

## AS Mathematics and Statistics

## General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits **all the marks** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.
2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).
3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.
4. Use of notation different from those in the marking scheme should not be penalized.
5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
6. Marks may be deducted for poor presentation (*pp*). The symbol  $\textcircled{pp-1}$  should be used to denote 1 mark deducted for *pp*. At most deducted 1 mark from Section A and 1 mark from Section B for *pp*. In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.
7. Marks may be deducted for numerical answers with inappropriate degree of accuracy (*a*). The symbol  $\textcircled{a-1}$  should be used to denote 1 mark deducted for *a*. At most deducted 1 mark from Section A and 1 mark from Section B for *a*. In any case, do not deduct any marks for *a* in those steps where candidates could not score any marks.
8. Marks entered in the Page Total Box should be the NET total scored on that page.
9. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles.

Solution	Marks
<p>1. <math>\frac{dx}{dt} = \frac{10}{t^3} - 6e^{-3t}</math>  <math>\frac{dy}{dt} = -\frac{20}{t^3} + 2e^{2t}</math></p> $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \left(\frac{dy}{dt}\right) \left(\frac{1}{\frac{dx}{dt}}\right) = \frac{-\frac{20}{t^3} + 2e^{2t}}{\frac{10}{t^3} - 6e^{-3t}}$ <p>For <math>\frac{dy}{dx} = -2</math></p> $\frac{-\frac{20}{t^3} + 2e^{2t}}{\frac{10}{t^3} - 6e^{-3t}} = -2$ $e^{5t} = 6$ $t = \frac{1}{5} \ln 6 (\approx 0.3584)$	<p>1M+1A (1M for <math>(e^{at})' = ae^{at}</math>)</p> <p>1M for Chain Rule and Inverse Function Rule</p> <p>1M</p> <p>1A <math>a^{-1}</math> for r.t. 0.358 -----(5)</p>
<p>2. (a) At <math>t = 2</math>, <math>r(2) = 5.5035(\text{m})</math>  <math>\frac{dV}{dr} = 4\pi r^2</math>  <math>\frac{dr}{dt} = \frac{18 \times 2e^{-t}}{(3+2e^{-t})^2} = \frac{36e^{-t}}{(3+2e^{-t})^2}</math></p> <p>At <math>t = 2</math>, <math>\frac{dV}{dr} = 380.6109</math>  <math>\frac{dr}{dt} = 0.45545</math></p> <p><math>\therefore \frac{dV}{dt} = \frac{dV}{dr} \frac{dr}{dt}</math></p> <p>At <math>t = 2</math>, <math>\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}</math>  <math>= 380.6109 \times 0.45545</math>  <math>= 173.35 (\text{m}^3/\text{h})</math></p>	<p>1A</p> <p>1M</p> <p>1A (Accept : 173.31–173.39) <math>a^{-1}</math> for more than 2 d.p.</p>
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{36e^{-t}}{(3+2e^{-t})^2}$ $\frac{dV}{dt} = 4\pi r^2 \cdot \frac{36e^{-t}}{(3+2e^{-t})^2}$ $= \frac{144\pi r^2 e^{-t}}{(3+2e^{-t})^2}$ <p>At <math>t = 2</math>,  <math>r \approx 5.50346</math></p> <p><math>\therefore \frac{dV}{dt} \approx 173.35 (\text{m}^3/\text{h})</math></p>	<p>1M</p> <p>1A</p> <p>(Accept : <math>\frac{dV}{dt} = \frac{46656\pi e^{-t}}{(3+2e^{-t})^4}</math>)</p> <p>1A (Accept : 173.31–173.39) <math>a^{-1}</math> for more than 2 d.p.</p>

Solution

Marks

(b)  $\lim_{t \rightarrow \infty} r(t) = \lim_{t \rightarrow \infty} \frac{18}{3 + 2e^{-t}} = 6 \text{ (m)}$

$\therefore$  the volume of the balloon will be

$$V = \frac{4}{3} \pi (6)^3 + 5\pi$$

$$= 293 \pi$$

$$= 920.49 \text{ (m}^3\text{)}$$

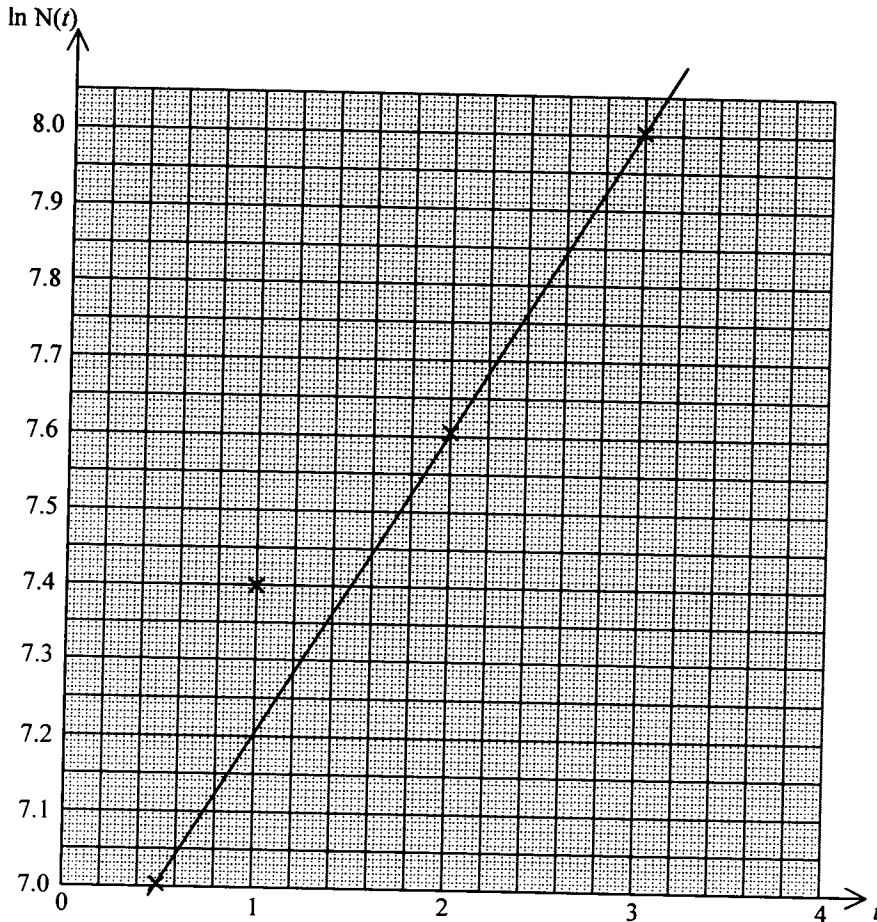
1M

1A  $a-1$  for more than 2 d.p.  
------(5)

