

Solution	Marks																																										
<p>1. Since $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and $P(A \cap B) = P(A)P(B)$ $\therefore 0.7 = 0.4 + P(B) - 0.4P(B)$ $P(B) = 0.5$</p>	<p>1M 1M 1A 1A -----(4)</p>																																										
<p>2. (a) Since $u = e^{2x}$, $\therefore \frac{du}{dx} = 2e^{2x}$.</p> $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \left(\frac{1}{u} - 2u\right) \cdot 2u = 2 - 4u^2 = 2 - 4e^{4x}$	<p>1A 1M+1A 1M for $\left(\frac{1}{u} - 2u\right) \cdot 2u$</p>																																										
$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \left(\frac{1}{u} - 2u\right) \cdot 2e^{2x} = \left(\frac{1}{e^{2x}} - 2e^{2x}\right) \cdot 2e^{2x} = 2 - 4e^{4x}$	<p>1M+1A</p>																																										
$y = \ln u - u^2 + c$ $y = \ln e^{2x} - (e^{2x})^2 + c$ $y = 2x - e^{4x} + c$ $\frac{dy}{dx} = 2 - 4e^{4x}$	<p>1M</p>																																										
<p>(b) Using (a), $y = \int (2 - 4e^{4x}) dx$ $= 2x - e^{4x} + c$ for some constant c. Putting $x=0$ and $y=1$, we have $c=2$. $\therefore y = 2x - e^{4x} + 2$</p>	<p>1A f.t.</p>																																										
<p>3. (a) $a=8, b=6, c=5$ $a=8, b=7, c=5$</p>	<p>1M</p>																																										
<p>(b)</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th></th> <th>Min</th> <th>Q₁</th> <th>Median</th> <th>Q₃</th> <th>Max</th> </tr> </thead> <tbody> <tr> <td>Before replacement</td> <td>18</td> <td>23</td> <td>22.75</td> <td>36</td> <td>52</td> </tr> <tr> <td>After replacement</td> <td>12</td> <td>23</td> <td>22.75</td> <td>38</td> <td>52</td> </tr> </tbody> </table>		Min	Q ₁	Median	Q ₃	Max	Before replacement	18	23	22.75	36	52	After replacement	12	23	22.75	38	52	<p>1A -----(5)</p> <p>1A+1A 1A for anyone correct 1A for all (award 1A for $a=18, b=36, c=65$)</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th colspan="6">Using $a=8, b=7, c=5$ will give</th> </tr> <tr> <th></th> <th>Min</th> <th>Q₁</th> <th>Q₂</th> <th>Q₃</th> <th>Max</th> </tr> </thead> <tbody> <tr> <td>Before</td> <td>18</td> <td>22.75</td> <td>36</td> <td>52</td> <td>66</td> </tr> <tr> <td>After</td> <td>12</td> <td>22.75</td> <td>36.5</td> <td>52</td> <td>68</td> </tr> </tbody> </table> <p>1M any box-and-whisker diagram with correct scale 1A all correct, same scale -----(6)</p>	Using $a=8, b=7, c=5$ will give							Min	Q ₁	Q ₂	Q ₃	Max	Before	18	22.75	36	52	66	After	12	22.75	36.5	52	68
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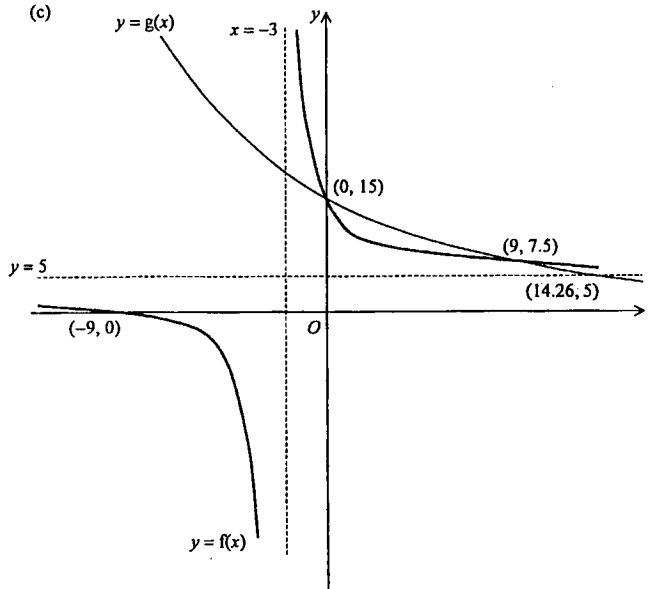
