香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2004年香港高級程度會考
HONG KONG ADVANCED LEVEL EXAMINATION 2004

數學及統計學　高級補充程度
MATHEMATICS AND STATISTICS　AS-LEVEL

本評卷參考乃香港考試及評核局專為今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員／教師應嚴詞拒絕，因學生極可能將評卷參考視為標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。
After the examinations, marking schemes will be available for reference at the teachers' centre.

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AS Mathematics and Statistics

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits *all the marks* allocated to that part, unless a particular method has been specified in the question. Makers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

6. Marks may be deducted for poor presentation (*pp*). The symbol (*pp-1*) should be used to denote 1 mark deducted for *pp*. At most deducted 1 mark from Section A and 1 mark from Section B for *pp*. In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.

7. Marks may be deducted for numerical answers with inappropriate degree of accuracy (*a*). The symbol (*a-1*) should be used to denote 1 mark deducted for *a*. At most deducted 1 mark from Section A and 1 mark from Section B for *a*. In any case, do not deduct any marks for *a* in those steps where candidates could not score any marks.

8. Marks entered in the Page Total Box should be the NET total scored on that page.

9. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles.

Solution	Marks
1. (a) $\mathrm{P}(A \cup B)$	
$= \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$	
$= 0.75 + 0.8 - k$	1M can be absorbed
$= 1.55 - k$	1A
(b) (i) $\because \mathrm{P}(A \cup B) \le 1$	1M can be absorbed (either one)
$\therefore 1.55 - k \le 1$ (by (a))	
Therefore, we have $k \ge 0.55$.	1 (either one)
$\because \mathrm{P}(A \cap B) \le 1$	
$\therefore k \le 1$	
Thus, we have $0.55 \le k \le 1$.	1
(ii) $\mathrm{P}(A' \cup B')$	
$= 1 - \mathrm{P}(A \cap B)$ (since $A' \cup B'$ is the complementary event of $A \cap B$)	
$= 1 - k$	1M accept $k = 1 - \mathrm{P}(A' \cup B')$
$\le 1 - 0.55$ (by (b)(i))	
$= 0.45$	1
Thus, we have $\mathrm{P}(A' \cup B') \le 0.45$.	

$\mathrm{P}(A')$	
$= 1 - \mathrm{P}(A)$	
$= 1 - 0.75$	1M (either one)
$= 0.25$	
$\mathrm{P}(B')$	
$= 1 - \mathrm{P}(B)$	
$= 1 - 0.8$	
$= 0.2$	
$\mathrm{P}(A' \cup B')$	
$= \mathrm{P}(A') + \mathrm{P}(B') - \mathrm{P}(A' \cap B')$	
$= 0.25 + 0.2 - \mathrm{P}(A' \cap B')$	
$= 0.45 - \mathrm{P}(A' \cap B')$	
≤ 0.45 (since $\mathrm{P}(A' \cap B') \ge 0$)	1

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Solution	Marks
2. (a) $\frac{dN}{dt}$	
$=\frac{6}{(e^{\frac{t}{4}}+e^{\frac{-t}{4}})^2}$	
$=\frac{6}{(e^{\frac{-t}{4}}(e^{\frac{t}{2}}+1))^2}$	
$=\frac{6}{e^{\frac{-t}{2}}(e^{\frac{t}{2}}+1)^2}$	
$=\frac{6e^{\frac{t}{2}}}{(e^{\frac{t}{2}}+1)^2}$	1 must show steps
Let $u=e^{\frac{t}{2}}+1$.	
Then, we have $\frac{du}{dt}=\frac{1}{2}e^{\frac{t}{2}}$.	
Also, $dt=\frac{2du}{u-1}$. Now,	
N	
$=\int\frac{6e^{\frac{t}{2}}}{(e^{\frac{t}{2}}+1)^2}\,dt$	
$=\int\frac{12(u-1)}{u^2(u-1)}\,du$	
$=\int\frac{12}{u^2}\,du$	1A accept $\frac{dN}{du}=\frac{12}{u^2}$.
So, we have $N=\frac{-12}{u}+C$ where C is a constant.	1A
Now, $N=\frac{-12}{e^{\frac{t}{2}}+1}+C$.	
Using the condition that $N=8$ when $t=0$, we have $8=-6+C$. Hence, $C=14$.	1M for finding C
Thus, $N=14-\frac{12}{e^{\frac{t}{2}}+1}$.	1A
(b) The required number of fish	
$=\lim_{t\to\infty}(14-\frac{12}{e^{\frac{t}{2}}+1})$	
$=14-\lim_{t\to\infty}\frac{12}{e^{\frac{t}{2}}+1}$	
$=14$ thousands	1A
	----------(6)

