

AS Mathematics and Statistics

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits *all the marks* allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.
2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).
3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.
4. Use of notation different from those in the marking scheme should not be penalized.
5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
6. Marks may be deducted for poor presentation (*pp*). The symbol $\textcircled{pp-1}$ should be used to denote 1 mark deducted for *pp*. At most deducted 1 mark from Section A and 1 mark from Section B for *pp*. In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.
7. Marks may be deducted for numerical answers with inappropriate degree of accuracy (*a*). The symbol $\textcircled{a-1}$ should be used to denote 1 mark deducted for *a*. At most deducted 1 mark from Section A and 1 mark from Section B for *a*. In any case, do not deduct any marks for *a* in those steps where candidates could not score any marks.
8. Marks entered in the Page Total Box should be the NET total scored on that page.
9. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles.

Solution	Marks
<p>1. (a) (i) $(1+ax)^{-3} = 1 - 3ax + \frac{(-3)(-4)}{2!}(ax)^2 + \frac{(-3)(-4)(-5)}{3!}(ax)^3 + \dots$ $= 1 - 3ax + 6a^2x^2 - 10a^3x^3 + \dots$</p>	<p>1M for any two terms correct 1A</p>
<p>(ii) $-10a^3 = \frac{80}{27}$ $a = \frac{-2}{3}$</p>	<p>1A accept $a \approx -0.6667$</p>
<p>(b) (i) $(3-2x)^{-3} = \frac{1}{27} \left(1 - \frac{2}{3}x\right)^{-3}$ $= \frac{1}{27} \left[1 + 3\left(\frac{2}{3}\right)x + 6\left(\frac{2}{3}\right)^2x^2 + \frac{80}{27}x^3 + \dots\right]$ $= \frac{1}{27} \left(1 + 2x + \frac{8}{3}x^2 + \frac{80}{27}x^3 + \dots\right)$ $= \frac{1}{27} + \frac{2}{27}x + \frac{8}{81}x^2 + \frac{80}{729}x^3 + \dots$</p>	<p>1A 1A</p>
<p>(ii) The range of values of x is $\left \frac{-2}{3}x \right < 1$ i.e. $\frac{-3}{2} < x < \frac{3}{2}$</p>	<p>1M can be absorbed 1A accept $x < \frac{3}{2}$ ------(7)</p>
<p>2. (a) (i) Let $u = 1 + 3te^{-\frac{t}{100}}$. Then, $\frac{du}{dt} = 3e^{-\frac{t}{100}} - \frac{3t}{100}e^{-\frac{t}{100}}$ $= \frac{3}{100}(100-t)e^{-\frac{t}{100}}$</p>	<p>1M for Product Rule + 1A</p>
<p>(ii) $\theta = \int \frac{12(100-t)e^{-\frac{t}{100}}}{25(1+3te^{-\frac{t}{100}})} dt$ $= \int \frac{16}{u} du$ $= 16 \ln(1 + 3te^{-\frac{t}{100}}) + C$ When $t = 0$, $\theta = 16$, we have $C = 16$ $\therefore \theta = 16 \ln(1 + 3te^{-\frac{t}{100}}) + 16$</p>	<p>1A 1M for finding C 1A</p>

Solution

Marks

$$(b) \frac{d\theta}{dt} = \frac{12(100-t)e^{-\frac{t}{100}}}{25(1+3te^{-\frac{t}{100}})}$$

$$\left. \begin{cases} > 0 & \text{if } 0 \leq t < 100 \\ = 0 & \text{if } t = 100 \\ < 0 & \text{if } t > 100 \end{cases} \right\}$$

1M for testing

$\therefore \theta$ attains its greatest value when $t = 100$

Note that $\theta(100) = 16 \ln(1 + 300e^{-1}) + 16$

$$\approx 91.40484176$$

$$\leq 95$$

Thus, the temperature of the surface of the vessel will not get higher than 95°C .

