\frac{te^{0.02t}-100e^{0.02t}-3750}{(2e^{0.02t}+3t)^2}\right)$ | 1A ( accept $\frac{4}{25}\left(\frac{te^{0.02t}-100e^{0.02t}-3750}{(2e^{0.02t}+3t)^2}\right)$ ) |
| $=\frac{4}{25}\left(\frac{(t-100)e^{0.02t}-3750}{(2e^{0.02t}+3t)^2}\right)$ | |
| (ii) Note that $\frac{d^2N}{dt^2}=\frac{4}{25}\left(\frac{(t-100)e^{0.02t}-3750}{(2e^{0.02t}+3t)^2}\right)$ ( by (c)(i) ) . | |
| Hence, we have $\frac{d^2N}{dt^2}<0$ for $59\le t\le 92$ . | 1M for considering the sign of numerator ---- either |
| So, $\frac{dN}{dt}$ decreases during the 3rd month after the start of the plan. | 1A f.t. |
| Also note that $\frac{dN}{dt}=\frac{4(50-t)}{2e^{0.02t}+3t}<0$ for $59\le t\le 92$ . | ---- |
| Therefore, $N$ decreases during the 3rd month after the start of the plan. | 1A f.t. ---(4) |



| Solution | Marks |
|---|---|
| 10. (a) The required probability | |
| $=\frac{6.2^0e^{-6.2}}{0!}+\frac{6.2^1e^{-6.2}}{1!}+\frac{6.2^2e^{-6.2}}{2!}+\frac{6.2^3e^{-6.2}}{3!}+\frac{6.2^4e^{-6.2}}{4!}$ | 1M for the 5 cases + 1M for Poisson probability |
| ~~$\approx 0.259177368$~~ | |
| $\approx 0.2592$ | 1A $a-1$ for r.t. 0.259 ---(3) |
| (b) (i) Let $X$ litres be the amount of the petrol for refuelling a car. | |
| Then, $X\sim N(23.2, 6^2)$ . | |
| The required probability | |
| $=P(X\ge 25)$ | |
| $=P(Z\ge\frac{25-23.2}{6})$ | 1M ( accept $P(Z>\frac{25-23.2}{6})$ ) |
| $=P(Z\ge 0.3)$ | |
| $=0.3821$ | 1A $a-1$ for r.t. 0.382 |
| (ii) The required probability | |
| $=C_2^4(0.3821)^2(1-0.3821)^4(0.3821)$ | 1M for $C_2^4p^2(1-p)^4p$ |
| | + 1M for $p=$ (b)(i) ---- |
| ~~$\approx 0.086935732$~~ | |
| $\approx 0.0869$ | 1A $a-1$ for r.t. 0.087 |
| (iii) The required probability | |
| $=\frac{6.2^3e^{-6.2}}{3!}(0.3821)^3$ | 1M for $\frac{6.2^3e^{-6.2}}{3!}p^3$ ---- either one |
| ~~$\approx 0.004497054178$~~ | |
| $\approx 0.0045$ | 1A $a-1$ for r.t. 0.004 |
| (iv) The required probability | |
| $=C_3^4(0.3821)^3(1-0.3821)+(0.3821)^4$ | 1M for $C_3^4p^3(1-p)+p^4$ ---- |
| ~~$\approx 0.159198667$~~ | |
| $\approx 0.1592$ | 1A $a-1$ for r.t. 0.159 |
| (v) The required probability | |
| $=\frac{0.004497054+0.159198667\left(\frac{6.2^4e^{-6.2}}{4!}\right)}{0.259177368}$ | 1M for numerator using (b)(iii) and (b)(iv) + 1M for denominator using (a) |
| ~~[illegible]~~ | |
| $\approx 0.0941$ | 1A $a-1$ for r.t. 0.094 ---(12) |