Solution	Marks
3. (a) Since $2\pi rh + 2\pi r^2 = 162\pi$, we have	1A (either one)
$rh + r^2 = 81$.	
Therefore,	
The required capacity	
$= \pi r^2 h + \frac{2}{3}\pi r^3$	
$= \pi r(81 - r^2) + \frac{2}{3}\pi r^3$	
$= (81\pi r - \frac{1}{3}\pi r^3)$ cm^3	1
(b) Let $f(r) = 81\pi r - \frac{1}{3}\pi r^3$ for all $r \geq 0$. Then, we have	
$\frac{df(r)}{dr} = 81\pi - \pi r^2$.	1A
$\frac{df(r)}{dr} = 0$ when $r = 9$	1M
$\frac{df(r)}{dr} \begin{cases} > 0 & \text{if } 0 \leq r < 9 \\ = 0 & \text{if } r = 9 \\ < 0 & \text{if } r > 9 \end{cases}$	1M for testing
So, $f(r)$ attains its greatest value when $r = 9$.	1A
Note that $f(9) = 486\pi$ ≈ 1526.81403 ≤ 1600	
Thus, by (a), the capacity of the container cannot be greater than 1 600 cm^3.	1A
Let $f(r) = 81\pi r - \frac{1}{3}\pi r^3$ for all $r \geq 0$. Then, we have	
$\frac{df(r)}{dr} = 81\pi - \pi r^2$.	1A
$\frac{df(r)}{dr} = 0$ when $r = 9$	1M
$\frac{d^2 f(r)}{dr^2} = -2\pi r < 0$ for any $r > 0$	1M for testing
Note that there is only one local maximum.	
So, $f(r)$ attains its greatest value when $r = 9$.	1A
Note that $f(9) = 486\pi$ ≈ 1526.81403 ≤ 1600	
Thus, by (a), the capacity of the container cannot be greater than 1 600 cm^3.	1A
	----------(7)

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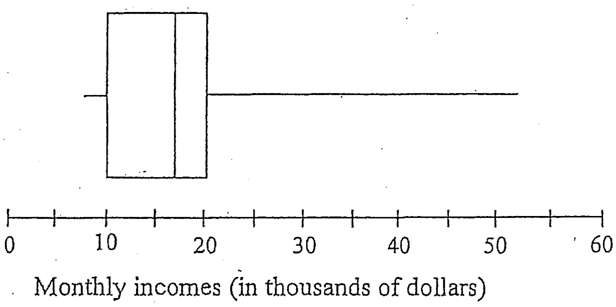
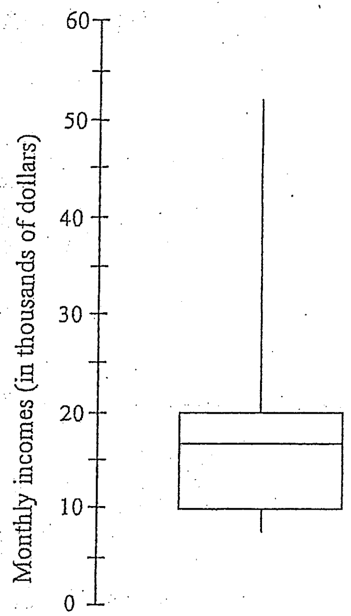
Solution	Marks
4. (a) (i) $(x+y+z)^2$	
$= x^2+y^2+z^2+2xy+2yz+2xz$	1A
(ii) Note that	
$(x+y+z)^4$	
$= (x^2+y^2+z^2+2xy+2yz+2xz)(x^2+y^2+z^2+2xy+2yz+2xz)$	
Thus, we have	
the coefficients of x^3y , x^3z , xy^3 , y^3z , xz^3 and yz^3	
$= (1)(2)+(2)(1)$	1M can be absorbed
$= 4$	1A
(b) (i) The required probability	
$= 1-p^4-q^4-r^4$	1M for complementary probability + 1A
The required probability	
$= (p+q+r)^4-p^4-q^4-r^4$	1M
$= 1-p^4-q^4-r^4$	1A
The required probability	
$= (p+q+r)^4-p^4-q^4-r^4$	1M
$= (p^2+q^2+r^2+2pq+2qr+2pr)(p^2+q^2+r^2+2pq+2qr+2pr)-p^4-q^4-r^4$	
$= p^4+q^4+r^4+4p^3q+4p^3r+4pq^3+4q^3r+4pr^3+4qr^3+$ $6p^2q^2+6p^2r^2+6q^2r^2+12p^2qr+12pq^2r+12pqr^2-p^4-q^4-r^4$	
$=4p^3q+4p^3r+4pq^3+4q^3r+4pr^3+4qr^3+6p^2q^2+6p^2r^2+6q^2r^2+12p^2qr+12pq^2r+12pqr^2$	1A
(ii) The required probability	
$= 4(p^3q+p^3r+pq^3+q^3r+pr^3+qr^3)$	1A for $(p^3q+p^3r+pq^3+q^3r+pr^3+qr^3)$ + 1A for all being correct
The required probability	
$= 4p^3(1-p)+4q^3(1-q)+4r^3(1-r)$	1A for $(p^3(1-p)+q^3(1-q)+r^3(1-r))$ + 1A for all being correct
The required probability	
$= 1-(p^4+q^4+r^4+6p^2q^2+6p^2r^2+6q^2r^2+12p^2qr+12pq^2r+12pqr^2)$	1A for $(p^4+q^4+r^4+6p^2q^2+6p^2r^2+6q^2r^2+12p^2qr+12pq^2r+12pqr^2)$ + 1A for all being correct
	----------(7)