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<p>4. (a) $(1+ax)^{-\frac{1}{n}} = 1 + \left(-\frac{1}{n}\right)(ax) + \frac{1}{2}\left(-\frac{1}{n}\right)\left(-\frac{1}{n}-1\right)(ax)^2 + \dots$ $= 1 - \frac{a}{n}x + \frac{(n+1)a^2}{2n^2}x^2 + \dots$ Solving $\frac{a}{n} = \frac{4}{3}$ and $\frac{(n+1)a^2}{2n^2} = \frac{32}{9}$, we have $9(n+1)\left(\frac{4n}{3}\right)^2 = 64n^2$ $n+1=4$ $n=3$ and $a=4$</p>	<p>1M+1A 1M for any 2 terms correct</p>												
<p>(b) The expansion is valid for $-\frac{1}{4} < x < \frac{1}{4}$. $x < \frac{1}{4}$</p>	<p>1M -----(6)</p>												
<p>5. (a)</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>t</th> <th>0</th> <th>1.5</th> <th>3</th> <th>4.5</th> <th>6</th> </tr> </thead> <tbody> <tr> <td>R</td> <td>8</td> <td>7.88177</td> <td>7.54717</td> <td>7.04846</td> <td>6.45161</td> </tr> </tbody> </table> $\int_0^6 R dt \approx \frac{1.5}{2} [8 + 6.45161 + 2(7.88177 + 7.54717 + 7.04846)]$ ≈ 44.5548 <p>\therefore The total bonus for the first 6 months is 44.5548 thousand dollars.</p>	t	0	1.5	3	4.5	6	R	8	7.88177	7.54717	7.04846	6.45161	<p>1M correct to 4 d.p. 1M 1A</p>
t	0	1.5	3	4.5	6								
R	8	7.88177	7.54717	7.04846	6.45161								
<p>(b) $\frac{dR}{dt} = \frac{-2400t}{(t^2+150)^2}$ $\frac{d^2R}{dt^2} = \frac{7200(t^2-50)}{(t^2+150)^3}$ < 0 for $0 \leq t \leq 6$ \therefore The graph of R is concave downward in the interval $0 \leq t \leq 6$. The approximation in (a) is an underestimate.</p>	<p>1A 1A 1M -----(6)</p>												

Solution	Marks
<p>6. (a) The probability that the heaviest student is in the selection</p> $= \frac{C_2^9}{C_3^{10}}$ $= C_1^3 \left(\frac{1}{10} \right)$ $= 1 - \frac{9}{10} \times \frac{8}{9} \times \frac{7}{8}$ $= \frac{3}{10} \quad \boxed{0.3}$	<p>1M denominator, maybe awarded in (b) or (c) below</p> <p>1M numerator</p> <p>1M C_1^3</p> <p>1M $\frac{1}{10}$</p> <p>1M denominator</p> <p>1M numerator with fraction subtracted by 1</p> <p>1A</p>
<p>(b) The probability that the heaviest one out of the 3 selected students is the 4th heaviest among the ten students</p> $= \frac{C_2^6}{C_3^{10}}$ $= C_1^3 \left(\frac{1}{10} \right) \left(\frac{6}{9} \right) \left(\frac{5}{8} \right)$ $= \frac{1}{8} \quad \boxed{0.125}$	<p>1M numerator</p> <p>1M for $\left(\frac{1}{10} \right) \left(\frac{6}{9} \right) \left(\frac{5}{8} \right)$</p> <p>1A</p>
<p>(c) The probability that the 2 heaviest student are not both selected</p> $= 1 - \frac{C_1^8}{C_3^{10}}$ $= \left(\frac{8}{10} \right) \left(\frac{7}{9} \right) \left(\frac{6}{8} \right) + C_1^3 \left(\frac{2}{10} \right) \left(\frac{8}{9} \right) \left(\frac{7}{8} \right)$ $= \frac{14}{15} \quad \boxed{0.9333}$	<p>1M for numerator and probability subtracted by 1</p> <p>1M Sum of the two cases</p> <p>1A $a-1$ for r.t. 0.933</p> <p>------(7)</p>