3.  $N(t) = 900 a^{kt}$   
 $\ln N(t) = (k \ln a)t + \ln 900$

1A

$t$	0.5	1.0	2.0	3.0
$N(t)$	1100	1630	2010	2980
$\ln N(t)$	7.0031	7.3963	7.6059	7.9997



1M

At  $t = 1.0$ ,  $N(t) = 1630$  is incorrect,  
 $\ln N(2.5) \approx 7.8$   
 $\therefore N(2.5) \approx 2440$

1A

1A  $a-1$  for more than 3 s.f.  
(Accept :  $N(2.5) \in [2420, 2470]$ )  
------(4)

Solution	Marks
<p>4. (a) <math>S = \int_0^{10} \frac{8100t}{(3t+10)^3} dt.</math></p> <p>Let <math>u = 3t + 10.</math>  <math>du = 3dt.</math>                      When <math>t = 0, u = 10.</math>                      When <math>t = 10, u = 40.</math></p> $S = \int_{10}^{40} \frac{8100 \left( \frac{u-10}{3} \right) \cdot \frac{1}{3} du}{u^3}$ $= 900 \int_{10}^{40} \left( \frac{1}{u^2} - \frac{10}{u^3} \right) du$ $= 900 \left[ -\frac{1}{u} + \frac{5}{u^2} \right]_{10}^{40}$ $= \frac{405}{16} = 25.3125$ <p>The percentage of smoke removed is 25.3125%.</p>	<p>1A</p> <p>1M change of variable</p> <p>1A</p>
$S = \int_0^{10} \frac{8100t}{(3t+10)^3} dt$ $= 900 \int_0^{10} \left[ \frac{1}{(3t+10)^2} - \frac{10}{(3t+10)^3} \right] d(3t+10)$ $= 900 \left[ -\frac{1}{3t+10} + \frac{5}{(3t+10)^2} \right]_0^{10}$ $= 25.3125$	<p>1A</p> <p>1M change of variable</p> <p>1A</p>
$S = \int \frac{8100t}{(3t+10)^3} dt = 900 \left[ -\frac{1}{3t+10} + \frac{5}{(3t+10)^2} \right] + C$ <p>When <math>t = 0, S = 0.</math> Hence, we have <math>C = 45.</math></p> <p>So, <math>S = 900 \left[ -\frac{1}{3T+10} + \frac{5}{(3T+10)^2} \right] + 45.</math></p> <p>When <math>t = 10, S = 25.3125.</math></p>	<p>1M+1M for change of variable</p> <p>1A</p>
<p>(b) <math>S = \int_0^T \frac{8100t}{(3t+10)^3} dt</math></p> $= 900 \left[ -\frac{1}{u} + \frac{5}{u^2} \right]_{10}^{3T+10}$ $= 900 \left[ -\frac{1}{3T+10} + \frac{5}{(3T+10)^2} + 0.05 \right]$ $\lim_{T \rightarrow \infty} S = \lim_{T \rightarrow \infty} 900 \left[ -\frac{1}{3T+10} + \frac{5}{(3T+10)^2} + 0.05 \right]$ $= 45$ <p><math>\therefore</math> 45% of smoke will be removed.</p>	<p>1M taking limit and in terms of <math>T</math></p> <p>1A</p> <p>-----(5)</p>

Solution	Marks
<p>5. (a) The required probability</p> $= \frac{6}{8} \times \frac{1}{3} + \frac{4}{7} \times \frac{1}{3} + \frac{2}{7} \times \frac{1}{3}$ $= \frac{15}{28} \quad (\approx 0.5357)$ <p>(b) P(the boy is selected from group A   a boy is selected)</p> $= \frac{\frac{6}{8} \times \frac{1}{3}}{\frac{15}{28}}$ $= \frac{7}{15} \quad (\approx 0.4667)$	<p>1A</p> <p>1A a-1 for r.t. 0.536</p> <p>1M + 1A (1A for numerator)</p> <p>1A a-1 for r.t. 0.467</p> <p>------(5)</p>
<p>6. Let <math>N</math> be the number of passengers arriving the bus stop in an hour and <math>M</math> be the number of male passengers.</p> <p>(a) <math>P(N = 4) = \frac{5^4}{4!} e^{-5}</math></p> $\approx 0.17547 \approx 0.1755$ <p>(b) <math>P(M = 2 \text{ and } N = 4)</math></p> $= C_2^4 (0.65)^2 (1 - 0.65)^2 \cdot 0.17547$ $\approx 0.0545$	<p>1A</p> <p>1A a-1 for r.t. 0.175</p> <p>1M for binomial distribution</p> <p>1M for multiplication rule</p> <p>1A a-1 for r.t. 0.055</p> <p>------(5)</p>
<p>7. (a) Mean = 61</p> <p>(b) After deleting two marks, there are two modes. One deleted mark must be 54.</p> <p>Let the other deleted mark be <math>x</math>.</p> $54 + x = 22 \times 61 - 20 \times (61 + 1.2) (= 98)$ $x = 44$ <p>(c) There are 5 students with marks more than 75.</p> <p>The required probability is</p> $\frac{C_2^5}{C_2^{20}} = \frac{1}{19} (\approx 0.0526)$	<p>1A</p> <p>1A</p> <p>1A</p> <p>1M for numerator</p> <p>1A a-1 for r.t. 0.053</p>
<p>There are 5 students with marks more than 75.</p> <p>The required probability is</p> $\frac{5}{20} \times \frac{4}{19} = \frac{1}{19} (\approx 0.0526)$	<p>1M for multiplication rule</p> <p>1A a-1 for r.t. 0.053</p>
	<p>------(5)</p>