Solution	Marks
5. (a) 	1A for any correct box-and-whisker diagram 1A for correct scale pp−1 for omitting the title
	1A for any correct box-and-whisker diagram 1A for correct scale pp−1 for omitting the title
(b) (i) Let \$$X$ be the monthly income of a randomly selected university graduate from the group. Then, we have $X \sim \mathrm{N}(17940, 4700^2)$.	
The required probability $= \mathrm{P}(X < 17000)$ $= \mathrm{P}(Z < \frac{17000-17940}{4700})$	
$= \mathrm{P}(Z < -0.2)$	1A
$= 0.4207$	1A $a-1$ for r.t. 0.421
(ii) Since the distribution is skewed to the right side,	1M accept skewed to one side or not symmetrical
the model proposed by the student is not appropriate.	1M ----------(6)

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Solution	Marks
6. (a) The required probability	
$=1-(\frac{1}{10})^2$	1M for complementary probability
$=\frac{99}{100}$	1A (accept 0.99)
The required probability	
$=1-10(\frac{1}{10})^3$	1M for complementary probability
$=\frac{99}{100}$	1A (accept 0.99)
(b) The required probability	
$=(\frac{3}{10})(\frac{2}{10})(\frac{1}{10})$	1A for the numerators
$=\frac{3}{500}$	1A (accept 0.006)
The required probability	
$=\frac{3!}{10^3}$	1A for the numerators
$=\frac{3}{500}$	1A (accept 0.006)
(c) The required probability	
$=C_2^3(\frac{4}{10})^2(1-\frac{4}{10})$	1M for $p^2(1-p)$ + 1A for all being correct
$=\frac{36}{125}$	1A (accept 0.288)
The required probability	
$=3(\frac{4}{10})(\frac{3}{10})(\frac{6}{10})+3(\frac{4}{10})(\frac{1}{10})(\frac{6}{10})$	1M for the two cases + 1A for all being correct
$=\frac{36}{125}$	1A (accept 0.288)
The required probability	
$=3(\frac{C_1^4C_1^3C_1^6}{10^3})+3(\frac{C_1^4C_1^6}{10^3})$	1M for the two cases + 1A for all being correct
$=\frac{36}{125}$	1A (accept 0.288)
The required probability	
$=3!(\frac{C_2^4}{10^2})(\frac{6}{10})+3(\frac{4}{10})(\frac{1}{10})(\frac{6}{10})$	1M for the two cases + 1A for all being correct
$=\frac{36}{125}$	1A (accept 0.288)
	----------(7)

Solution	Marks

7. (a) (i) $\because \lim_{x\to -3^-} \dfrac{2x+5}{2x+6} = +\infty$ and $\lim_{x\to -3^+} \dfrac{2x+5}{2x+6} = -\infty$