<p>7. (a) The required probability</p> $= \frac{0.39 \times 0.58}{0.48 \times 0.65 + 0.39 \times 0.58 + 0.13 \times 0.5}$ $= 0.375 \quad (p)$	<p>1A numerator</p> <p>1A denominator</p> <p>1A</p>
<p>(b) The required probability</p> $= C_2^5 (0.375)^2 (1 - 0.375)^3$ $= 0.3433$	<p>1M binomial, for any p</p> <p>1M $C_2^5 p^2 (1-p)^3$, for p in (a)</p> <p>1A $a-1$ for r.t. 0.343</p> <p>------(6)</p>

Solution	Marks
<p>8. (a) (i) Since $G(0) = 9$,</p> $\therefore 2a - 12 + (a + 12) = 9$ $a = 3$	<p>1</p>
<p>(ii) $G(x) = 6 - 12e^{-kx} + 15e^{-2kx}$</p> $G'(x) = 12ke^{-kx} - 30ke^{-2kx}$ $= 6ke^{-kx} (2 - 5e^{-kx})$ $G'(x) = 0 \text{ when } e^{-kx} = \frac{2}{5} \text{ or } x = \frac{1}{k} \ln \frac{5}{2} \quad \boxed{\frac{0.9163}{k}}$ <p>and $G'(x) \begin{cases} < 0 & \text{when } 0 \leq x < \frac{1}{k} \ln \frac{5}{2} \\ > 0 & \text{when } x > \frac{1}{k} \ln \frac{5}{2} \end{cases}$</p> <p>$\therefore G(x)$ is minimum when $e^{-kx} = \frac{2}{5}$.</p>	<p>1A</p> <p>1A</p> <p>1M</p>
<p>$G''(x) = -12k^2 e^{-kx} + 60k^2 e^{-2kx}$</p> <p>When $e^{-kx} = \frac{2}{5}$, $G''(x) = \frac{24}{5} k^2 > 0$</p> <p>Since $G(x)$ has only one stationary point for $x \geq 0$,</p> <p>$G(x)$ is minimum when $e^{-kx} = \frac{2}{5}$.</p>	<p>1M</p>
<p>(ii) $G(x) = 6 - 12e^{-kx} + 15e^{-2kx}$</p> $= 15(e^{-2kx} - \frac{4}{5}e^{-kx}) + 6$ $= 15(e^{-kx} - \frac{2}{5})^2 + \frac{18}{5}$ <p>$G(x)$ is minimum when $e^{-kx} = \frac{2}{5}$.</p>	<p>1M+1A</p> <p>1M</p>
<p>The minimum CDO = $\left[6 - 12 \left(\frac{2}{5} \right) + 15 \left(\frac{2}{5} \right)^2 \right]$ mg/L</p> $= 3.6 \text{ mg/L}$	<p>1A f.t.</p> <p>------(5)</p>
<p>(b) (i) Solving $G(x) = 4.5$, we have</p> $6 - 12e^{-kx} + 15e^{-2kx} = 4.5$ $10(e^{-kx})^2 - 8e^{-kx} + 1 = 0$ $e^{-kx} = \frac{4 \pm \sqrt{6}}{10}$ $x = -\frac{1}{k} \ln \frac{4 \pm \sqrt{6}}{10}$ <p>Hence $-\frac{1}{k} \ln \frac{4 - \sqrt{6}}{10} + \frac{1}{k} \ln \frac{4 + \sqrt{6}}{10} = 2.85$</p> $\frac{1}{k} \ln \frac{4 + \sqrt{6}}{4 - \sqrt{6}} = 2.85$ $k = 0.5 \quad (1 \text{ d.p.})$	<p>1M</p> <p>1A</p> <p>1M+1A</p> <p>1A</p>

Solution	Marks
(ii) $G'(x) = 6e^{-0.5x} - 15e^{-x}$ $G''(x) = -3e^{-0.5x} + 15e^{-x}$ $= 3e^{-0.5x}(5e^{-0.5x} - 1)$ $G''(x) = 0$ when $x = -\frac{1}{0.5} \ln \frac{1}{5} (\approx 3.2)$ and $G''(x) \begin{cases} < 0 & \text{when } x > -\frac{1}{0.5} \ln \frac{1}{5} \\ > 0 & \text{when } 0 \leq x < -\frac{1}{0.5} \ln \frac{1}{5} \end{cases}$ $G'''(x) = 1.5e^{-0.5x}(1 - 10e^{-0.5x})$ When $e^{-0.5x} = \frac{1}{5}$, $G'''(x) = -0.3 < 0$ Since $G'(x)$ has only one stationary point for $x \geq 0$, $G'(x)$ is greatest when $e^{-0.5x} = \frac{1}{5}$.	IM IM IM