Solution	Marks
<p>8. (a) The sample space is  <math>\{1R 2W, 1R 1W 1Y, 1R 2Y, 2R 1W, 2R 1Y, 1W 2Y, 2W 1Y, 3W, 3Y\}</math></p> <p>(b) The probability is  <math display="block">\frac{C_1^2 \cdot C_2^{11}}{C_3^{13}} = \frac{5}{13} \approx 0.3846</math></p>	<p>1A withhold this mark if not given in set notation</p> <p>1M+1A a-1 for r.t. 0.385</p>
<p>The probability is  <math display="block">C_1^3 \left(\frac{2}{13}\right) \left(\frac{11}{12}\right) \left(\frac{10}{11}\right) = \frac{5}{13} \approx 0.3846</math></p>	<p>1M+1A a-1 for r.t. 0.385</p>
<p>(c) <math>P(\text{one of the others is white} \mid \text{one is red})</math>  <math display="block">= \frac{P(\text{one is red and one is white})}{P(\text{one is red})}</math> <math display="block">= \frac{\frac{C_1^2 \cdot C_1^5 \cdot C_1^6}{C_3^{13}}}{\frac{C_1^2 \cdot C_2^{11}}{C_3^{13}}}</math> <math display="block">= \frac{6}{11} \approx 0.5455</math></p>	<p>1M for conditional probability</p> <p>1M for numerator</p> <p>1A a-1 for r.t. 0.545</p>
<p><math>P(\text{one of the others is white} \mid \text{one is red})</math>  <math display="block">= C_1^2 \left(\frac{5}{11}\right) \left(\frac{6}{10}\right)</math> <math display="block">= \frac{6}{11}</math> <math display="block">\approx 0.5455</math></p>	<p>1M for <math>C_1^2</math> + 1M for <math>\left(\frac{5}{11}\right) \left(\frac{6}{10}\right)</math></p> <p>1A a-1 for r.t. 0.545</p>
	<p>------(6)</p>

Solution

Marks

9. (a) (i)

$t$	0	0.5	1.0	1.5	2	2.5
$\frac{dM}{dt}$	4	4.78496	5.84320	7.24875	9.10480	11.55161

$$M = \int_0^{2.5} \frac{12e^{\frac{2}{3}t}}{3+t} dt \approx \frac{0.5}{2} [4 + 11.55161 + 2(4.78496 + 5.8432 + 7.24875 + 9.1048)] = 17.3788 \text{ (m mol/L)}$$

1M  
1A a-1 for r.t. 17.379

(ii)  $\therefore \frac{dM}{dt} = \frac{12e^{\frac{2}{3}t}}{3+t}$ ,

$$\frac{d}{dt} \left( \frac{12e^{\frac{2}{3}t}}{3+t} \right) = 12 \left[ \frac{2}{3} \cdot \frac{e^{\frac{2}{3}t}}{3+t} - \frac{e^{\frac{2}{3}t}}{(3+t)^2} \right] = \frac{4(3+2t)e^{\frac{2}{3}t}}{(3+t)^2}$$

1A need not simplify

and  $\frac{d^2}{dt^2} \left( \frac{12e^{\frac{2}{3}t}}{3+t} \right) = \frac{8(9+6t+2t^2)}{3(3+t)^3} e^{\frac{2}{3}t}$

1A need simplification

$\therefore \frac{d^2}{dt^2} \left( \frac{dM}{dt} \right) > 0$  (for  $0 \leq t \leq 2.5$ )

So,  $\frac{dM}{dt}$  is concave upward on  $[0, 2.5]$ .

Hence it is over-estimate.

1  
------(5)

(b) (i)  $\frac{1}{3+t} = \frac{1}{3} \left( 1 - \frac{1}{3}t + \frac{1}{9}t^2 - \frac{1}{27}t^3 + \dots \right)$   
 $= \frac{1}{3} - \frac{1}{9}t + \frac{1}{27}t^2 - \frac{1}{81}t^3 + \dots$

1A

$$e^{\frac{2}{3}t} = 1 + \frac{2}{3}t + \frac{1}{2!} \left( \frac{2}{3}t \right)^2 + \frac{1}{3!} \left( \frac{2}{3}t \right)^3 + \dots$$

$$= 1 + \frac{2}{3}t + \frac{2}{9}t^2 + \frac{4}{81}t^3 + \dots$$

1M any three terms

1A

$$\frac{12e^{\frac{2}{3}t}}{3+t} = 12 \left( \frac{1}{3} - \frac{1}{9}t + \frac{1}{27}t^2 - \frac{1}{81}t^3 + \dots \right) \left( 1 + \frac{2}{3}t + \frac{2}{9}t^2 + \frac{4}{81}t^3 + \dots \right)$$