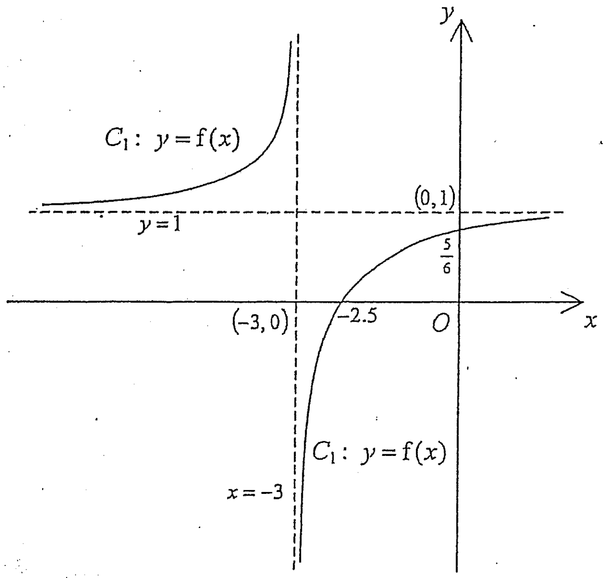
$\therefore$ the equation of the vertical asymptote to C_1 is $x=-3$. — 1A

$\because \lim_{x\to\pm\infty} \dfrac{2x+5}{2x+6} = \lim_{x\to\pm\infty} \dfrac{2+\frac{5}{x}}{2+\frac{6}{x}} = 1$

$\therefore$ the equation of the horizontal asymptote to C_1 is $y=1$. — 1A

(ii) The x-intercept of C_1 is -2.5 .

The y-intercept of C_1 is $\dfrac{5}{6}$.



1A for all asymptotes of C_1
1A for all intercepts of C_1
1A for shape and position of C_1

---------(5)

(b) The equation of the vertical asymptote to C_2 is $x=-3$.
The equation of the horizontal asymptote to C_2 is $y=-1$.
The x-intercept of C_2 is -2.5 .
The y-intercept of C_2 is $\dfrac{-5}{6}$.
Also, for $f(x)=g(x)$, we have

$$\frac{2x+5}{2x+6} = \frac{-(2x+5)}{2x+6}$$

$x=-2.5$ (since $x \neq -3$)

Therefore, the coordinates of the point of intersection are $(-2.5, 0)$.

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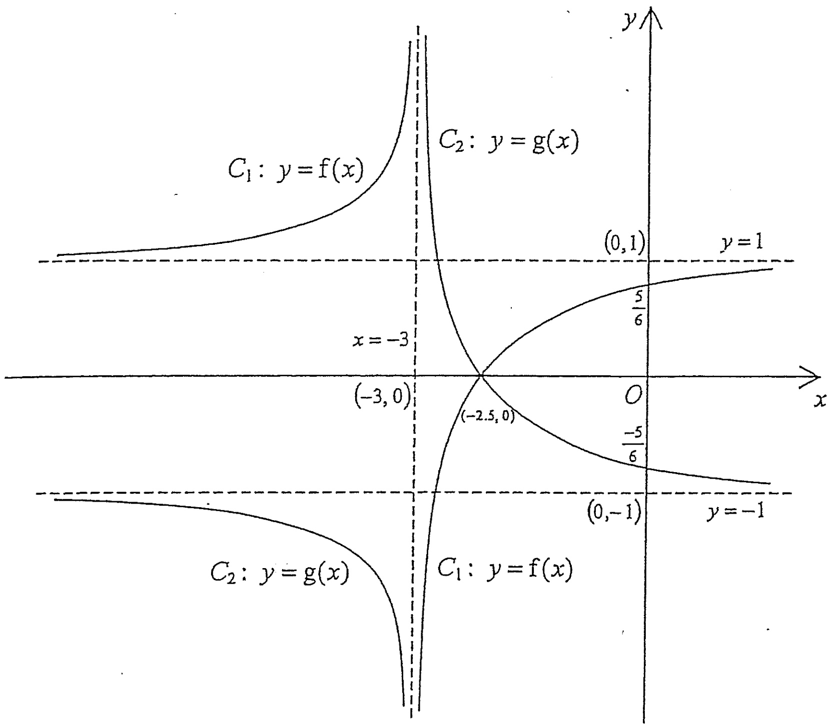
Solution	Marks
	1A for all asymptotes of C_2 1M for shape and position of C_2 1A for all intercepts of C_2 1A for the intersection point
	---------(4)
(c) $\int_{-2.5}^{\lambda}(\mathrm{f}(x)-\mathrm{g}(x))\,dx = 2\lambda+5+\ln(\lambda-7)$	1A
$2\int_{-2.5}^{\lambda}\frac{2x+5}{2x+6}\,dx = 2\lambda+5+\ln(\lambda-7)$	
$2\int_{-2.5}^{\lambda}(1-\frac{1}{2x+6})\,dx = 2\lambda+5+\ln(\lambda-7)$	1M for division
$2\left[x-\frac{1}{2}\ln(2x+6)\right]_{-2.5}^{\lambda} = 2\lambda+5+\ln(\lambda-7)$	1A for correct integration
$2\lambda-\ln(2\lambda+6)+5 = 2\lambda+5+\ln(\lambda-7)$	
$\ln((2\lambda+6)(\lambda-7))=0$	1M for $\ln \mathrm{h}(\lambda)=$ constant
$(2\lambda+6)(\lambda-7)=1$	
$2\lambda^2-8\lambda-43=0$	1A
$\lambda=\frac{8\pm2\sqrt{102}}{4}$	
$\lambda=2\pm\frac{1}{2}\sqrt{102}$	
Since $\lambda>7$, we have $\lambda=2+\frac{1}{2}\sqrt{102}$.	1A
	---------(6)