| Solution | Marks |
|---|---|
| Let $X$ minutes be the time needed for Peter to go to the train station platform. Then, $X \sim N(17.5, 2^2)$. | |
| (a) The required probability | |
| $= P(13 < X \le 19)$ | |
| $= P(\frac{13-17.5}{2} < Z \le \frac{19-17.5}{2})$ | 1M ( accept $P(\frac{13-17.5}{2} \le Z < \frac{19-17.5}{2})$ ) |
| $= P(-2.25 < Z \le 0.75)$ | |
| $= 0.4878 + 0.2734$ | |
| $= 0.7612$ | 1A $a-1$ for r.t. 0.761 |
| | ———(2) |
| (b) The required probability | |
| $=(0.02)(0.0122)+(0.15)(0.7612)+(0.35)(0.2144)+(1)(0.0122)$ | 1M for $(0.02)p_1+(0.15)p_2+(0.35)p_3$ + 1M for $(1)(1-p_1-p_2-p_3)$ |
| $= 0.201664$ | 1A |
| $\approx 0.2017$ | $a-1$ for r.t. 0.202 |
| | ———(3) |
| (c) The required probability | |
| $=\frac{(0.15)(0.7612)}{0.201664}$ | 1M for $\frac{(0.15)(a)}{(b)}$ |
| $\approx 0.566189305$ | |
| $\approx 0.5662$ | 1A ( accept 0.5661 ) $a-1$ for r.t. 0.566 |
| | ———(2) |
| (d) The required probability | |
| $= C_2^5(0.201664)^2(1-0.201664)^3$ | 1M for $C_2^5(b)^2(1-(b))^3$ |
| $\approx 0.206925443$ | |
| $\approx 0.2069$ | 1A ( accept 0.2070 ) $a-1$ for r.t. 0.207 |
| | ———(2) |
| (e) The required probability | |
| $=\frac{C_2^5[(0.15)(0.7612)]^2[(0.0122)(1-0.02)+(0.2144)(1-0.35)]^3}{0.206925443}$ | 1M for $\frac{C_2^5 p^2 q^3}{(d)}$ + 1A |
| $\approx 0.002$[illegible] | |
| $\approx 0.0022$ | 1A $a-1$ for r.t. 0.002 |
| The required probability | |
| $=(0.566189305)^2\left(\frac{(0.0122)(1-0.02)+(0.2144)(1-0.35)}{1-0.201664}\right)^3$ | 1M for $(c)^2 r^3$ + 1A |
| $\approx$ [illegible] | |
| $\approx 0.0022$ | 1A $a-1$ for r.t. 0.002 |
| | ———(3) |
| (f) Suppose Peter leaves home $t$ minutes before 7:00 a.m. | |
| Then, we have $P(X \le 13+t) \ge 0.95$. | 1M withhold 1M for equality or strict inequality |
| So, we have $P(Z \le \frac{13+t-17.5}{2}) \ge 0.95$. | |
| Therefore, we have $\frac{t-4.5}{2} \ge 1.645$. | 1A ( accept $\frac{t-4.5}{2} \ge z$, $1.64 \le z \le 1.65$ ) |
| Hence, we have $t \ge 7.79$. | |
| Thus, the required time is 6:52 a.m. | 1A |
| | ———(3) |