$$= 4 + \frac{4}{3}t + \frac{4}{9}t^2 + \frac{4}{81}t^3 + \dots$$

1A for the first three terms or the term  $t^3$

1A for all being correct

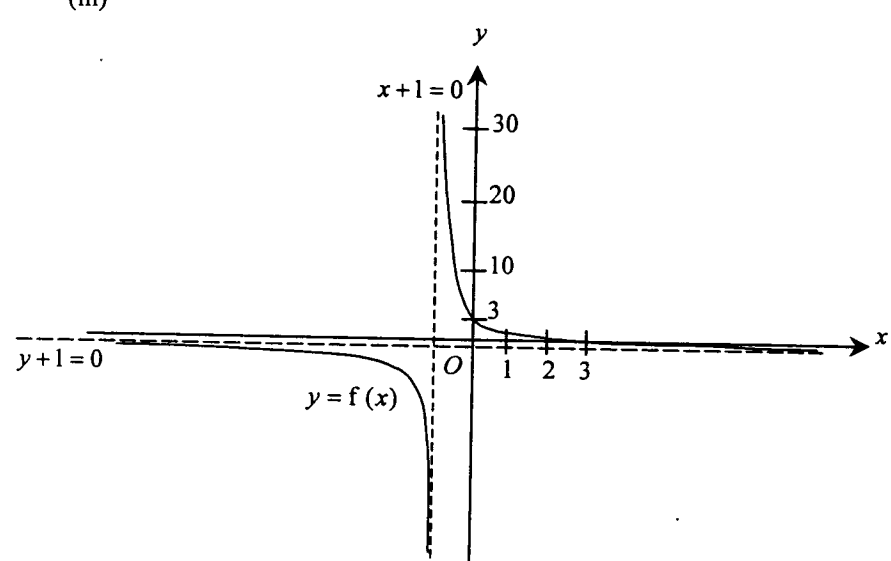
(ii)  $\int_0^{2.5} \frac{12e^{\frac{2}{3}t}}{3+t} dt \approx \int_0^{2.5} \left( 4 + \frac{4}{3}t + \frac{4}{9}t^2 + \frac{4}{81}t^3 \right) dt$   
 $= \left[ 4t + \frac{2}{3}t^2 + \frac{4}{27}t^3 + \frac{1}{81}t^4 \right]_0^{2.5}$   
 $= 16.9637 \text{ (m mol/L)}$

1M

1A a-1 for r.t. 16.964  
------(7)

Solution	Marks
<p>(c) The expansion is valid only when</p> $-1 < \frac{t}{3} < 1$ $-3 < t < 3$ <p>Hence <math>0 \leq t &lt; 3</math> (as <math>t \geq 0</math>)</p> <p><math>\therefore</math> this method is not valid to estimate the amount of lactic acid for <math>t \geq 3</math>.</p>	<p>2A</p> <p>1A</p> <p>-----(3)</p>



	Solution	Marks
<p>10. (a) (i) <math>f(0) = g(0)</math>  <math>\Rightarrow b = 3</math> ..... (1)</p> <p><math>f(3) = g(3)</math>  <math>\Rightarrow \frac{3a+b}{3c+1} = 0</math>  <math>\Rightarrow 3a+b = 0</math> ..... (2)</p> <p><math>f(-2) = g(-2)</math>  <math>\Rightarrow \frac{-2a+b}{-2c+1} = -5</math>  <math>\Rightarrow 2a-b+10c = 5</math> ..... (3)</p> <p>Using (1) and (2),  <math>a = -1</math></p> <p>Using (3), <math>c = 1</math></p>		<p>1M for using any two of the conditions <math>f(0) = g(0)</math>, <math>f(3) = g(3)</math> and <math>f(-2) = g(-2)</math></p> <p>1A for all correct values of <math>a</math>, <math>b</math> and <math>c</math></p>
<p>(ii) <math>f(x) = \frac{-x+3}{x+1}</math></p> <p><math>\therefore \lim_{x \rightarrow \pm\infty} f(x) = \lim_{x \rightarrow \pm\infty} \frac{-1 + \frac{3}{x}}{1 + \frac{1}{x}} = -1</math></p> <p>The horizontal asymptote is <math>y+1=0</math></p> <p>The vertical asymptote is <math>x+1=0</math></p>		<p>1A</p> <p>1A</p>
<p>(iii)</p> 		<p>1A Shape, asymptotes and <math>y</math>-intercept</p> <p>----- (5)</p>

Solution	Marks																								
<p>(b) (i) <math>g'(x) = -(x+1)^3 - 3(x-3)(x+1)^2</math>  <math>= -4(x+1)^2(x-2)</math>  <math>g''(x) = -8(x+1)(x-2) - 4(x+1)^2</math>  <math>= -12(x+1)(x-1)</math>  <math>g'(x) = 0 \Rightarrow x = -1</math> or <math>2</math>  <math>g''(2) = -36 &lt; 0</math></p>	<p>1M</p>																								
<p><math>g'(x) = -(x+1)^3 - 3(x-3)(x+1)^2</math>  <math>= -4(x+1)^2(x-2)</math>  <math>g'(x) = 0 \Rightarrow x = -1</math> or <math>2</math></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;"><math>x &lt; -1</math></td> <td style="padding: 2px 5px;"><math>-1 &lt; x &lt; 2</math></td> <td style="padding: 2px 5px;"><math>x &gt; 2</math></td> </tr> <tr> <td style="padding: 2px 5px;"><math>g'(x)</math></td> <td style="padding: 2px 5px;">+</td> <td style="padding: 2px 5px;">+</td> <td style="padding: 2px 5px;">-</td> </tr> <tr> <td style="padding: 2px 5px;"><math>g(x)</math></td> <td style="padding: 2px 5px;">↗</td> <td style="padding: 2px 5px;">↗</td> <td style="padding: 2px 5px;">↘</td> </tr> </table>	$x$	$x < -1$	$-1 < x < 2$	$x > 2$	$g'(x)$	+	+	-	$g(x)$	↗	↗	↘	<p>1M</p>												
$x$	$x < -1$	$-1 < x < 2$	$x > 2$																						
$g'(x)$	+	+	-																						
$g(x)$	↗	↗	↘																						
<p><math>\therefore g(x)</math> is maximum when <math>x = 2</math> and <math>g(2) = 27</math>  <math>\therefore</math> the maximum point is at <math>(2, 27)</math>.</p> <p><math>g''(x) = 0 \Rightarrow x = -1</math> or <math>1</math></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;"><math>x &lt; -1</math></td> <td style="padding: 2px 5px;"><math>x = -1</math></td> <td style="padding: 2px 5px;"><math>-1 &lt; x &lt; 1</math></td> <td style="padding: 2px 5px;"><math>x = 1</math></td> <td style="padding: 2px 5px;"><math>x &gt; 1</math></td> </tr> <tr> <td style="padding: 2px 5px;"><math>x + 1</math></td> <td style="padding: 2px 5px;">-</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">+</td> <td style="padding: 2px 5px;">+</td> <td style="padding: 2px 5px;">+</td> </tr> <tr> <td style="padding: 2px 5px;"><math>x - 1</math></td> <td style="padding: 2px 5px;">-</td> <td style="padding: 2px 5px;">-</td> <td style="padding: 2px 5px;">-</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">+</td> </tr> <tr> <td style="padding: 2px 5px;"><math>g''(x)</math></td> <td style="padding: 2px 5px;">-</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">+</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">-</td> </tr> </table> <p><math>g(-1) = 0, g(1) = 16</math>  <math>\therefore (-1, 0)</math> and <math>(1, 16)</math> are points of inflexion of <math>g(x)</math>.</p>	$x$	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$x > 1$	$x + 1$	-	0	+	+	+	$x - 1$	-	-	-	0	+	$g''(x)$	-	0	+	0	-	<p>1A</p> <p>1M for testing any one point</p> <p>1A for both points</p>
$x$	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$x > 1$																				
$x + 1$	-	0	+	+	+																				
$x - 1$	-	-	-	0	+																				
$g''(x)$	-	0	+	0	-																				
<p>(ii)</p>	<p>1M for the shape</p> <p>1A for all being correct</p> <p>-----(6)</p>																								