Solution	Marks
8. (a) The total fuel consumption	
$= \int_0^{15} \mathrm{f}(t)\,\mathrm{d}t$	1A withhold 1A for omitting this step
$\approx \frac{15-0}{10}\left(\mathrm{f}(0)+\mathrm{f}(15)+2\left(\mathrm{f}(3)+\mathrm{f}(6)+\mathrm{f}(9)+\mathrm{f}(12)\right)\right)$	1M for trapezoidal rule
≈ 27.40558785	
≈ 27.4036 litres	1A $a-1$ for r.t. 27.404 ----------(3)
(b) The total fuel consumption	
$= \int_0^{15} \mathrm{g}(t)\,\mathrm{d}t$	
$= \int_0^{15} \frac{1}{145} t(15-t)^2\,\mathrm{d}t$	
$= \frac{1}{145}\int_0^{15} (225t - 30t^2 + t^3)\,\mathrm{d}t$	1A
$= \frac{1}{145}\left[\frac{225t^2}{2} - 10t^3 + \frac{t^4}{4}\right]_0^{15}$	1A for correct integration
$= \frac{3375}{116}$ litres	1A
≈ 29.09482759	
≈ 29.0948 litres	$a-1$ for r.t. 29.095 ----------(3)
(c) $\mathrm{f}(t) = \frac{1}{4} t(15-t) e^{\frac{-t}{4}}$	
$\frac{\mathrm{d}\mathrm{f}(t)}{\mathrm{d}t} = \frac{1}{4}(15-2t)\,e^{\frac{-t}{4}} - \frac{1}{16} t(15-t)\,e^{\frac{-t}{4}}$	1M for Product Rule or Chain Rule
$= \frac{1}{16}(t^2 - 23t + 60)\,e^{\frac{-t}{4}}$	1A must be simplified
$= \frac{1}{16}(t-3)(t-20)\,e^{\frac{-t}{4}}$	
$\frac{\mathrm{d}\mathrm{f}(t)}{\mathrm{d}t} \begin{cases} >0 & \text{if } 0 \le t < 3 \\ =0 & \text{if } t = 3 \\ <0 & \text{if } 3 < t \le 15 \end{cases}$	1M for testing + 1A
So, we have the greatest value	
$= \mathrm{f}(3)$	
$= 9e^{\frac{-3}{4}}$	1A provided the testing is correct
≈ 4.251298975	
≈ 4.2513	$a-1$ for r.t. 4.251