\therefore 3.2 km downstream from the location of discharge of the waste, the rate of change of the CDO is greatest.	1A
(iii) Solving $G(x) = 5.5$, we have $30e^{-x} - 24e^{-5x} + 1 = 0$ $e^{-0.5x} = \frac{12 \pm \sqrt{114}}{30}$ $x = -\frac{1}{0.5} \ln \frac{12 \pm \sqrt{114}}{30}$ $x \approx 0.6$ or 6.2 \therefore The river will return to be healthy 6.2 km downstream from the location of discharge of waste.	1A 1
Since $\lim_{x \rightarrow \infty} G(x) = \lim_{x \rightarrow \infty} (6 - 12e^{-0.5x} + 15e^{-x}) = 6 > 5.5$ \therefore The river will return to be healthy. Solving $G(x) = 5.5$, we have $x \approx 0.6$ or 6.2 \therefore The river will return to be healthy 6.2 km downstream from the location of discharge of waste.	I 1A -----(10)

Solution	Marks
9. (a) (i) $\ln P'(t) = -kt + \ln \frac{0.04ak}{1-a}$ From the graph, $-k \approx \frac{-8 - (-3.5)}{18 - 0}$, $k \approx 0.25$ $\ln \frac{0.04ak}{1-a} \approx -3.5$, $a \approx 0.7512 \approx 0.75$ $P'(t) \approx 0.03e^{-0.25t}$ $P(t) \approx -0.12e^{-0.25t} + c$ for some constant c Since $P(0) = 0.09$, $\therefore c \approx 0.21$ Hence $P(t) \approx -0.12e^{-0.25t} + 0.21$	1A 1A $a-1$ for more than 2 d.p. 1A $a-1$ for more than 2 d.p. IM 1A
(ii) $\mu = P(3) \approx 0.1533$	1A $\mu \in [0.1530, 0.1533]$
(iii) Stabilized PPI in town A = $\lim_{t \rightarrow \infty} P(t) = 0.21$	IM+1A ----- (8)
(b) (i) Suppose $b = 0.09$.	
(I) $Q'(t) = 0.24(3t+4)^{-\frac{3}{2}}$ $Q(t) = \frac{1}{3}(0.24)(-2)(3t+4)^{-\frac{1}{2}} + c$ for some constant c $= -0.16(3t+4)^{-\frac{1}{2}} + c$ Since $Q(0) = 0.09$, $\therefore c = 0.17$ If $Q(t) = \mu \approx 0.1533$ $-0.16(3t+4)^{-\frac{1}{2}} + 0.17 \approx 0.1533$ $(3t+4)^{\frac{1}{2}} \approx \frac{0.16}{0.0167}$ $\therefore 3t+4 > 0$ $\therefore t \approx 29.3$ i.e. the PPI will reach the value of μ . Since $Q(0) = 0.09$, $\lim_{t \rightarrow \infty} Q(t) = 0.17$ and Q is continuous and strictly increasing ($Q'(t) > 0$), $\therefore Q$ can reach any value between 0.09 and 0.17 including $\mu \approx 0.1533$.	1A 1A IM $t \in [28.2, 29.3]$ I IM I
(II) Stabilized PPI in town B = $\lim_{t \rightarrow \infty} Q(t) = 0.17$ \therefore The stabilized PPI will be reduced by 0.04.	1A 1A
(ii) $0.05 < b (< 1)$. Otherwise, $Q'(t) \leq 0$ and the PPI will not increase. It follows that the epidemic will not break out.	I ----- (7)