Solution	Marks
<p>(c) Using the graphs, the area is</p> $\int_0^3 \left[ -(x-3)(x+1)^3 - \frac{-x+3}{x+1} \right] dx$ $= \int_0^3 \left\{ -(x^4 - 6x^2 - 8x - 3) - \left[ -1 + \frac{4}{x+1} \right] \right\} dx$ $= \int_0^3 \left( -x^4 + 6x^2 + 8x + 4 - \frac{4}{x+1} \right) dx$ $= \left[ -\frac{1}{5}x^5 + 2x^3 + 4x^2 + 4x - 4 \ln(x+1) \right]_0^3$ $= \frac{267}{5} - 4 \ln 4$ $= 47.8548$	<p>1M for <math>\int_0^3 [f(x) - g(x)] dx</math> or <math>\int_0^3 [g(x) - f(x)] dx</math></p> <p>1A + 1A for each of <math>\int f(x) dx</math> and <math>\int g(x) dx</math></p> <p>1A</p> <p><math>\alpha-1</math> for r.t. 47.855</p>
<div style="border: 1px solid black; padding: 10px;"> <math display="block">\int \frac{-x+3}{x+1} dx = -x + 4 \ln(x+1) + C</math> <math display="block">\int (x-3)(x+1)^3 dx = \frac{x^5}{5} - 2x^3 - 4x^2 - 3x + C</math> <math display="block">\int_0^3 [-(x-3)(x+1)^3] dx = \frac{252}{5} = 50.4</math> <math display="block">\int_0^3 \frac{-x+3}{x+1} dx = -3 + 4 \ln 4 \approx 2.5452</math> <p>The required area = <math>\frac{267}{5} - 4 \ln 4</math></p> </div>	<p>1A</p> <p>1A</p> <p>1M+1A (<math>\approx 47.8548</math>)</p> <p><math>\alpha-1</math> for r.t. 47.855</p>
	<p>----- (4)</p>

Solution	Marks
<p>11. (a) (i) <math>G = \int \frac{2t-8}{t^2-8t+20} dt</math>  <math>= \ln(t^2-8t+20) + C</math>                      When <math>t = 0</math>, <math>G = 50</math>.  <math>C = 50 - \ln 20</math>  <math>G = \ln(t^2-8t+20) + 50 - \ln 20</math></p>	<p>1A 1A 1A</p>
<p>(ii) For <math>G = 50</math>,  <math>\ln(t^2-8t+20) + 50 - \ln 20 = 50</math>  <math>t^2-8t+20 = 20</math>  <math>t^2-8t = 0</math>  <math>t = 0</math> or <math>t = 8</math>.                      At the end of the 8th week, the weekly sale is the same as at the start of the promotion plan.</p>	<p>1M 1A ------(5)</p>
<p>(b) (i) <math>\therefore \frac{dG}{dt} = \frac{2t-8}{t^2-8t+20} = \frac{2(t-4)}{[(t-4)^2+4]}</math>  <math>\therefore \frac{dG}{dt} = 0</math> when <math>t = 4</math>                      Since <math>\frac{dG}{dt} &lt; 0</math> when <math>t &lt; 4</math>                      and <math>\frac{dG}{dt} &gt; 0</math> when <math>t &gt; 4</math>,  <math>\therefore G</math> is least at <math>t = 4</math>.                      At the end of the 4th week, the weekly sale is least.</p> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 200px;"> <math>G = \ln(t^2-8t+20) + C</math>  <math>= \ln[(t-4)^2+4] + C</math> </div>	<p>1M 1A</p>
<p>(ii) <math>G(6) - G(5) = (\ln 8 + 50 - \ln 20) - (\ln 5 + 50 - \ln 20)</math>  <math>= \ln \frac{8}{5} \approx 0.4700</math> (thousand dollars)</p>	<p>1A <math>a-1</math> for more than 4 d.p. or r.t. 0.470</p>
<p>(iii) <math>G(t+1) - G(t) &lt; 0.2</math>  <math>\{\ln[(t+1)^2-8(t+1)+20] + 50 - \ln 20\}</math>  <math>-\{\ln(t^2-8t+20) + 50 - \ln 20\} &lt; 0.2</math>  <math>\ln \frac{t^2-6t+13}{t^2-8t+20} &lt; 0.2</math>  <math>(e^{0.2}-1)t^2 - (8e^{0.2}-6)t + (20e^{0.2}-13) &gt; 0</math>  <math>t &lt; 3.94316</math> or <math>t &gt; 13.09015</math>  <math>\therefore \frac{dG}{dt} &lt; 0</math> when <math>0 &lt; t &lt; 4</math>, <math>G</math> is decreasing  <math>\therefore t &lt; 3.94316</math> is rejected.  <math>\therefore t = 14</math>.                      Thus the promotion plan will be terminated at the end of the 15th week.</p>	<p>1M 1A  <math>0.22140t^2 - 3.77122t + 11.42806 &gt; 0</math>  <math>t &lt; 3.94315</math> or <math>t &gt; 13.09037</math>                      1A must show reasons                      -----(6)</p>