Solution	Marks
$f(t)=\frac{1}{4}t(15-t)e^{\frac{-t}{4}}$	
$\frac{df(t)}{dt}=\frac{1}{4}(15-2t)e^{\frac{-t}{4}}-\frac{1}{16}t(15-t)e^{\frac{-t}{4}}$	1M for Product Rule or Chain Rule
$=\frac{1}{16}(t^2-23t+60)e^{\frac{-t}{4}}$	1A must be simplified
$=\frac{1}{16}(t-3)(t-20)e^{\frac{-t}{4}}$	
For $\frac{df(t)}{dt}=0$, we have $t=3$ or $t=20$ (rejected since $0\le t\le 15$).	
$\frac{d^2f(t)}{dt^2}=\frac{-1}{64}(t-3)(t-20)e^{\frac{-t}{4}}+\frac{1}{16}(2t-23)e^{\frac{-t}{4}}$	
$=\frac{-1}{64}(t^2-31t+152)e^{\frac{-t}{4}}$	
$\left.\frac{d^2f(t)}{dt^2}\right\|_{t=3}=\frac{-17}{16}e^{\frac{-3}{4}}\approx -0.501889462<0$	1M for testing + 1A
So, we have the greatest value	
$=f(3)$	
$=9e^{\frac{-3}{4}}$	1A provided the testing is correct
≈ 4.251298975	
≈ 4.2513	$a-1$ for r.t. 4.251
	----------(5)
(d) (i) $\frac{d^2f(t)}{dt^2}$	
$=\frac{-1}{64}(t-3)(t-20)e^{\frac{-t}{4}}+\frac{1}{16}(2t-23)e^{\frac{-t}{4}}$	
$=\frac{-1}{64}(t^2-31t+152)e^{\frac{-t}{4}}$	1A must be simplified
(ii) $\left.\frac{d^2f(t)}{dt^2}\right\|_{t=0}=\frac{-19}{8}<0$ $\left.\frac{d^2f(t)}{dt^2}\right\|_{t=15}=\frac{11}{8}e^{\frac{-15}{4}}>0$	1M for testing two values of t in [0, 15] or for factorizing $\frac{d^2f(t)}{dt^2}e^{\frac{t}{4}}$.
Therefore, by considering $\frac{d^2f(t)}{dt^2}$, we cannot determine whether the estimate in (a) is an over-estimate or an under-estimate.	1A
Thus, by considering $\frac{d^2f(t)}{dt^2}$, we cannot determine whether the total fuel consumption from $t=0$ to $t=15$ when using driving tactic A will be less than that of using driving tactic B.	1M
	----------(4)

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Solution	Marks
9. (a) (i) $\frac{dy}{dx}=-\alpha\beta^{-x}$	
$-\frac{dy}{dx}=\alpha\beta^{-x}$	
$\ln(-\frac{dy}{dx})=\ln\alpha-(\ln\beta)x$	1A do not accept $\ln\alpha-\ln\beta x$
$-0.125=-\ln\beta$	
$\beta\approx 1.133148453$	
$\beta\approx 1.133$ (correct to 3 decimal places)	1A
(ii) $\beta^{-x}=e^{-\lambda x}$ for all $x\geq 0$	
$\lambda=\ln\beta$	
$\lambda=0.125$	1A accept $\lambda\approx 0.1249$ a–1 for r.t. 0.125
$\frac{dy}{dx}=-\alpha\beta^{-x}$	
$\frac{dy}{dx}=-\alpha e^{-\lambda x}$	1M can be absorbed
$y=-\alpha\int e^{-\lambda x}dx$	1M for finding y by integration
$y=\frac{\alpha}{\lambda}e^{-\lambda x}+C$ where C is a constant.	1A for correct integration
Note that $y(0)=76$ and $y(2)=59.2$. Then, we have	
$\frac{\alpha}{\lambda}+C=76$ and $\frac{\alpha}{\lambda}e^{-2\lambda}+C=59.2$.	1M
So, we have $\frac{\alpha}{\lambda}(1-e^{-2\lambda})=16.8$. Hence, we have	
$\alpha\approx 9.493704497$	
$\alpha\approx 9.5$ (correct to 1 decimal place)	1A
$\beta^{-x}=e^{-\lambda x}$ for all $x\geq 0$	
$\lambda=\ln\beta$	
$\lambda=0.125$	1A accept $\lambda\approx 0.1249$ a–1 for r.t. 0.125
$\frac{dy}{dx}=-\alpha\beta^{-x}$	
$\frac{dy}{dx}=-\alpha e^{-\lambda x}$	1M can be absorbed
$[y]_0^2=-\alpha\int_0^2 e^{-\lambda x}dx$	1M for integrating from $x=0$ to $x=2$ on both sides
$y(2)-y(0)=\frac{\alpha}{\lambda}\left[e^{-\lambda x}\right]_0^2$	1A for correct integration
So, we have $\frac{\alpha}{\lambda}(1-e^{-2\lambda})=16.8$. Hence, we have	1M
$\alpha\approx 9.493704497$	
$\alpha\approx 9.5$ (correct to 1 decimal place)	1A
	----------(8)