1A

----- (7)

$$3. (a) \frac{dy}{dx} = \frac{4(4x^2 + 3x + 9)^{\frac{1}{2}} - \frac{1}{2}(4x^2 + 3x + 9)^{-\frac{1}{2}}(8x + 3)(4x - 9)}{4x^2 + 3x + 9}$$

1M for quotient rule +
1M for chain rule + 1A

$$= \frac{8(4x^2 + 3x + 9) - (8x + 3)(4x - 9)}{2(4x^2 + 3x + 9)^{\frac{3}{2}}}$$

$$= \frac{84x + 99}{2(4x^2 + 3x + 9)^{\frac{3}{2}}}$$

$$\text{When } x = 10, \frac{dy}{dx} = \frac{84(10) + 99}{2(4(10)^2 + 3(10) + 9)^{\frac{3}{2}}}$$

$$\approx 0.051043308$$

$$\approx 0.0510$$

1A a-1 for r.t. 0.051

(b) The required amount

$$= \lim_{x \rightarrow \infty} \left(3 + \frac{4x - 9}{\sqrt{4x^2 + 3x + 9}} \right)$$

$$= 3 + \lim_{x \rightarrow \infty} \frac{4 - \frac{9}{x}}{\sqrt{4 + \frac{3}{x} + \frac{9}{x^2}}}$$

$$= 3 + 2$$

$$= 5 \text{ grams}$$

1M can be absorbed

1A

----- (6)

Solution	Marks
4. (a) $P(A \cap B) = P(A) P(B A)$ $P(A \cap B) = 0.4P(A)$ $P(A \cap B) = 0.4a$ $P(A \cap B) = P(B) P(A B) = 0.5P(B)$ $0.4a = 0.5P(B)$ $P(B) = \frac{0.4}{0.5}a$ $P(B) = 0.8a$	1M can be absorbed 1A either one 1A
$P(A \cap B) = P(A) P(B A)$ $P(A \cap B) = 0.4P(A)$ $P(A \cap B) = 0.4a$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $0.84 = a + P(B) - 0.4a$ $P(B) = 0.84 - 0.6a$	1M can be absorbed 1A 1M can be absorbed 1A
$P(A \cap B) = P(B) P(A B)$ $P(B) = 2 P(A \cap B)$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $0.84 = a + 2 P(A \cap B) - P(A \cap B)$ $P(A \cap B) = 0.84 - a$ $P(B) = 1.68 - 2a$	1M can be absorbed 1M can be absorbed 1A accept $P(A \cap B) = 1.8a - 0.84$ 1A accept $P(B) = 0.8a$
(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $0.84 = a + 0.8a - 0.4a$ (by (a)) $0.84 = 1.4a$ $a = 0.6$	1M can be absorbed 1A
$P(A \cap B) = P(B) P(A B) = 0.5P(B)$ $0.4a = 0.5P(B)$ $P(B) = \frac{0.4}{0.5}a$ $P(B) = 0.8a$ $0.8a = 0.84 - 0.6a$ (by (a)) $1.4a = 0.84$ $a = 0.6$	1A
$P(A \cap B) = P(A) P(B A)$ $P(A \cap B) = 0.4 P(A)$ $0.84 - a = 0.4a$ $a = 0.6$	1A
(c) $\therefore P(A) P(B) = (0.6)(0.8)(0.6) = 0.288 \neq 0.24 = (0.4)(0.6) = P(A \cap B)$ $\therefore A$ and B are not independent.	1M 1A
$\therefore P(A B) = 0.5 \neq 0.6 = P(A)$ $\therefore A$ and B are not independent.	1M 1A -----(7)