Solution

Marks

$$G(t) - G(t-1) < 0.2$$

$$\{\ln(t^2 - 8t + 20) + 50 - \ln 20\} - \{\ln[(t-1)^2 - 8(t-1) + 20] + 50 - \ln 20\} < 0.2$$

$$\ln \frac{t^2 - 8t + 20}{t^2 - 10t + 29} < 0.2$$

$$(e^{0.2} - 1)t^2 - (10e^{0.2} - 8)t + (29e^{0.2} - 20) > 0$$

$$0.22140t^2 - 4.21403t + 15.42068 > 0$$

$$t < 4.94316 \text{ or } t > 14.09015$$

1M

1A

(c)  $\frac{dG}{dt} = \frac{2t-8}{t^2-8t+20}$

$$\frac{d^2G}{dt^2} = \frac{2(t^2-8t+20) - (2t-8)(2t-8)}{(t^2-8t+20)^2}$$

$$= -\frac{2(t-2)(t-6)}{(t^2-8t+20)^2}$$

1A

$$\frac{d^2G}{dt^2} = 0 \text{ when } t=2 \text{ or } t=6. \quad \therefore t_2 = 6$$

1A

Although  $G$  keeps increasing,

1A

$\frac{dG}{dt}$  increases immediately before  $t=6$ ,

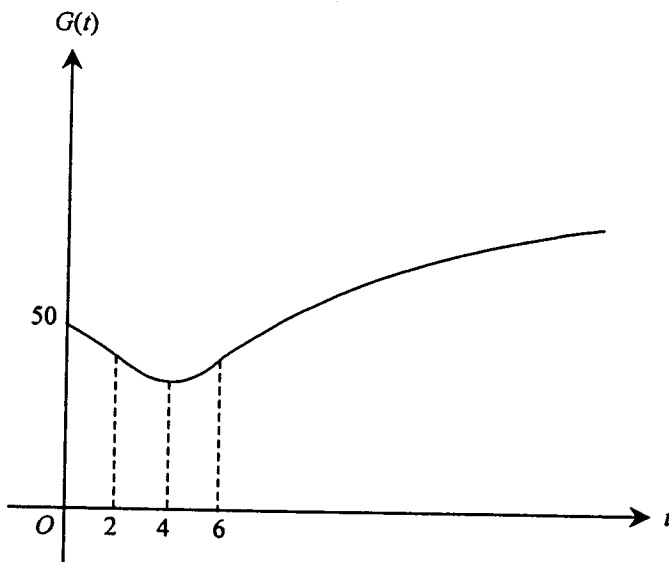
$\frac{dG}{dt}$  decreases immediately after  $t=6$ .

1A

}

----- (4)

For reference only



	$G(t)$	$\Delta G(t)$
0	50.0	
1.0	49.5692	- 0.4308
2.0	49.0837	- 0.4855
3.0	48.6137	- 0.47
4.0	48.3906	- 0.2231
5.0	48.6137	+ 0.2231
6.0	49.0837	+ 0.47
7.0	49.5692	+ 0.4855
8.0	50.0	+ 0.4308
9.0	50.3716	+ 0.3716
10.0	50.6931	+ 0.3215
11.0	50.9746	+ 0.2815
12.0	51.2238	+ 0.2492
13.0	51.4469	+ 0.2231
14.0	51.6487	+ 0.2018
15.0	51.8326	+ 0.1839
16.0	52.0015	+ 0.1689
17.0	52.1576	+ 0.1561
18.0	52.3026	+ 0.145
19.0	52.438	+ 0.1354
20.0	52.5649	+ 0.1269

Solution	Marks
<p>12. (a) Let <math>\lambda_1</math> be the sample mean of car accidents at the road junction in a month.</p> $\lambda_1 = \frac{0 \times 12 + 1 \times 15 + 2 \times 9 + 3 \times 4}{40} = 1.125$ <p>Let <math>X</math> be the number of car accidents at the road junction in a month. For researcher A ,</p> $a = 40 \cdot P(X = 3)$ $= 40 \times \frac{1.125^3}{3!} e^{-1.125}$ $\approx 3.08$ <p>(b) For researcher B , let the mean be <math>\lambda_2</math> . Then</p> <p>(i) <math>12.05 = 40 \cdot P(X = 0)</math></p> $12.05 = 40 e^{-\lambda_2}$ $\lambda_2 = -\ln \frac{12.05}{40}$ $\approx 1.1998$ <p>(ii) <math>b = 40 \cdot P(X = 2) \approx 40 \times \frac{1.1998^2}{2!} e^{-1.1998} (\approx 8.6732) \approx 8.67</math></p>	<p>1A (Accept <math>\lambda_1 = 1.1247</math>)</p> <p>1M</p> <p>1A (<math>\approx 3.08166</math>) a-1 for more than 2 d.p. ------(3)</p> <p>1A a-1 for more than 4 d.p. (accept <math>\lambda_2 \approx 1.1999, 1.2000</math> or 1.2)</p> <p>1A a-1 for more than 2 d.p. (accept <math>b = 8.68</math>)</p>
<p>Equivalent forms:</p> $\frac{b}{12.05} = \frac{\lambda^2}{2}, \quad \frac{b}{14.46} = \frac{\lambda}{2}, \quad \frac{b}{3.47} = \frac{3}{\lambda}$	<p>Accept <math>b = 8.68</math></p>
<p>(c) For the number of car accidents is 4 or more, the expected number of months observed by researcher A is</p> $40 - (12.99 + 14.61 + 8.22 + 3.08) \approx 1.10$ <p>Let</p> $\text{TSE}_1 = \text{Total sum of errors for model fitted by researcher A}$ $= \sum  f_0 - f_{E_1} $ $=  12 - 12.99  +  15 - 14.61  +  9 - 8.22  +  4 - 3.08  +  0 - 1.10 $ $\approx 4.18$ <p>For the number of car accidents is 4 or more, the expected number of months observed by researcher B is</p> $40 - (12.05 + 14.46 + 8.67 + 3.47) \approx 1.35$ $\text{TSE}_2 = \text{Total sum of errors for model fitted by researcher B}$ $= \sum  f_0 - f_{E_2} $ $\approx  12 - 12.05  +  15 - 14.46  +  9 - 8.67  +  4 - 3.47  +  0 - 1.35 $ $= 2.8$ <p>As <math>\text{TSE}_2 &lt; \text{TSE}_1</math> , researcher B fits the data of car accidents better than researcher A does.</p>	<p>------(2)</p> <p>1M (either A or B)</p> <p>1M + 1M (1M for the first 4 terms 1M for the last term) (either A or B)</p> <p>1A Sum of the first 4 terms 3.08 (for both A and B)</p> <p>Sum of the first 4 terms 1.45</p> <p>1M (accept using the first 4 terms)</p> <p>------(5)</p>