| Solution | Marks |
|---|---|
| 12. (a) Let $X$ be the number of computers sold in a day. | |
| Also let $\bar{X}$ be the sample mean of the number of computers sold in a day. | |
| $\bar{X}=\frac{(0)(6)+(1)(10)+(2)(6)+(3)(2)+(4)(1)}{25}=1.28$ | 1A |
| For the Poisson distribution, | |
| $a=(25)P(X=1)$ | |
| $=(25)\left(\frac{1.28^1}{1!}\right)e^{-1.28}$ | 1M |
| $\approx 8.897193614$ | |
| $\approx 8.90$ | 1A $a-1$ for r.t. 8.9 |
| For the binomial distribution, | |
| $8p=1.28$ | |
| $p=0.16$ | 1M |
| $b=(25)P(X=3)$ | |
| $=(25)C_3^8(0.16)^3(0.84)^5$ | 1M |
| $\approx 2.398194562$ | |
| $\approx 2.40$ | 1A $a-1$ for r.t. 2.4 |
| | ———(6) |
| (b) For the number of computers sold is 5 or more, the expected number of days by the Poisson distribution is | |
| $25-(6.95+8.90+5.69+2.43+0.78)=0.25$ | 1M - - - - - - - - - |
| Let $SE_1$ be the sum of errors for model fitted by the Poisson distribution. So, | |
| $SE_1$ | |
| $=\lvert 6-6.95\rvert+\lvert 10-8.90\rvert+\lvert 6-5.69\rvert+\lvert 2-2.43\rvert+\lvert 1-0.78\rvert+\lvert 0-0.25\rvert$ | 1M + 1M (1M for the first 5 terms, 1M for the last term) |
| $=3.26$ | 1A - - - - - - - - - |
| For the number of computers sold is 5 or more, the expected number of days by the binomial distribution is | |
| $25-(6.20+9.44+6.30+2.40+0.57)=0.09$ | - - - - - either one - - - - - |
| Let $SE_2$ be the sum of errors for model fitted by the binomial distribution. So, | |
| $SE_2$ | |
| $=\lvert 6-6.20\rvert+\lvert 10-9.44\rvert+\lvert 6-6.30\rvert+\lvert 2-2.40\rvert+\lvert 1-0.57\rvert+\lvert 0-0.09\rvert$ | - - - - - either one - - - - - |
| $=1.98$ | - - - - - both - - - - - |
| Since $SE_2 < SE_1$, the binomial distribution fits the data better. | 1M |
| | ———(5) |
| (c) (i) Let \$ $Y$ be the price of a computer. Then, $Y \sim N(7\,580, 800^2)$. | |
| The required probability | |
| $=P(Y > 8\,500)$ | |
| $=P(Z > \frac{8\,500-7\,580}{800})$ | 1M ( accept $P(Z \ge \frac{8\,500-7\,580}{800})$ ) |
| $=P(Z > 1.15)$ | |
| $=0.1251$ | 1A $a-1$ for r.t. 0.125 |

| Solution | Marks |
|---|---|
| (ii) The required probability | |
| $= \left(C_3^5(0.16)^3(1-0.16)^2\right)\left(C_1^3(0.1251)^1(1-0.1251)^2\right)$ | 1M |
| [redacted] | |
| $\approx 0.0276$ | 1A a–1 for r.t. 0.028 |
| The required probability | |
| $= \frac{2.598194563}{25}\left(C_1^3(0.1251)^1(1-0.1251)^2\right)$ | 1M |
| [redacted] | |
| $\approx 0.0276$ | 1A a–1 for r.t. 0.028 |
| | ———(4) |



## 考生表現

甲部（必答題）

| 題號 | 一般表現 |
|---|---|
| 1 | 良好。部分考生未能應用容許 r 爲有理數的更一般的公式，而只考慮 r 爲正整數的公式。 |
| 2 | 平平。部分考生仍然混淆了定積分與不定積分。 |
| 3 | 良好。部分考生未能應用鏈式法則。 |
| 4 | 平平。很多考生都懂得計算較耗時，但他們未能顯示明白背後的概念；若他們明白，則可省卻很多運算。 |
| 5 | 良好。部分考生混淆了「互斥事件」的定義與「獨立事件」的定義。 |
| 6 | 平平。部分考生仍未熟習數出有關事件的數目及分辨有關事件是否獨立。 |

乙部（6 題選答 4 題）

| 題號 | 選題百分率 | 一般表現 |
|---|---|---|
| 7(a) | 81 | 甚佳。 |
| (b) | | 良好。部分考生未能適當地利用方程表示漸近線。 |
| (c) | | 平平。部分考生沒有標明圖像的交點。 |
| (d) | | 平平。很多考生於運用積分法求面積時感到困難。 |
| 8(a) | 37 | 甚佳。 |
| (b) | | 良好。部分考生未能證明該駐點爲極大點。 |
| (c)(i) | | 平平。考生於前部分表現理想，但後部分表現卻不符理想。只有部分考生能求得所減少的氣體排放總量。 |
| (ii) | | 欠佳。由於未能解 (c)(i) 部，故此很多考生未能完成本部。 |