Solution		Marks																														
5. (a)	<table border="1"> <thead> <tr> <th>Stem (tens)</th> <th colspan="4">Leaf (units)</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>2</td> <td>1</td> <td>5</td> <td>9</td> <td></td> </tr> <tr> <td>3</td> <td>6</td> <td>7</td> <td>8</td> <td></td> </tr> <tr> <td>4</td> <td>1</td> <td>7</td> <td>9</td> <td>9</td> </tr> <tr> <td>5</td> <td>2</td> <td>4</td> <td>7</td> <td></td> </tr> </tbody> </table>	Stem (tens)	Leaf (units)				1	2	3	4	5	2	1	5	9		3	6	7	8		4	1	7	9	9	5	2	4	7		<p>1M+1A</p>
Stem (tens)	Leaf (units)																															
1	2	3	4	5																												
2	1	5	9																													
3	6	7	8																													
4	1	7	9	9																												
5	2	4	7																													
(b) (i)	<p>Revised median = 49 minutes</p> <p>Revised Q_1 = 25 minutes</p> <p>Revised Q_3 = 60 minutes</p> <p>Revised interquartile range = 60 - 25 = 35 minutes</p>	<p>1A do not accept 48 minutes</p> <p>accept 24, 26 or 27</p> <p>1A accept 36, 34 or 33</p>																														
(ii)	<p>The revised mean is larger than the original mean. The revised range is larger than the original range.</p>	<p>1A 1A</p>																														
<p>The change in the revised mean is positive. The change in the revised range is positive.</p>		<p>1A 1A</p>																														
		----- (6)																														
6.	Let $\$X$ be the amount of a business transaction. Then, $X \sim N(215, 50^2)$.																															
(a)	<p>The required probability = $P(X > 300)$ = $P\left(Z > \frac{300-215}{50}\right)$ = $P(Z > 1.7)$ = 0.0446</p>	<p>1A accept $P\left(Z \geq \frac{300-215}{50}\right)$</p> <p>1A $\alpha-1$ for r.t. 0.045</p>																														
(b)	<p>The required probability = $C_2^7 (0.0446)^2 (1-0.0446)^5$ ≈ 0.033251802 ≈ 0.0333</p>	<p>1M for Binomial probability</p> <p>1A $\alpha-1$ for r.t. 0.033</p>																														
(c)	<p>The required probability $\approx (0.033251802)(0.0446)$ (by (b)) ≈ 0.001483030369 ≈ 0.0015</p>	<p>1A $\alpha-1$ for r.t. 0.001</p>																														
(d)	<p>$P(X > K) = 64.8\%$ $P\left(Z > \frac{K-215}{50}\right) = 0.648$ $\frac{K-215}{50} = -0.38$ $K = 196$</p>	<p>1M accept $\frac{K-215}{50} = 0.38$</p> <p>1A</p> <p>----- (7)</p>																														

Solution

Marks

7. (a) $\therefore \lim_{x \rightarrow \frac{7}{2}^-} \frac{20-4x}{7-2x} = +\infty$ and $\lim_{x \rightarrow \frac{7}{2}^+} \frac{20-4x}{7-2x} = -\infty$

\therefore the equation of the vertical asymptote to C_1 is $x = \frac{7}{2}$.

$\therefore \lim_{x \rightarrow \pm\infty} \frac{20-4x}{7-2x} = \lim_{x \rightarrow \pm\infty} \frac{\frac{20}{x} - 4}{\frac{7}{x} - 2} = 2$

\therefore the equation of the horizontal asymptote to C_1 is $y = 2$.

1A

1A

------(2)

(b) $\therefore g(-3) = 0$

$\therefore a - 3b = 0$

$\therefore g(0) = 4$

$\therefore \frac{a}{3} = 4$

So, $a = 12$ and $b = 4$

Also, $\lim_{x \rightarrow \pm\infty} \frac{12+4x}{3+cx} = 2$

$\therefore \lim_{x \rightarrow \pm\infty} \frac{\frac{12}{x} + 4}{\frac{3}{x} + c} = 2$

Thus, $c = 2$

Therefore, $a = 12$, $b = 4$ and $c = 2$.

1A for both correct

1M can be absorbed

1A

------(3)

(c) $C_1: y = f(x)$, where $f(x) = \frac{20-4x}{7-2x}$

$C_2: y = g(x)$, where $g(x) = \frac{12+4x}{3+2x}$

When $x = 0$, $f(x) = \frac{20}{7}$

When $f(x) = 0$, $x = 5$

So, the x -intercept of C_1 is 5 and the y -intercept of C_1 is $\frac{20}{7}$.

$\therefore \lim_{x \rightarrow \frac{-3}{2}^-} g(x) = \lim_{x \rightarrow \frac{-3}{2}^-} \frac{12+4x}{3+2x} = -\infty$ and $\lim_{x \rightarrow \frac{-3}{2}^+} g(x) = \lim_{x \rightarrow \frac{-3}{2}^+} \frac{12+4x}{3+2x} = +\infty$

\therefore the equation of the vertical asymptote to C_2 is $x = \frac{-3}{2}$.