Solution	Marks
10. (a) $f(0) = g(0) \Rightarrow k = \frac{45}{3} = 15$ $f(9) = g(9) \Rightarrow \frac{15}{a} = \frac{90}{12} \Rightarrow a = 2$	1A 1A -----(2)
(b) Since $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} \frac{5x+45}{x+3} = -\infty$ and $\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \frac{5x+45}{x+3} = +\infty$, $\therefore x = -3$ is a vertical asymptote. Since $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} \frac{5 + \frac{45}{x}}{1 + \frac{3}{x}} = 5$, $\therefore y = 5$ is a horizontal asymptote.	1A 1A -----(2)
(c) 	1A asymptotes 1A shape and position 1A intercepts and intersections -----(3)
(d) (i) $A = \int_0^9 \frac{5x+45}{x+3} dx$ $= \int_0^9 \left(5 + \frac{30}{x+3}\right) dx$ $= [5x + 30 \ln(x+3)]_0^9$ $= 45 + 30 \ln 4$ 86.5888	1A 1A ignore limits 1A $a-1$ for r.t. 86.589

Solution	Marks
(ii) Let $u = 2^{-\frac{1}{9}x}$, then $\ln u = -\frac{\ln 2}{9}x$ and $dx = -\frac{9}{\ln 2} \cdot \frac{1}{u} \cdot du$. $\int_a^{a+9} 15 \cdot 2^{-\frac{1}{9}x} dx$ $= \int_{2^{-a/9}}^{2^{-(a+9)/9}} 15u \left(-\frac{9}{\ln 2} \cdot \frac{1}{u}\right) du$ $= \left[-\frac{135}{\ln 2} u\right]_{2^{-a/9}}^{2^{-(a+9)/9}}$ $= \frac{135}{\ln 2} \left(2^{-\frac{a}{9}} - 2^{-\frac{a}{9}-1}\right)$ $= \frac{135}{2 \ln 2} \cdot 2^{-\frac{a}{9}}$	1M 1M change of variable and limits 1M ignore limits 1A
Let $u = 2^{-\frac{1}{9}x}$, then $\ln u = -\frac{\ln 2}{9}x$ and $dx = -\frac{9}{\ln 2} \cdot \frac{1}{u} \cdot du$. $\int 15 \cdot 2^{-\frac{1}{9}x} dx = \int 15u \left(-\frac{9}{\ln 2} \cdot \frac{1}{u}\right) du$ $= -\frac{135}{\ln 2} \cdot 2^{-\frac{1}{9}x} + c$ for some constant c. $\int_a^{a+9} 15 \cdot 2^{-\frac{1}{9}x} dx = \frac{135}{\ln 2} \left(2^{-\frac{a}{9}} - 2^{-\frac{a}{9}-1}\right)$ $= \frac{135}{2 \ln 2} \cdot 2^{-\frac{a}{9}}$	1M 1M 1M 1A
$\int_a^{a+9} 15 \cdot 2^{-\frac{1}{9}x} dx = 15 \int_a^{a+9} e^{-\frac{1}{9}x \ln 2} dx$ $= \frac{-9 \times 15}{\ln 2} \left[e^{-\frac{1}{9}x \ln 2} \right]_a^{a+9}$ $= \frac{-135}{\ln 2} \left[2^{-\frac{1}{9}x} \right]_a^{a+9}$ $= \frac{135}{\ln 2} \left(2^{-\frac{a}{9}} - 2^{-\frac{a}{9}-1} \right)$ $= \frac{135}{2 \ln 2} \cdot 2^{-\frac{a}{9}}$	1M 1M ignore limits 1M 1A
If $\frac{135}{2 \ln 2} \cdot 2^{-\frac{a}{9}} = 45 + 30 \ln 4$, then $-\frac{a}{9} \ln 2 = \ln \left[(45 + 30 \ln 4) \frac{2 \ln 2}{135} \right]$ $\alpha \approx 1.5253$	1A $a-1$ for r.t. 1.525 ------(8)

Solution	Marks
11. Let X_A and X_B be the numbers of persons entered the building using entrances A and B respectively within a 15-minute period.	
