HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG ADVANCED LEVEL EXAMINATION 2005

MATHEMATICS AND STATISTICS AS-LEVEL

8.30 am – 11.30 am (3 hours)
This paper must be answered in English

1. This paper consists of Section A and Section B.
2. Answer ALL questions in Section A, using the AL(E) answer book.
3. Answer any FOUR questions in Section B, using the AL(C)2 answer book.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

SECTION A (40 marks)
Answer ALL questions in this section.
Write your answers in the AL(E) answer book.

1. Let r be a rational number. The binomial expansion of $(1+ax)^r$ in ascending powers of x is $1+\frac{3}{2}x+\frac{27}{8}x^2+bx^3+\cdots$, where a and b are constants.

   (a) Find the values of r , a and b .

   (b) State the range of values of x for which the expansion is valid.

(6 marks)

2. (a) Using the trapezoidal rule with 4 sub-intervals, estimate $\int_0^8 te^{\frac{t}{5}}\,dt$.

   (b) A researcher modelled the rate of change of the number of certain insects under controlled conditions by

$$\frac{dx}{dt}=4te^{\frac{t}{5}}+\frac{200}{t+1} ,$$

   where x is the number of insects and $t\ (\geq 0)$ is the time measured in weeks. It is known that $x=100$ when $t=0$.

   Using the result of (a), estimate the number of insects when $t=8$. Give your answer correct to 2 significant figures.

(7 marks)

3. Let $w=\sqrt{\frac{(x-1)^3}{(x+2)(2x+1)}}$, where $x>1$.

   (a) Express $\ln w$ in the form $a\ln(x-1)+b\ln(x+2)+c\ln(2x+1)$, where a , b and c are constants.

   Hence find $\frac{dw}{dx}$.

   (b) Suppose $w=2^y$.

   Express $\frac{dy}{dw}$ in terms of w .

   Hence express $\frac{dy}{dx}$ in terms of x .

(7 marks)

4. The stem-and-leaf diagram below shows the distribution of heights in cm of 32 students:

Stem (tens)	Leaf (units)
14	5 5 6 6
15	1 2 2 4 4 5 5 7 7 7 7 7 9
16	0 2 2 5 6 7 8 8 9
17	0 1 2 3 4 4

It is found that three records less than 150 cm are incorrect. Each of them should be 10 cm greater than the original record. Find the change in each of the following statistics after correcting the three records:

   (a) the mean,

   (b) the median,

   (c) the mode,

   (d) the range,

   (e) the interquartile range.

(6 marks)

5. A and B are two events. Suppose that $P(A \mid B') = \frac{5}{12}$, $P(B \mid A') = \frac{8}{15}$ and $P(B) = \frac{2}{5}$, where A' and B' are complementary events of A and B respectively. Let $P(A) = a$, where $0 < a < 1$.

(a) Find $P(A \cap B')$.

(b) Express $P(A' \cap B)$ in terms of a .

(c) Using the fact that $A' \cup B$ is the complementary event of $A \cap B'$, or otherwise, find the value of a .

(d) Are A and B mutually exclusive? Explain your answer.

(7 marks)

6. Mrs. Wong has 12 bottles of fruit juice in her kitchen:

1 bottle of grape juice,
6 bottles of apple juice and
5 bottles of orange juice.

Mrs. Wong randomly chooses 4 bottles to serve her friends, Ann, Billy, Christine and Donald.

(a) Find the probability that exactly 2 bottles of orange juice are chosen by Mrs. Wong.

(b) Suppose that each of the four friends randomly selects a bottle of fruit juice from the 4 bottles offered by Mrs. Wong.

(i) If only 2 of the bottles of fruit juice offered by Mrs. Wong are orange juice, find the probability that both Ann and Billy select orange juice.

(ii) Find the probability that fewer than 4 of the bottles of fruit juice offered by Mrs. Wong are orange juice and both Ann and Billy select orange juice.

(7 marks)

SECTION B (60 marks)
Answer any FOUR questions in this section. Each question carries 15 marks.
Write your answers in the AL(C)2 answer book.

7. Define $f(x) = \frac{2x+a}{x+2}$ for all $x \neq -2$ and let $g(x) = -x^2 + x + b$, where a and b are constants.

Let C_1 and C_2 be the curves $y = f(x)$ and $y = g(x)$ respectively.

It is given that C_1 and C_2 have a common y-intercept and $f(3) = g(3)$.

(a) Find the values of a and b .

(3 marks)

(b) Find the equations of the horizontal asymptote(s) and vertical asymptote(s) to C_1 .

(2 marks)

(c) Sketch C_1 and C_2 on the same diagram and indicate the asymptote(s) to C_1 . Also indicate the intercept(s) and the points of intersection of C_1 and C_2 .

(6 marks)

(d) Find the area enclosed by C_1 and C_2 .

(4 marks)

8. A researcher studied the soot reduction effect of a petrol additive on soot emission of a car. Let t be the number of hours elapsed after the petrol additive has been used and $\mathrm{r}(t)$, measured in ppm per hour, be the rate of change of the amount of soot reduced. The researcher suggested that $\mathrm{r}(t)$ can be modelled by $\mathrm{r}(t) = \alpha t e^{-\beta t}$, where α and β are positive constants.

(a) Express $\ln \frac{\mathrm{r}(t)}{t}$ as a linear function of t.

(1 mark)

(b) It is given that the slope and the intercept on the vertical axis of the graph of the linear function obtained in (a) are -0.50 and 2.3 respectively. Find the values of α and β correct to 1 significant figure.

Hence find the greatest rate of change of the amount of soot reduced after the petrol additive has been used. Give your answer correct to 1 significant figure.