Also, $f(x) = g(x)$

$$\frac{20-4x}{7-2x} = \frac{12+4x}{3+2x}$$

$$\frac{5-x}{7-2x} = \frac{3+x}{3+2x}$$

$$(x-5)(2x+3) = (x+3)(2x-7)$$

$$-7x - 15 = -x - 21$$

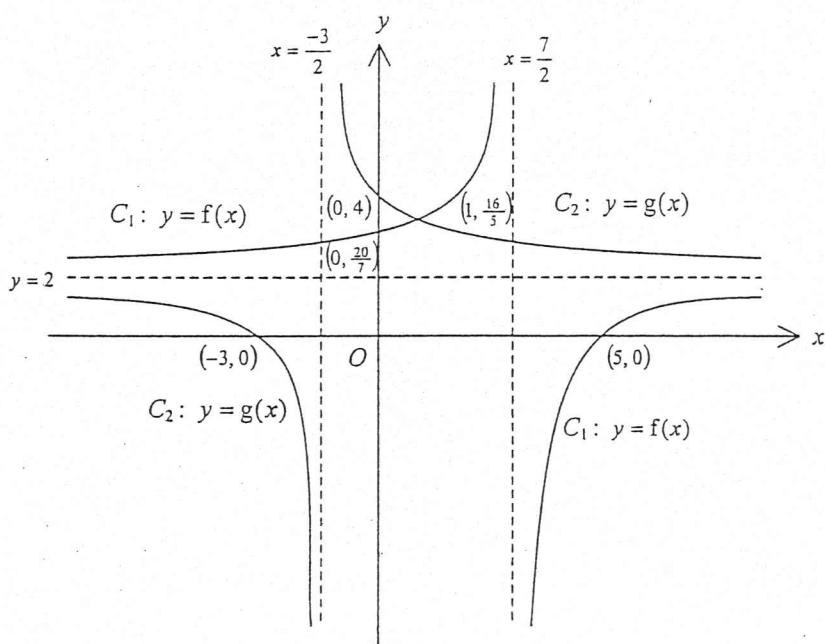
$$-6x = -6$$

$$x = 1$$

Thus, the point of intersection is $(1, \frac{16}{5})$.

Solution

Marks



- 1A all asymptotes
- 1A shape of C_1
- 1A shape of C_2
- 1A intercepts of C_1
- 1A intersection point

----- (5)

(d) $\int_1^\lambda (f(x) - g(x)) dx = 3 \ln 3$

$$\int_1^\lambda \left(\frac{20-4x}{7-2x} - \frac{12+4x}{3+2x} \right) dx = 3 \ln 3$$

$$\int_1^\lambda \left(\left(2 + \frac{6}{7-2x} \right) - \left(2 + \frac{6}{3+2x} \right) \right) dx = 3 \ln 3$$

$$6 \int_1^\lambda \left(\frac{1}{7-2x} - \frac{1}{3+2x} \right) dx = 3 \ln 3$$

$$\frac{-6}{2} [\ln(7-2x) + \ln(3+2x)]_1^\lambda = 3 \ln 3$$

$$[\ln((7-2x)(3+2x))]_1^\lambda = -\ln 3$$

$$\ln[(7-2\lambda)(3+2\lambda)] - \ln 25 = -\ln 3$$

$$(7-2\lambda)(3+2\lambda) = \frac{25}{3}$$

$$3(2\lambda+3)(2\lambda-7) + 25 = 0$$

$$3(4\lambda^2 - 8\lambda - 21) + 25 = 0$$

$$12\lambda^2 - 24\lambda - 38 = 0$$

$$6\lambda^2 - 12\lambda - 19 = 0$$

$$\lambda = \frac{12 \pm \sqrt{600}}{12}$$

$$\lambda = 1 \pm \frac{5}{6}\sqrt{6}$$

$$\lambda = 1 + \frac{5}{6}\sqrt{6} \quad \left(\because 0 < \lambda < \frac{7}{2} \right)$$

1A accept $\int_\lambda^1 (g(x) - f(x)) dx = 3 \ln 3$

1M for either one correct

1A for integration

1M

1A

----- (5)