(a) (i) $P(X_A = 0) = \frac{(3.2)^0 e^{-3.2}}{0!} = e^{-3.2} \quad \boxed{0.0408} \quad (p_1)$	1A a-1 for r.t. 0.041
(ii) $P(X_B = 0) = \frac{(2.7)^0 e^{-2.7}}{0!} = e^{-2.7} \quad \boxed{0.0672} \quad (p_2)$	1A a-1 for r.t. 0.067
(iii) $P(X_A + X_B \geq 1) = 1 - P(X_A = 0 \text{ and } X_B = 0)$ $= 1 - P(X_A = 0)P(X_B = 0)$ $= 1 - e^{-3.2}e^{-2.7}$ $= 1 - e^{-5.9} \quad \boxed{0.9973}$	1M $1 - (p_1)(p_2)$ 1A a-1 for r.t. 0.997
(iv) $P(X_A + X_B = 2)$ $= P(X_A = 2)P(X_B = 0) + P(X_A = 1)P(X_B = 1) + P(X_A = 0)P(X_B = 2)$ $= \frac{(3.2)^2 e^{-3.2}}{2!} \cdot e^{-2.7} + \frac{3.2 e^{-3.2}}{1!} \cdot \frac{2.7 e^{-2.7}}{1!} + e^{-3.2} \cdot \frac{(2.7)^2 e^{-2.7}}{2!}$ $= 17.405e^{-5.9} \quad \boxed{0.0477}$	1M for the 3 cases 1A 1A a-1 for r.t. 0.048 ------(7)
(b) (i) Since k is the most probable number of persons entered the building within a 15-minute period, $\therefore P(X = k - 1) \leq P(X = k)$ and $P(X = k + 1) \leq P(X = k)$ Hence $\frac{\lambda^{k-1} e^{-\lambda}}{(k-1)!} \leq \frac{\lambda^k e^{-\lambda}}{k!}$ $k \leq \lambda$ and $\frac{\lambda^{k+1} e^{-\lambda}}{(k+1)!} \leq \frac{\lambda^k e^{-\lambda}}{k!}$ $\lambda \leq k + 1$ $\lambda - 1 \leq k$	1M+1M 1 1
(ii) From (b)(i), $k = 5$. The probability required $= C_2^4 [P(X = k)]^2 [1 - P(X = k)]^2 [P(X = k)]$ $= C_2^4 \left(\frac{(5.9)^5 e^{-5.9}}{5!} \right)^2 \left(1 - \frac{(5.9)^5 e^{-5.9}}{5!} \right)^2 \left(\frac{(5.9)^5 e^{-5.9}}{5!} \right)$ ≈ 0.0183	1A 1M for binomial 1M for all 1A a-1 for r.t. 0.018 ------(8)

Solution	Marks
12. Let E_X and E_Y be the lifetimes of brand X and brand Y CFLs respectively.	
(a) $P(E_X < 8200) = 0.1151 \Rightarrow P\left(\frac{E_X - \mu}{400} < \frac{8200 - \mu}{400}\right) = 0.0808$ $\Rightarrow \frac{8200 - \mu}{400} = -1.4$ $\Rightarrow \mu = 8760$ $P(E_Y < 8200) = 0.1587 \Rightarrow P\left(\frac{E_Y - 8800}{\sigma} < \frac{8200 - 8800}{\sigma}\right) = 0.1587$ $\Rightarrow \frac{8200 - 8800}{\sigma} = -1.00$ $\Rightarrow \sigma = 600$ $a_1 = 0.3811, a_2 = 0.0548$ $b_1 = 0.2120, b_2 = 0.2586, b_3 = 0.2120$ $\boxed{b_1 = 0.2109, b_2 = 0.2608, b_3 = 0.2109}$	1A for either 1A 1A 1A 1A 1A $b_1 = b_3 \in [0.2101, 0.2120]$ $b_2 \in [0.2586, 0.2624]$ ------(5)
(b) The mean of the lifetimes of the 2 brands only differ a little but the standard deviation of the lifetimes of brand X CFLs is significantly smaller than that of brand Y . I shall choose brand X because the lifetimes of its CFLs are more reliable. I shall choose brand Y because there will be a bigger chance of getting a long life CFL. I shall choose brand Y because the mean lifetime is larger.	1M 1M 1M ------(1)