(6 marks)

(c) Using the values of α and β obtained in (b) correct to 1 significant figure,

(i) find $\frac{\mathrm{d}}{\mathrm{d}t}\left((t+\frac{1}{\beta})e^{-\beta t}\right)$ and hence find, in terms of T, the total amount of soot reduced when the petrol additive has been used for T hours;

(ii) estimate the total amount of soot reduced when the petrol additive has been used for a very long time.

[Note : Candidates may use $\lim_{T\to\infty}(Te^{-\beta T}) = 0$ without proof.]

(8 marks)

9. A web administrator, David, launches a promotion plan to increase the *daily number of visits* to his web site. The rate of change of the *daily number of visits* to the web site can be modelled by

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{k(50-t)}{2e^{0.02t}+3t},$$

where N is the *daily number of visits* recorded at the end of a day in thousands of visits, $t\ (\geq 0)$ is the number of days elapsed since the start of the plan and k is a positive constant.

David finds that at the start of the plan (i.e. $t = 0$), $\frac{\mathrm{d}N}{\mathrm{d}t} = 100$ and $N = 10$.

(a) (i) Let $v = 2 + 3te^{-0.02t}$, find $\frac{\mathrm{d}v}{\mathrm{d}t}$.

(ii) Prove that $k = 4$ and hence express N in terms of t .

(7 marks)

(b) David claims that the *daily number of visits* to his web site will be greater than 500 thousand on a certain day after the start of the plan. Do you agree? Explain your answer.

(4 marks)

(c) (i) Find $\frac{\mathrm{d}^2N}{\mathrm{d}t^2}$.

(ii) Describe the behaviour of N and $\frac{\mathrm{d}N}{\mathrm{d}t}$ during the 3rd month after the start of the plan.

(4 marks)

10. In a city, the number of cars entering a filling station for petrol per hour can be modelled by a Poisson distribution with a mean of 6.2 cars per hour.

(a) Find the probability that there are fewer than 5 cars entering the filling station for petrol in a certain hour.

(3 marks)

(b) The manager of the filling station models the amount of petrol for refuelling a car by a normal distribution with a mean of 23.2 litres and a standard deviation of 6 litres.

(i) Find the probability that the amount of petrol for refuelling a car is at least 25 litres.

(ii) Find the probability that the 9th car entering the filling station for petrol is the 3rd car which has been refuelled with at least 25 litres.

(iii) Find the probability that there are exactly 3 cars entering the filling station for petrol in a certain hour and each of them will be refuelled with at least 25 litres.

(iv) If there are exactly 4 cars entering the filling station for petrol in a certain hour, find the probability that more than 2 of them will each be refuelled with at least 25 litres.

(v) Given that there are fewer than 5 cars entering the filling station for petrol in a certain hour, find the probability that more than 2 of them will each be refuelled with at least 25 litres.

(12 marks)

11. Every school day, Peter leaves home at 7:00 a.m. to go to the train station to take a train to his school. The time needed for him to go to the train station platform follows a normal distribution with a mean of 17.5 minutes and a standard deviation of 2 minutes.

The following table shows the departure times for trains A, B and C and the probabilities for Peter to be late when taking trains A, B and C respectively:

Train	Departure time	Probability for Peter to be late
A	7:13 a.m.	0.02
B	7:19 a.m.	0.15
C	7:22 a.m.	0.35

Peter takes the earliest departing train when he arrives at the train station platform. Assume that the time needed for him to get on the train from the platform is negligible. It is certain that he will be late if he cannot catch any one of the trains A, B and C.

(a) Find the probability that Peter takes train B to the school on a certain morning.

(2 marks)

(b) Find the probability that Peter is late on a certain morning.

(3 marks)

(c) Given that Peter is late on a certain morning, find the probability that Peter takes train B to the school on this morning.

(2 marks)

(d) Find the probability that Peter is late on exactly 2 mornings in a certain week of 5 school days.

(2 marks)

(e) Given that Peter is late on exactly 2 mornings in a certain week of 5 school days, find the probability that he takes train B to the school only on these 2 mornings.

(3 marks)

(f) If Peter tries to leave home earlier so that the probability of his getting on train A is at least 0.95, what is the latest time that he should leave home? Give your answer correct to the nearest minute.

(3 marks)

12. Mary, a shopkeeper of a computer shop, wants to model the distribution of the number of computers sold in a day. She tabulated the sales data of 25 days as shown in the following table and fitted the data by a Poisson distribution and a binomial ($n = 8$) distribution. In the fitting process, she used the sample mean of the data as the mean of each of the distributions.

Number of computers sold	Observed number of days	Expected number of days *	
		Poisson distribution	Binomial distribution
0	6	6.95	6.20
1	10	a	9.44
2	6	5.69	6.30
3	2	2.43	b
4	1	0.78	0.57

* Correct to 2 decimal places.

(a) Find the values of a and b correct to 2 decimal places.
(6 marks)

(b) The absolute values of the differences between the observed numbers of days and the expected numbers of days are regarded as errors. The distribution with a smaller sum of errors will fit the data better. Which distribution, Poisson or binomial, fits the data better? Explain your answer.
(5 marks)

(c) Assume the distribution that fits the data better in (b) is adopted and the price of a computer follows a normal distribution with a mean of $7580 and a standard deviation of $800 .

(i) Find the probability that the price of a computer is more than $8 500 .

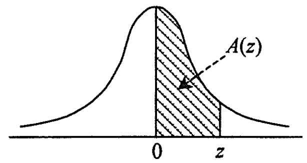
(ii) Find the probability that 3 computers are sold on a particular day and only one of them is of a price of more than $8 500 .
(4 marks)

END OF PAPER

Table: Area under the Standard Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note: An entry in the table is the proportion of the area under the entire curve which is between $z = 0$ and a positive value of z. Areas for negative values of z are obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$