Solution	Marks
<p>8. (a) $f(t) = 5 + 2^{-kt+h}$ $\ln(f(t) - 5) = -(k \ln 2)t + h \ln 2$</p>	<p>1A do not accept $-k \ln 2t + h \ln 2$ -----(1)</p>
<p>(b) $-k \ln 2 = -0.35$ $k \approx 0.504943264$ $k \approx 0.5$ (correct to 1 decimal place)</p>	<p>1A</p>
<p>$h \ln 2 = 1.39$ $h \approx 2.005346107$ $h \approx 2.0$ (correct to 1 decimal place)</p>	<p>1A -----(2)</p>
<p>(c) The total amount $= \int_2^{12} g(t) dt$ $\approx \frac{12-2}{10} (g(2) + g(12) + 2(g(4) + g(6) + g(8) + g(10)))$ ≈ 75.77699747 ≈ 75.7770 thousand barrels</p>	<p>1M for trapezoidal rule 1A -----(2)</p>
<p>(d) (i) $2^t = e^{at}$ for all $t \geq 0$ $t \ln 2 = at$ for all $t \geq 0$ $a = \ln 2$</p>	<p>1A accept $a \approx 0.6931$</p>
<p>(ii) $g(t) = 5 + \ln(t+1) + 2^{\frac{-t}{2}+2}$ $= 5 + \ln(t+1) + 4e^{\left(\frac{-\ln 2}{2}\right)t}$ $\frac{dg(t)}{dt} = \frac{1}{t+1} + (4)\left(\frac{-\ln 2}{2}\right)e^{\left(\frac{-\ln 2}{2}\right)t}$ $= \frac{1}{t+1} - 2(\ln 2)e^{\left(\frac{-\ln 2}{2}\right)t}$ $\frac{d^2g(t)}{dt^2} = \frac{-1}{(t+1)^2} - (2 \ln 2)\left(\frac{-\ln 2}{2}\right)e^{\left(\frac{-\ln 2}{2}\right)t}$ $= (\ln 2)^2 e^{\left(\frac{-\ln 2}{2}\right)t} - \frac{1}{(t+1)^2}$ $= (\ln 2)^2 2^{\frac{-t}{2}} - \frac{1}{(t+1)^2}$</p>	<p>1A for the first term + 1M for Chain Rule 1M for the second term 1A for all being correct</p>
<p>$\therefore p(t) = (\ln 2)^2 2^{\frac{-t}{2}}$</p>	<p>1A accept $p(t) = (\ln 2)^2 e^{\left(\frac{-\ln 2}{2}\right)t}$</p>

Solution

Marks

(iii) $\therefore p(2) = (\ln 2)^2 2^{-1} \approx 0.240226506 \approx 0.2402$

$q(2) = \frac{1}{9} \approx 0.111111111 \approx 0.1111$

$\therefore p(2) > q(2)$

It is known that $y = p(t)$ and $y = q(t)$ have no intersection, where $2 \leq t \leq 12$.

So, we have $p(t) > q(t)$ for all $2 \leq t \leq 12$.

$\therefore \frac{d^2 g(t)}{dt^2} > 0$ on $[2, 12]$

Thus, the estimate is an over-estimate of I .

1M for testing

1A

1M

1M

----- (10)

