2003-ASL
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HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG ADVANCED LEVEL EXAMINATION 2003

MATHEMATICS AND STATISTICS AS-LEVEL

8.30 am – 11.30 am (3 hours)

This paper must be answered in English

1. This paper consists of Section A and Section B.
2. Answer ALL questions in Section A, using the AL(E) answer book.
3. Answer any FOUR questions in Section B, using the AL(C)2 answer book.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

SECTION A (40 marks)
Answer ALL questions in this section.
Write your answers in the AL(E) answer book.

1. (a) Let $|ax| < 1$.

(i) Expand $(1+ax)^{-3}$ in ascending powers of x as far as the term in x^3 .

(ii) If the coefficient of x^3 in the expansion of $(1+ax)^{-3}$ is $\frac{80}{27}$, find the value of a .

(b) (i) Using the results in (a), or otherwise, expand $(3-2x)^{-3}$ in ascending powers of x as far as the term in x^3 .

(ii) State the range of values of x for which the expansion of $(3-2x)^{-3}$ is valid.

(7 marks)

2. After a fixed amount of hot liquid is poured into a vessel, the rate of change of the temperature θ of the surface of the vessel can be modelled by

$$\frac{d\theta}{dt} = \frac{12(100-t)e^{\frac{-t}{100}}}{25(1+3te^{\frac{-t}{100}})} ,$$

where θ is measured in °C and $t\ (\geq 0)$ is the time measured in seconds. Initially $(t = 0)$, the temperature of the surface of the vessel is 16°C .

(a) (i) Let $u = 1 + 3te^{\frac{-t}{100}}$, find $\frac{du}{dt}$.

(ii) Using the result of (i), or otherwise, express θ in terms of t .

(b) Will the temperature of the surface of the vessel get higher than 95°C ? Explain your answer briefly.

(7 marks)

3. A chemical X is continuously added to a solution to form a substance Y. The total amount of Y formed is given by

$$y = 3 + \frac{4x-9}{\sqrt{4x^2+3x+9}} ,$$

where x grams and y grams are the total amount of X added and the total amount of Y formed respectively.

(a) Find $\frac{dy}{dx}$ when 10 grams of X is added to the solution.

(b) Estimate the total amount of Y formed if X is indefinitely added to the solution.

(6 marks)

4. A and B are two events. Suppose that $P(A|B) = 0.5$, $P(B|A) = 0.4$ and $P(A \cup B) = 0.84$. Let $P(A) = a$, where $a > 0$.

(a) Express $P(A \cap B)$ and $P(B)$ in terms of a .

(b) Using the results of (a), or otherwise, find the value of a .

(c) Are A and B independent events? Explain your answer briefly.

(7 marks)

5. A researcher conducted a study on the time (in minutes) spent on using the Internet by university students. Thirty questionnaires were sent out and only 19 were returned. The results are as follows:

12	13	14	15	15	21	25	29
36	37	38	41	47	49	49	49
52	54	57					

(a) Construct a stem and leaf diagram for these data.

(b) Suppose that the researcher has received eight more questionnaires. Three of them show that the time spent on using the Internet is one hour. The others show that the time spent is more than one hour.

(i) Find the revised median and the revised interquartile range of the time spent.

(ii) Describe briefly the change in the mean and the change in the range of the time spent.

(6 marks)

6. The amount of money involved in a business transaction follows a normal distribution with mean \$215 and standard deviation \$50 . Any transaction with an amount more than \$300 is classified as a Type A transaction.

(a) Find the probability that a transaction will be classified as Type A.

(b) Find the probability that in 7 randomly selected transactions, exactly 2 transactions will be classified as Type A.

(c) Find the probability that the 8th randomly selected transaction is the 3rd transaction which is classified as Type A.

(d) It is known that 64.8% of the transactions each exceeds $\$K$. Find K .

(7 marks)

SECTION B (60 marks)
Answer any FOUR questions in this section. Each question carries 15 marks.
Write your answers in the AL(C)2 answer book.

7. Define $f(x)=\dfrac{20-4x}{7-2x}$ for all $x\neq\dfrac{7}{2}$ and $g(x)=\dfrac{a+bx}{3+cx}$ for all $x\neq\dfrac{-3}{c}$, where a, b and c are positive constants.

Let C_1 and C_2 be the curves $y=f(x)$ and $y=g(x)$ respectively.

It is given that the x-intercept and the y-intercept of C_2 are -3 and 4 respectively. Also, it is known that C_1 and C_2 have a common horizontal asymptote.

(a) Find the equations of the vertical asymptote(s) and horizontal asymptote(s) to C_1.

(2 marks)

(b) Find the values of a, b and c.

(3 marks)

(c) Sketch C_1 and C_2 on the same diagram and indicate the asymptote(s), intercept(s) and the point(s) of intersection of the two curves.

(5 marks)

(d) If the area enclosed by C_1, C_2 and the straight line $x=\lambda$, where $0<\lambda<\dfrac{7}{2}$, is $3\ln 3$ square units, find the exact value(s) of λ.

(5 marks)

8. According to the past production record, an oil company manager modelled the rate of change of the amount of oil production in thousand barrels by

$$f(t)=5+2^{-kt+h},$$

where h and k are positive constants and $t\ (\geq 0)$ is the time measured in months.

(a) Express $\ln(f(t)-5)$ as a linear function of t.