Solution	Marks
<p>(d) (i) P(the number of car accidents at the road junction in a month is 3 and one of which involves a bus)</p> $= P(X = 3 \text{ and one of which involves a bus})$ $= P(\text{one accident involves a bus} \mid X = 3) P(X = 3)$ $= C_1^3 \cdot 0.3 \times (1 - 0.3)^2 \times \frac{\lambda_2^3}{3!} e^{-\lambda_2}$ $= 3 \times 0.3 \times 0.7^2 \times \frac{1.1998^3}{3!} e^{-1.1998} (\approx 0.038254)$ $\approx 0.0382$	<p>1M for multiplication rule ( for (i) or (ii) )</p> <p>1M for <math>C_1^3 \cdot 0.3 \times (1 - 0.3)^2</math></p> <p>0.0383 if <math>\lambda_2 \approx 1.200</math></p> <p>1A (Accept 0.0383) a-1 for r.t. 0.038</p>
<p>P(the number of car accidents at the road junction in a month is 3 and one of which involves a bus)</p> $= P(X = 3 \text{ and one of which involves a bus})$ $= P(\text{one accident involves a bus} \mid X = 3) P(X = 3)$ $= C_1^3 \cdot 0.3 \times (1 - 0.3)^2 \times \frac{3.47}{40}$ $\approx 0.0383$	<p>1M for multiplication rule</p> <p>1M for <math>C_1^3 \cdot 0.3 \times (1 - 0.3)^2</math></p> <p>1A a-1 for r.t. 0.038</p>
<p>(ii) P(the number of car accidents at the road junction in a month is 3 and only the third car accident involves a bus)</p> $= P(X = 3 \text{ and only the third car accident involves a bus})$ $= \frac{1}{3} P(X = 3 \text{ and one of which involves a bus})$ $\approx 0.0127$	<p>1A a-1 for r.t. 0.013 (Accept 0.0128)</p>
<p>P(the number of car accidents at the road junction in a month is 3 and only the third car accident involves a bus)</p> $= P(X = 3 \text{ and only the third car accident involves a bus})$ $= P(\text{only the third car accident involves a bus} \mid X = 3) P(X = 3)$ $= (1 - 0.3)^2 \times 0.3 \times \frac{\lambda_2^3}{3!} e^{-\lambda_2}$ $= 0.7^2 \times 0.3 \times \frac{1.200^3}{3!} e^{-1.200}$ $\approx 0.0128$	<p>1A a-1 for r.t. 0.013</p>
<p>(iii) <math>P(X = 3 \text{ and the third car accident involves a bus} \mid X = 3 \text{ and only one of which involves a bus})</math></p> $= \frac{1}{3}$	<p>1M</p>
<p><math>P(X = 3 \text{ and the third car accident involves a bus} \mid X = 3 \text{ and one of which involves a bus})</math></p> $= \frac{0.0128}{0.0383} \approx 0.3342$	<p>1M</p>
	<p>-----(5)</p>

Solution	Marks
<p>13. Let <math>Xg</math> be the weight of a bag of self raising flour in the batch.</p>	
<p>(a) (i) <math>P(\text{a bag of flour is underweight}) = P(X &lt; 376)</math>  <math>= P\left(\frac{X-400}{10} &lt; \frac{376-400}{10}\right)</math>  <math>= P(Z &lt; -2.4)</math>  <math>\approx 0.0082</math></p>	<p>1M -----                  1A either one</p>
<p>(ii) <math>P(\text{a bag of flour is overweight}) = P(X &gt; 424)</math>  <math>= P\left(\frac{X-400}{10} &gt; \frac{424-400}{10}\right)</math>  <math>= P(Z &gt; 2.4)</math>  <math>\approx 0.0082</math></p>	<p>-----                  1A -----(3)</p>
<p>(b) (i) <math>P(\text{a bag of flour is substandard})</math>  <math>= P(X &lt; 376) + P(X &gt; 424)</math>  <math>\approx 0.0082 + 0.0082 = 0.0164</math></p>	<p>1A</p>
<p>Let <math>Y</math> be the number of substandard bags in the sample.  <math>P(\text{there is no substandard bags in the sample}) = P(Y = 0)</math>  <math>= C_0^{50} 0.0164^0 \times (1 - 0.0164)^{50}</math>  <math>= 0.9836^{50} \approx 0.4374</math></p>	<p>1M                  1A (0.43745) <math>a-1</math> for r.t. 0.437</p>
<p>(ii) <math>P(Y \leq 2)</math>  <math>= P(Y = 0) + P(Y = 1) + P(Y = 2)</math>  <math>= C_0^{50} 0.0164^0 \times 0.9836^{50} + C_1^{50} 0.0164 \times 0.9836^{49}</math>  <math>\qquad\qquad\qquad + C_2^{50} 0.0164^2 \times 0.9836^{48}</math>  <math>\approx 0.43745 + 0.36469 + 0.14897</math>  <math>\approx 0.9511</math></p>	<p>1M                  1A (0.95111) <math>a-1</math> for r.t. 0.951                  -----(5)</p>
<p>(c) Let <math>W</math> be the number of underweight bags in the sample.</p>	
<p>(i) <math>P(W = 0, Y = 1)</math>  <math>= P(W = 0   Y = 1) \cdot P(Y = 1)</math>  <math>= \frac{1}{2} \times C_1^{50} (0.0164)(0.9836)^{49}</math>  <math>\approx 0.1823</math></p>	<p>1M + 1M for <math>\frac{1}{2}</math> and cond. prob.                  1A <math>a-1</math> for r.t. 0.182</p>
<p>(ii) The required probability is <math>P(W = 0, Y \leq 2)</math>  <math>= P(W = 0, Y = 0) + P(W = 0, Y = 1) + P(W = 0, Y = 2)</math>  <math>= P(Y = 0) + P(W = 0, Y = 1) + P(W = 0   Y = 2) \cdot P(Y = 2)</math>  <math>\approx 0.43745 + 0.18235 + \left(\frac{1}{2}\right)^2 \cdot C_2^{50} (0.0164)^2 (0.9836)^{48}</math>  <math>\approx 0.6570</math></p>	<p>1M                  1M for the last term                  1A (0.65704) (Accept 0.6569)  <math>a-1</math> for r.t. 0.657</p>
<p>(iii) The required probability is <math>P(W = 0   Y \leq 2)</math>  <math>= \frac{P(W = 0, Y \leq 2)}{P(Y \leq 2)}</math>  <math>\approx \frac{0.65704}{0.95111}</math>  <math>\approx 0.6908</math></p>	<p>1M                  (Accept 0.6907)                  -----(7)</p>