Solution	Marks
<p>9. (a) $\therefore P(0) = 5.9$ $\therefore a + \frac{1}{5}(0 - 0 - 8) = 5.9$ So, $a = 7.5$</p> <p>$P(t) = 7.5 + \frac{1}{5}(t^2 - 8t - 8)e^{-kt}$ $\therefore P(8) - P(4) = 1.83$ $\therefore -1.6e^{-8k} + 4.8e^{-4k} = 1.83$ $160(e^{-4k})^2 - 480e^{-4k} + 183 = 0$ $e^{-4k} \approx 2.551784198$ or $e^{-4k} \approx 0.448215801$ $k \approx -0.2341982$ or $k \approx 0.200620116$</p> <p>$\therefore k > 0$ $\therefore k \approx 0.2$ (correct to 1 decimal place)</p>	<p>1A</p> <p>1M+1A 1M can be absorbed</p> <p>1A ------(5)</p>
<p>(b) $P(t) = \frac{15}{2} + \frac{1}{5}(t^2 - 8t - 8)e^{-0.2t}$</p> <p>(i) $\frac{dP(t)}{dt} = \frac{-1}{25}[(t^2 - 8t - 8) - 5(2t - 8)]e^{-0.2t}$ $= \frac{-1}{25}(t^2 - 18t + 32)e^{-0.2t}$ $= \frac{-1}{25}(t - 2)(t - 16)e^{-0.2t}$</p> <p>For $\frac{dP(t)}{dt} = 0$, we have $t = 2$ or $t = 16$.</p> <p>$\frac{dP(t)}{dt} \begin{cases} < 0 & \text{if } 0 \leq t < 2 \\ = 0 & \text{if } t = 2 \\ > 0 & \text{if } 2 < t < 16 \end{cases}$</p> <p>So, the minimum pH value occurred at $t = 2$.</p> <p>$\frac{dP(t)}{dt} \begin{cases} > 0 & \text{if } 2 < t < 16 \\ = 0 & \text{if } t = 16 \\ < 0 & \text{if } t > 16 \end{cases}$</p> <p>So, the maximum pH value occurred at $t = 16$.</p>	<p>1M for Product Rule or Chain Rule</p> <p>1A independent of the obtained value of a</p> <p>1M+1A</p> <p>1M+1A accept max at $t = 0$ and at $t = 16$</p>

Solution

Marks

$$\begin{aligned} \frac{dP(t)}{dt} &= \frac{-1}{25} [(t^2 - 8t - 8) - 5(2t - 8)] e^{-0.2t} \\ &= \frac{-1}{25} (t^2 - 18t + 32) e^{-0.2t} \\ &= \frac{-1}{25} (t - 2)(t - 16) e^{-0.2t} \end{aligned}$$

For $\frac{dP(t)}{dt} = 0$, we have $t = 2$ or $t = 16$.

$$\begin{aligned} \frac{d^2P(t)}{dt^2} &= \frac{1}{125} [t^2 - 18t + 32 - 5(2t - 18)] e^{-0.2t} \\ &= \frac{1}{125} (t^2 - 28t + 122) e^{-0.2t} \end{aligned}$$

$$\left. \frac{d^2P(t)}{dt^2} \right|_{t=2} \approx 0.375379225 > 0$$

So, the minimum pH value occurred at $t = 2$.

$$\left. \frac{d^2P(t)}{dt^2} \right|_{t=16} \approx -0.022826834 < 0$$

So, the maximum pH value occurred at $t = 16$.

1M for Product Rule or Chain Rule

1A independent of the obtained value of a

1M+1A

1M+1A accept max at $t = 0$ and at $t = 16$

$$\begin{aligned} \text{(ii)} \quad \frac{d^2P}{dt^2} &= \frac{1}{125} [t^2 - 18t + 32 - 5(2t - 18)] e^{-0.2t} \\ &= \frac{1}{125} (t^2 - 28t + 122) e^{-0.2t} \end{aligned}$$

$$\therefore \frac{d^2P}{dt^2} = \frac{1}{125} (t - (14 - \sqrt{74})) (t - (14 + \sqrt{74})) e^{-0.2t}$$

$$5 < 14 - \sqrt{74} < 6 \quad \text{and} \quad 22 < 14 + \sqrt{74} < 23$$

$$\therefore \frac{d^2P}{dt^2} > 0 \quad \text{for all } t \geq 23.$$

1A

1

----- (8)

(c) The required pH value

$$= \lim_{t \rightarrow \infty} \left(7.5 + \frac{1}{5} (t^2 - 8t - 8) e^{-0.2t} \right)$$

$$= 7.5 + \frac{1}{5} \lim_{t \rightarrow \infty} (t^2 e^{-0.2t}) - \frac{8}{5} \lim_{t \rightarrow \infty} (t e^{-0.2t}) - \frac{8}{5} \lim_{t \rightarrow \infty} e^{-0.2t}$$

$$= 7.5 + \frac{1}{5} (0) - \frac{8}{5} (0) - \frac{8}{5} (0) \quad \left(\because \lim_{t \rightarrow \infty} (t e^{-0.2t}) = \left(\lim_{t \rightarrow \infty} \frac{1}{t} \right) \left(\lim_{t \rightarrow \infty} t^2 e^{-0.2t} \right) = (0)(0) = 0 \right)$$

$$= 7.5$$

1A for $\lim_{t \rightarrow \infty} (t e^{-0.2t}) = 0$ (can be absorbed)