(1 mark)

(b) Given that the slope and the intercept on the vertical axis of the graph of the linear function in (a) are -0.35 and 1.39 respectively, find the values of h and k correct to 1 decimal place.

(2 marks)

(c) The manager decides to start a production improvement plan and predicts the rate of change of the amount of oil production in thousand barrels by

$$g(t)=5+\ln(t+1)+2^{-kt+h},$$

where h and k are the values obtained in (b) correct to 1 decimal place, and $t\ (\geq 0)$ is the time measured in months from the start of the plan.

Using the trapezoidal rule with 5 sub-intervals, estimate the total amount of oil production in thousand barrels from $t=2$ to $t=12$.

(2 marks)

(d) It is known that $g(t)$ in (c) satisfies

$$\frac{d^2g(t)}{dt^2}=p(t)-q(t)\ , \text{ where } q(t)=\frac{1}{(t+1)^2}.$$

(i) If $2^t=e^{at}$ for all $t\geq 0$, find a.

(ii) Find $p(t)$.

(iii) It is known that there is no intersection between the curve $y=p(t)$ and the curve $y=q(t)$, where $2\leq t\leq 12$. Determine whether the estimate in (c) is an over-estimate or under-estimate.

(10 marks)

9. A researcher monitors the process of using micro-organisms to decompose food waste to fertilizer. He records daily the pH value of the waste and models its pH value by

$$P(t) = a + \frac{1}{5}(t^2 - 8t - 8)\,e^{-kt} \;,$$

where $t\ (\geq 0)$ is the time measured in days, a and k are positive constants.

When the decomposition process starts (i.e. $t = 0$) , the pH value of the waste is 5.9 . Also, the researcher finds that $P(8) - P(4) = 1.83$.

(a) Find the values of a and k correct to 1 decimal place.

(5 marks)

(b) Using the value of k obtained in (a) ,

(i) determine on which days the maximum pH value and the minimum pH value occurred respectively ;

(ii) prove that $\frac{d^2P}{dt^2} > 0$ for all $t \geq 23$.

(8 marks)

(c) Estimate the pH value of the waste after a very long time.
[Note : Candidates may use $\lim_{t\to\infty} (t^2 e^{-kt}) = 0$ without proof.]

(2 marks)

10. A bank customer service centre records the number of incoming telephone calls in five-minute time intervals (FMTIs) . The following table lists the number of calls in a sample of 50 FMTIs .

Number of calls	0	1	2	3	4	5	6	7 or more
Frequency	5	12	14	10	6	2	1	0

(a) Find the sample mean and the sample standard deviation of the data in the table.

(2 marks)

(b) The manager of the bank believes that the number of calls in a FMTI follows a Poisson distribution and its mean can be estimated by the sample mean obtained in (a).

(i) Find the probability that there are fewer than 4 calls in a FMTI .

(ii) Find the probability that there are fewer than 4 calls each in exactly 2 FMTIs out of 6 consecutive FMTIs .

(6 marks)

(c) Assume the model in (b) is adopted and it is known that 55% of the calls are from male customers and 45% of the calls are from female customers.

(i) If there are 3 calls in a FMTI , find the probability that exactly 2 calls are from male customers.

(ii) Find the probability that there are 2 calls in a FMTI and they are both from male customers.

(iii) Given that there are fewer than 4 calls in a FMTI , find the probability that there are at least 2 calls and all of these calls are from male customers.

(7 marks)

11. In a game, two boxes A and B each contains n balls which are numbered $1, 2, \ldots, n$. A player is asked to draw a ball randomly from each box. If the number drawn from box A is greater than that from box B, the player wins a prize.

(a) Find the probability that the two numbers drawn are the same. (1 mark)

(b) Let p be the probability that a player wins the prize.

(i) Find, in terms of p only, the probability that the number drawn from box B is greater than that from box A.

(ii) Using the result of (i), express p in terms of n.

(iii) If the above game is designed so that at least 46% of the players win the prize, find the least value of n.

(6 marks)

(c) Two winners, John and Mary, are selected to play another game. They take turns to throw a fair six-sided die. The first player who gets a number '6' wins the game. John will throw the die first.

(i) Find the probability that John will win the game on his third throw.

(ii) Find the probability that John will win the game.

(iii) Given that Mary has won the game, find the probability that Mary did not win the game before her third throw.

(8 marks)

12. A teacher randomly selected 7 students from a class of 13 boys and 17 girls to form a group to take part in a flag-selling activity.

(a) Find the probability that the group consists of at least 1 boy and 1 girl. (3 marks)

(b) Given that the group consists of at least 1 boy and 1 girl, find the probability that there are more than 3 girls in the group. (3 marks)

(c) A group of 3 boys and 4 girls is formed. It is known that the amount of money collected by a boy and a girl in the activity can be modelled respectively by normal distributions with the following means and standard deviations:

Student	Mean	Standard deviation
Boy	\$673	\$100
Girl	\$708	\$100

Any student who collects more than \$800 receives a certificate.

(i) Find the probability that a particular boy in the group will receive a certificate.

(ii) Find the probability that exactly 1 boy and 1 girl in the group will receive certificates.

(iii) Given that the group has received 2 certificates, find the probability that exactly 1 boy and 1 girl received the certificates.

(9 marks)

END OF PAPER