Solution	Marks
<p>14. (a) Let <math>N</math> be the number of customers visiting the supermarket in one minute.</p> $P(N \leq 2) = \sum_{k=0}^2 \frac{6^k}{k!} e^{-6}$ $= e^{-6} + \frac{6}{1!} e^{-6} + \frac{6^2}{2!} e^{-6}$ $\approx 0.002479 + 0.01487 + 0.04462$ $\approx 0.0620$ <p><math>\therefore P(N &gt; 2) = 1 - P(N \leq 2) \approx 0.9380</math></p>	<p>1A</p> <p>1M+1A a-1 if 0.938 ------(3)</p>
<p>(b) (i) <math>X \sim N(\mu, \sigma^2)</math></p> $P(X < 100) = 0.063$ $P\left(Z < \frac{100 - \mu}{\sigma}\right) = 0.063$ $\frac{100 - \mu}{\sigma} \approx -1.53 \quad \dots\dots\dots (1)$ $P(X \geq 400) = 0.006$ $P\left(Z \geq \frac{400 - \mu}{\sigma}\right) = 0.006$ $\frac{400 - \mu}{\sigma} \approx 2.51 \quad \dots\dots\dots (2)$	<p>1A</p> <p>1A (Accept <math>\frac{200 - \mu}{\sigma} \in [-0.185, -0.18]</math>)</p>
<p>Solving (1) and (2), we get</p> $\mu \approx 213.6$ $\sigma \approx 74.26 \approx 74.3$	<p>1A a-1 for more than 1 d.p. (Accept <math>\mu \in [213.3, 213.8]</math>)</p> <p>1A a-1 for more than 1 d.p. (Accept <math>\sigma \in [74.1, 74.3]</math>)</p>
$a_1 = P(200 \leq X < 300)$ $= P\left(Z < \frac{300 - 213.6}{74.3}\right) - P\left(Z < \frac{200 - 213.6}{74.3}\right)$ $\approx 0.4484$ $\approx 0.448$	<p>1A a-1 for more than 3 d.p. (Accept <math>a_1 \in [0.448, 0.453]</math>)</p>
$a_2 = P(300 \leq X < 400)$ $\approx 0.117$	<p>1A a-1 for more than 3 d.p. (Accept <math>a_2 \in [0.115, 0.119]</math>)</p>
<p>(ii) For normal distribution, median = mean = 213.6</p>	<p>1M</p>
<p>(iii) <math>P(X &gt; 50 \mid X \leq 200)</math></p> $= \frac{P(50 \leq X < 200)}{P(X < 100) + P(100 \leq X < 200)}$ $= \frac{P(-2.20 \leq Z < -0.18)}{0.063 + 0.364}$ $\approx \frac{0.4861 - 0.0714}{0.427}$ $\approx 0.9712$	<p>1M</p>
	<p>1A a-1 for more than 4 d.p. (Accept probability <math>\in [0.9620, 0.9749]</math>)</p>

Solution	Marks
$\frac{P(X > 50   X \leq 200)}{P(50 \leq X < 200)}$ $= \frac{P(X < 100) + P(100 \leq X < 200)}{P(X < 200) - P(X \leq 50)}$ $= \frac{P(X < 200) - P(Z \leq -2.20)}{P(X \leq 200)}$ $= \frac{0.427 - 0.0139}{0.427}$ $\approx 0.9674$	<p>1M</p> <p>1A <math>\alpha</math>-1 for more than 4 d.p. (Accept probability <math>\in [0.9620, 0.9749]</math>)</p>
<p>(iv) The required probability</p> $= C_2^5 P(X < 200)^2 (1 - P(X < 200))^3 \cdot P(N = 5)$ $= 10(0.063 + 0.364)^2 (1 - (0.063 + 0.364))^3 \cdot \frac{6^5}{5!} e^{-6}$ $\approx 10(0.1823)(0.1881) \cdot (0.1606)$ $\approx 0.0551$	<p>1M for Binomial/Poisson probability 1M for the multiplication rule (Binomial <math>\times</math> Poisson)</p> <p>1A <math>\alpha</math>-1 for r.t. 0.055 (Accept probability <math>\in [0.0550, 0.0552]</math>) ----- (12)</p>