(c) (i) Let X_a, X_b and X_c be the lifetimes of lamps a, b and c resp. (I) The required probability $= P(X_a > 8200)[P(X_b > 8200 \text{ or } X_c > 8200)]$ $= [1 - P(E_X < 8200)]\{1 - [P(E_X < 8200)]^2\}$ $\approx (1 - 0.0808)(1 - 0.0808^2)$ $\approx 2(0.9192)^2(1 - 0.9192) + (0.9192)^3$ ≈ 0.9132 (II) The required probability $= \frac{P(X_a < 8200)P(X_b > 8200)P(X_c > 8200)}{1 - 0.9132}$ $= \frac{0.0808(1 - 0.0808)^2}{1 - 0.9132}$ $= 0.7865$	1M 1M 1M 1M+1M+1M 1A 1M for numerator 1M for denominator 1A

Solution	Marks
<p>(ii) Note that $P(E_X < 8200) \approx 0.0808$ and $P(E_Y < 8200) \approx 0.1578$.</p> <p>Since a brand X CFL is less likely than a brand Y CFL to have a lifetime less than 8200 hours, and lamp a is the most critical lamp for the lighting system to work (according to the result of (c)(i)(II)), \therefore Lamp a should be a brand X CFL. Hence I will put the brand Y CFL as lamp b or c.</p>	<p>I</p> <p>1A with explanation</p>
<p>Let X_a and Y_a be the lifetimes of lamp a when using brand X CFL and brand Y CFL respectively. Similar notations are used for the other two lamps.</p> $P(Y_a > 8200)[P(X_b > 8200 \text{ or } X_c > 8200)]$ $= (1 - 0.1587)(1 - 0.0808^2)$ $= 0.8358$ $P(X_a > 8200)[P(Y_b > 8200 \text{ or } Y_c > 8200)]$ $= (1 - 0.0808)[(1 - 0.0808) + (1 - 0.1587) - (1 - 0.0808)(1 - 0.1587)]$ $= 0.9074$ <p>Hence putting the brand Y CFL as lamp b or c will yield a better system.</p>	<p>I</p> <p>1A with explanation</p> <p>----- (9)</p>

Solution	Marks
<p>13. Let X be the number of Grade A potatoes in the 8 selected potatoes.</p> <p>(a) $P(X \leq 1 p = 0.65) \approx 0.0002 + 0.0033$ ≈ 0.0035 0.0036</p> <p>(b) (i) $P(X \leq 3 p = 0.65) \approx 0.0002 + 0.0033 + 0.0217 + 0.0808$ ≈ 0.1060 0.1061 (q)</p> <p>(ii) $P(X > 3 p = 0.2)$ $\approx 0.0459 + 0.0092 + 0.0011 + 0.0001 + 0.0000$ $\approx 1 - (0.1678 + 0.3355 + 0.2936 + 0.1468)$ ≈ 0.0563</p> <p>(c) The required probability $= C_2^3 q^2 (1 - q) + C_3^3 q^3$ $\approx C_2^3 (0.1060)^2 (1 - 0.1060) + C_3^3 (0.1060)^3$ ≈ 0.0313 0.0314</p> <p>(d) The probability that the farmer will wrongly reject the claim is 0.1060 whereas the probability that his wife will wrongly reject the claim is 0.0313. Therefore the farmer will have a bigger chance of rejecting the claim wrongly.</p> <p>(e) $P(X \leq 2 p = 0.65) \approx 0.0252$ $P(X \leq 3 p = 0.65) \approx 0.0252 + 0.0808 \approx 0.1060$ Since $P(X \leq 2 p = 0.65) < 0.05 < P(X \leq 3 p = 0.65)$ $\therefore k = 2$.</p>	<p>1M 1A ----- (2)</p> <p>1M 1A</p> <p>1M 1M</p> <p>1A ----- (5)</p> <p>1M for the 2 cases 1M for 1st term 1M for 2nd term 1M+1M+1M 1A ----- (4)</p> <p>1M ----- (1)</p> <p>1M+1A 1M for 0.05 as a value between</p> <p>1A independent ----- (3)</p>