Solution	Marks
(b) (i) By (a)(ii),	
~~$C \approx 0.050364028$~~	
$C \approx 0.0504$	accept $C \in [-0.08, 0.06]$
So, $y \approx 75.94963597e^{-0.125x} + 0.050364028$	accept $y \approx Be^{-0.125x} + C$ where $B \in [75.94, 76.08]$
When $y = 25.2$, we have	
$25.2 \approx 75.94963597e^{-0.125x} + 0.050364028$	1M for leaving x only
~~$x \approx 8.841816111$~~	
$x \approx 8.8$ (correct to 1 decimal place)	1A provided B and C both acceptable
Thus, the altitude of the mountain is 8.8 km above sea-level (correct to the nearest 0.1 km).	
(ii) $\frac{\alpha}{\lambda}e^{-\lambda h} - \frac{\alpha}{\lambda}e^{-2\lambda h} = 13$	1M for using $y(h) - y(2h) = 13$
$\frac{\alpha}{\lambda}(e^{-\lambda h})^2 - \frac{\alpha}{\lambda}e^{-\lambda h} + 13 = 0$	1M for transforming into a quadratic equation
$75.94963597(e^{-0.125h})^2 - 75.94963597e^{-0.125h} + 13 \approx 0$	
$e^{-0.125h} \approx 0.780773822$ or $e^{-0.125h} \approx 0.219226177$	
$h \approx 1.979758169$ or $h \approx 12.1412048$	1M for taking ln to find h
Note that $h \approx 12.1412048$ is rejected since $h > 8.8$ is impossible.	
Thus, we have $h \approx 2.0$ (correct to 1 decimal place) .	2A provided (b)(i) is correct
	----------(7)

Solution	Marks
10. (a) The required probability	
$= (0.075)(0.94)+(1-0.075)(0.14)$	1M for $(p(0.94)+(1-p)(0.14))$ +1A
$= 0.2$	1A
	----------(3)
(b) The required probability	
$= \frac{(1-0.075)(0.14)}{0.2}$	1M for denominator using (a) +1A
$= 0.6475$	1A (accept $\frac{259}{400}$) a–1 for r.t. 0.648
	----------(3)
(c) (i) $P(M=3)$	
$= (1-0.2)^2(0.2)$	1M for $(1-(a))^2(a)$
$= 0.128$	1A
(ii) μ	
$= \frac{1}{0.2}$	1M for $\frac{1}{(a)}$
$= 5$	
σ	
$= \sqrt{\frac{1-0.2}{0.2^2}}$	1M for $\sqrt{\frac{1-(a)}{(a)^2}}$
$= \sqrt{20}$	1A for both correct
$= 2\sqrt{5}$	
(iii) Putting $k=2\sqrt{5}$ in $P(-k\sigma \le M-\mu \le k\sigma) \ge 1-\frac{1}{k^2}$, we have	1A for $k=2\sqrt{5}$ or $k=\sqrt{20}$
$P(-2\sqrt{5}\sigma \le M-\mu \le 2\sqrt{5}\sigma) \ge 1-(\frac{1}{2\sqrt{5}})^2$.	
By (c)(ii), we have $P(-20 \le M-5 \le 20) \ge 0.95$.	
So, we have $P(-15 \le M \le 25) \ge 0.95$.	1M
Note that $P(-15 \le M < 1) = 0$.	1M for using $P(-l \le M < 1) = 0$ for any $l > 0$
Thus, we have	
$P(1 \le M \le 25)$	
$= P(-15 \le M \le 25) - P(-15 \le M < 1)$	
$= P(-15 \le M \le 25)$	
≥ 0.95	1 do not accept finding the value of $P(1 \le M \le 25)$ directly
	----------(9)