1M accept the required pH value = a

----- (2)

Solution	Marks
<p>10. (a) Sample mean</p> $= \frac{12(1)+14(2)+10(3)+6(4)+2(5)+1(6)}{5+12+14+10+6+2+1}$ $= 2.2$	<p>1A</p>
<p>Sample Standard deviation</p> $= \sqrt{\frac{12(1^2)+14(2^2)+10(3^2)+6(4^2)+2(5^2)+1(6^2)-(50)(2.2)^2}{5+12+14+10+6+2+1-1}}$ $= \sqrt{2}$ ≈ 1.414213562 ≈ 1.4142	<p>1A</p> <p>a-1 for r.t. 1.414 ------(2)</p>
<p>(b) (i) The required probability</p> $= \frac{2.2^0 e^{-2.2}}{0!} + \frac{2.2^1 e^{-2.2}}{1!} + \frac{2.2^2 e^{-2.2}}{2!} + \frac{2.2^3 e^{-2.2}}{3!}$ ≈ 0.819352421 ≈ 0.8194	<p>1M for the 4 cases + 1M for Poisson probability</p> <p>1A (accept 0.8193) a-1 for r.t. 0.819</p>
<p>(ii) The required probability</p> $\approx C_2^6 (0.819352421)^2 (1 - 0.819352421)^4$ ≈ 0.010724111 ≈ 0.0107	<p>1M for Binomial probability + 1M for using (b)(i)</p> <p>1A a-1 for r.t. 0.011 ------(6)</p>
<p>(c) (i) The required probability</p> $= C_2^3 (0.55)^2 (0.45)$ $= 0.408375$ ≈ 0.4084	<p>1M for (0.55)²(0.45)</p> <p>1A</p> <p>a-1 for r.t. 0.408</p>
<p>(ii) The required probability</p> $= \frac{2.2^2 e^{-2.2}}{2} (0.55^2)$ ≈ 0.081113452 ≈ 0.0811	<p>1M</p> <p>1A a-1 for r.t. 0.081</p>
<p>(iii) The required probability</p> $\approx \frac{\left(\frac{2.2^2 e^{-2.2}}{2!}\right)(0.55)^2 + \left(\frac{2.2^3 e^{-2.2}}{3!}\right)(0.55)^3}{0.819352421}$ ≈ 0.138925825 ≈ 0.1389	<p>1M for numerator + 1M for denominator using (b)(i)</p> <p>1A a-1 for r.t. 0.139 ------(7)</p>

Solution	Marks
11. (a) The required probability $= (1) \binom{1}{n}$ $= \frac{1}{n}$	1A
The required probability $= (n) \binom{1}{n} \binom{1}{n}$ $= \frac{1}{n}$	1A
	----- (1)
(b) (i) The required probability $= p$	1A
(ii) $p + p + \frac{1}{n} = 1$ $p = \frac{1}{2} \left(1 - \frac{1}{n}\right)$	1M 1A
(iii) $p \geq 0.46$ $\frac{1}{2} \left(1 - \frac{1}{n}\right) \geq 0.46$ $n \geq 12.5$ $\therefore n \text{ is a positive integer.}$ $\therefore \text{the least value of } n \text{ is } 13.$	1A can be absorbed 1M 1A ----- (6)
(c) (i) The required probability $= \left(\frac{5}{6}\right)^4 \frac{1}{6}$ $= \frac{625}{7776}$ ≈ 0.080375514 ≈ 0.0804	1M for $\left(\frac{5}{6}\right)^k \frac{1}{6}$ 1A $a-1$ for r.t. 0.080
(ii) The required probability $= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^6 \left(\frac{1}{6}\right) + \dots$ $= \frac{1}{6}$ $= \frac{1}{1 - \frac{25}{36}}$ $= \frac{6}{11}$ ≈ 0.545454545 ≈ 0.5455	1A must indicate infinite series and have at least 3 terms 1M for sum of GP 1A $a-1$ for r.t. 0.545

Solution	Marks
<p>(iii) The required probability</p> $= \frac{\left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^7 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^9 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^{11} \left(\frac{1}{6}\right) + \dots}{1 - \frac{6}{11}}$ $= \frac{\left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^5}{1 - \frac{25}{36}}$ $= \frac{\frac{5}{11}}{\frac{11}{11}}$ $= \frac{625}{1296}$ ≈ 0.482253086 ≈ 0.4823	<p>1M for denominator using 1-(c)(ii) + 1A for numerator</p> <p>1A</p> <p>$\alpha-1$ for r.t. 0.482</p>
<p>The required probability</p> $= \frac{\frac{5}{11} - \left(\frac{5}{6}\right) \left(\frac{1}{6}\right) - \left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right)}{1 - \frac{6}{11}}$ $= \frac{\frac{5}{11} - \frac{5}{36} - \frac{125}{1296}}{\frac{11}{11}}$ $= \frac{625}{1296}$ ≈ 0.482253086 ≈ 0.4823	<p>1M for denominator using 1-(c)(ii) + 1A for numerator</p> <p>1A</p> <p>$\alpha-1$ for r.t. 0.482</p>
<p>----- (8)</p>	

Solution

Marks

12. (a) The required probability

$$= 1 - \frac{C_7^{17} + C_7^{13}}{C_7^{30}}$$

$$= \frac{38743}{39150}$$

$$\approx 0.989604086$$

$$\approx 0.9896$$

1M for counting cases +
1A for correctness of probability

1A

a-1 for r.t. 0.990
------(3)

(b) The required probability

$$= \frac{C_4^{17} C_3^{13} + C_5^{17} C_2^{13} + C_6^{17} C_1^{13}}{C_7^{30}}$$

$$= \frac{38743}{39150}$$

$$= \frac{1498}{2279}$$

$$\approx 0.657305835$$

$$\approx 0.6573$$

1M for denominator using (a) +
1A for numerator

1A

a-1 for r.t. 0.657

The required probability

$$= \frac{C_4^{17} C_3^{13} + C_5^{17} C_2^{13} + C_6^{17} C_1^{13}}{C_7^{30} - C_7^{17} - C_7^{13}}$$

$$= \frac{1498}{2279}$$

$$\approx 0.657305835$$

$$\approx 0.6573$$

1M for denominator using (a) +
1A for numerator

1A

a-1 for r.t. 0.657

------(3)

(c) Let \$X\$ be the amount of money collected by a boy
and \$Y\$ be the amount of money collected by a girl.
Then, \$X \sim N(673, 100^2)\$ and \$Y \sim N(708, 100^2)\$.

(i) The required probability

$$= P(X > 800)$$

$$= P\left(Z > \frac{800 - 673}{100}\right)$$

$$= P(Z > 1.27)$$

$$= 0.102$$

1M accept \$Z \geq \frac{800 - 708}{100}\$

1A accept 0.1020

Solution	Marks
<p>(ii) $P(Y > 800)$ $= P\left(Z > \frac{800 - 708}{100}\right)$ $= P(Z > 0.92)$ $= 0.1788$</p> <p>The required probability $= \left(C_1^3 (0.102)(0.898)^2\right) \left(C_1^4 (0.1788)(0.8212)^3\right)$ ≈ 0.097734619 ≈ 0.0977</p>	<p>1A</p> <p>1M for Binomial probability + 1M for Binomial \times Binomial</p> <p>1A $a-1$ for r.t. 0.098</p>
<p>(iii) The required probability $\approx \frac{0.097734619}{0.097734619 + C_2^3 (0.102)^2 (0.898)(0.8212)^4 + (0.898)^3 C_2^4 (0.1788)^2 (0.8212)^2}$ ≈ 0.478730045 ≈ 0.4787</p>	<p>1M for numerator +1M for denominator</p> <p>1A (accept 0.4786) $a-1$ for r.t. 0.479 ------(9)</p>