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Solution	Marks
11. (a) The required probability	
$= (C_4^5(0.7)^4(0.3))(0.7)$	1M for binomial probability + 1M for multiplication rule
$= 0.252105$	1A
≈ 0.2521	a−1 for r.t. 0.252
	----------(3)
(b) Let X be the number of red coupons in the 10 packets of brand C potato chips.	
(i) The required probability	
$= \mathrm{P}(X \geq 4)$	1M
$= 1-(0.7)^{10} - C_1^{10}(0.7)^9(0.3) - C_2^{10}(0.7)^8(0.3)^2 - C_3^{10}(0.7)^7(0.3)^3$	1A
≈ 0.3503892816	
≈ 0.3504	1A a−1 for r.t. 0.350
(ii) The required probability	
$= \mathrm{P}(4 \leq X \leq 5)$	1M
$= C_4^{10}(0.7)^6(0.3)^4 + C_5^{10}(0.7)^5(0.3)^5$	
≈ 0.3030402942	
≈ 0.3030	1A a−1 for r.t. 0.303
(iii) The required probability	
$= \mathrm{P}(4 \leq X \leq 5 \mid X \geq 4)$	
$= \dfrac{\mathrm{P}(4 \leq X \leq 5)}{\mathrm{P}(X \geq 4)}$	
$\approx \dfrac{0.3030402942}{0.3503892816}$	1M for numerator using (b)(ii) + 1M for denominator using (b)(i)
≈ 0.864867478	
≈ 0.8649	1A (accept 0.8647 and 0.8648) a−1 for r.t. 0.865
	----------(8)
(c) (i) The required probability	
$= (\mathrm{P}(X \geq 4))^2$	
$\approx (0.3503892816)^2$ (by (b)(i))	1M for $((\text{b})(\text{i}))^2$
≈ 0.122772648	
≈ 0.1228	1A a−1 for r.t. 0.123
(ii) The required probability	
$= 2\,\mathrm{P}(X \geq 4)\,\mathrm{P}(X = 0)$	1M
$\approx 2(0.3503892816)(0.0282475249)$ (by (b)(i))	
≈ 0.019795259	
≈ 0.0198	1A a−1 for r.t. 0.020
	----------(4)

Solution	Marks
12. Let $\$X$ be the amount of money spent by a customer. Then, $X \sim N(428, 100^2)$.	
Also let Y be the number of customers visiting the store in a minute. Then, $X \sim P_0(4)$.	
(a) The required probability	
$= P(X \geq 300)$	
$= P(Z \geq \frac{300-428}{100})$	
$= P(Z \geq -1.28)$	1A accept $P(Z > -1.28)$
$= 0.8997$	1A a−1 for r.t. 0.900
	----------(2)
(b) The required probability	
$= 1 - P(Y=0) - P(Y=1)$	1M for complementary probability
$= 1 - \frac{4^0 e^{-4}}{0!} - \frac{4^1 e^{-4}}{1!}$	1A
$= 1 - 5e^{-4}$	1A
≈ 0.908421805556	
≈ 0.9084	a−1 for r.t. 0.908
	----------(3)
(c) The required probability	
$= P(Y=3)\,(C_2^3 (0.8997)^2 (1-0.8997))$	1M for $C_2^3 (a)^2 (1-(a))$ + 1M for multiplication rule
$= (\frac{4^3 e^{-4}}{3!})(C_2^3 (0.8997)^2 (1-0.8997))$	
≈ 0.047584819333	
≈ 0.0476	1A a−1 for r.t. 0.048
	----------(3)
(d) The required probability	
$= \dfrac{(\frac{4^2 e^{-4}}{2!})(C_2^2 (0.8997)^2) + (\frac{4^3 e^{-4}}{3!})(C_2^3 (0.8997)^2 (1-0.8997))}{\frac{4^2 e^{-4}}{2!} + \frac{4^3 e^{-4}}{3!}}$	1A for denominator + 1M for numerator
$\approx \frac{0.16619104895}{0.34189192592}$	
≈ 0.48609234776	
≈ 0.4861	1A a−1 for r.t. 0.486
	----------(3)
(e) $P(X \geq 600)$	
$= P(Z \geq \frac{600-428}{100})$	
$= P(Z \geq 1.72)$	
$= 0.0427$	1A
Let n be the number of customers visiting the store. Then, we have	
$1-(1-0.0427)^n \geq 0.99$	1M accept $(1-0.0427)^n \leq 0.01$; withhold 1M for using equality or strict inequality
$(0.9573)^n \leq 0.01$	
$n \ln 0.9573 \leq \ln 0.01$	1M for using ln or trial and error
$n \geq \frac{\ln 0.01}{\ln 0.9573}$	
$n \geq 105.5300874$	
Thus, the smallest number of customers visiting the store is 106.	1A
	----------(4)