

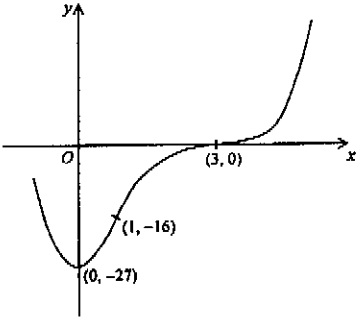
AS Mathematics and Statistics

General Marking Instructions

- It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits *all the marks* allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.
- In the marking scheme, marks are classified into the following three categories:
 'M' marks awarded for correct methods being used;
 'A' marks awarded for the accuracy of the answers;
 Marks without 'M' or 'A' awarded for correctly completing a proof or arriving at an answer given in a question.
 In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).
- For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.
- Use of notation different from those in the marking scheme should not be penalized.
- In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
- Marks may be deducted for poor presentation (*pp*). The symbol $\textcircled{pp-1}$ should be used to denote 1 mark deducted for *pp*. At most deducted 1 mark from Section A and 1 mark from Section B for *pp*. In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.
- Marks may be deducted for numerical answers with inappropriate degree of accuracy (*a*). The symbol $\textcircled{a-1}$ should be used to denote 1 mark deducted for *a*. At most deducted 1 mark from Section A and 1 mark from Section B for *a*. In any case, do not deduct any marks for *a* in those steps where candidates could not score any marks.
- Marks entered in the Page Total Box should be the NET total scored on that page.

| Solution | Marks |
|---|---|
| 1. $\ln(xy) = \frac{x}{y}$ $\ln x + \ln y = \frac{x}{y}$ $\frac{1}{x} + \frac{1}{y} \frac{dy}{dx} = \frac{y-x}{y^2} \frac{dy}{dx}$ (or $x \frac{dy}{dx} + y = \frac{y-x}{y^2} \frac{dy}{dx}$) $y^2 + xy \frac{dy}{dx} = xy - x^2 \frac{dy}{dx}$ $\frac{dy}{dx} = \frac{xy - y^2}{xy + x^2}$ | { 1M differentiation of $\ln x$ 1M chain rule 1M quotient/product rule 1 at least one step |
| Alternatively, $xy = e^{\frac{x}{y}}$ $x \frac{dy}{dx} + y = e^{\frac{x}{y}} \left(\frac{y-x}{y^2} \frac{dy}{dx} \right)$ $x \frac{dy}{dx} + y = xy \left(\frac{y-x}{y^2} \frac{dy}{dx} \right)$ $xy \frac{dy}{dx} + y^2 = xy - x^2 \frac{dy}{dx}$ $\frac{dy}{dx} = \frac{xy - y^2}{xy + x^2}$ | { 1M differentiation of e^x 1M chain rule 1M quotient rule 1 ----(4) |

| Solution | Marks |
|---|---|
| <p>2. (a) $(1+2x)^{\frac{1}{2}} = 1 + \frac{1}{2}(2x) + \frac{\frac{1}{2}(-\frac{1}{2})}{2}(2x)^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{3!}(2x)^3 + \dots$</p> $= 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \dots$ <p>$(1+8x^3)^{\frac{1}{2}} = 1 - \frac{1}{2}(8x^3) + \dots$</p> $= 1 - 4x^3 + \dots$ <p>(b) $(1-2x+4x^2)^{\frac{1}{2}} = \frac{(1+8x^3)^{\frac{1}{2}}}{(1+2x)^{\frac{1}{2}}}$</p> $= (1+8x^3)^{\frac{1}{2}}(1+2x)^{-\frac{1}{2}}$ $= (1-4x^3 + \dots)(1+x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \dots)$ $= 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 - 4x^3 + \dots$ $= 1 + x - \frac{1}{2}x^2 - \frac{7}{2}x^3 + \dots$ | <p>1M any 3 terms</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p> |
| <p>Alternatively,</p> $(1-2x+4x^2)^{\frac{1}{2}} = [1-2x(1-2x)]^{\frac{1}{2}}$ $= 1 + \left(-\frac{1}{2}\right)[-2x(1-2x)] + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}[-2x(1-2x)]^2$ $+ \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}[-2x(1-2x)]^3 + \dots$ $= 1 + x(1-2x) + \frac{3}{2}x^2(1-2x)^2 + \frac{5}{2}x^3(1-2x)^3 + \dots$ $= 1 + x - \frac{1}{2}x^2 - \frac{7}{2}x^3 + \dots$ | <p>1A</p> <p>1A</p> <p>(pp-1 for extra terms or missing '+' in all cases) ------(5)</p> |

| Solution | Marks |
|--|---|
| <p>3. Area of the shaded region = $\int_0^8 (1+x^{\frac{1}{3}} - e^{\frac{x}{8}}) dx$</p> $= \left[x + \frac{3}{4}x^{\frac{4}{3}} - 8e^{\frac{x}{8}} \right]_0^8$ $\approx 6.2537 \quad (\text{or } 28 - 8e)$ | <p>1A integrand accept $e^{\frac{x}{8}} - 1 - x^{\frac{1}{3}}$</p> <p>1A limits (pp-1 for missing dx)</p> <p>1A for $x + \frac{3}{4}x^{\frac{4}{3}}$</p> <p>1A for $-8e^{\frac{x}{8}}$</p> <p>1A $a-1$ for r.t. 6.254 ------(5)</p> |
| <p>4. (a) The graph of $f(x)$ is concave downward (or convex upward) when $1 < x < 3$. (or $1 \leq x \leq 3$ etc.)</p> <p>(b) The points of inflexion are $(1, -16)$ and $(3, 0)$.</p> <p>(c)</p> | <p>1A</p> <p>1A</p> |
|  | <p>1A minimum point</p> <p>1A shape and coordinates of points ------(4)</p> |

| Solution | | | | | Marks |
|--|-------------------|---------------------|----------------------|---------|-------|
| 5. (a) Let Q_1, Q_2, Q_3 be the 1st quartile, median and 3rd quartile resp., then | | | | | |
| Alternative methods ($n=21$) | | | | | |
| i. $Q_1 = \frac{r}{4}n$ th term | 75 kg (5.25th) | 84.5 kg (10.5th) | 89.5 kg (15.75th) | 14.5 kg | |
| ii. $Q_2 = \frac{r}{4}(n+1)$ th term | 75 kg (5.5th) | 85 kg (11th) | 91 kg (16.5th) | 16 kg | |
| iii. $Q_3 = \frac{r(n+2)}{4}$ th term | 75 kg (5.75th) | 85 kg (11th) | 90.5 kg (16.25th) | 15.5 kg | |
| iv. $Q_1 = \frac{r(n-1)+4}{4}$ th term | 75 kg (6th) | 85 kg (11th) | 90 kg (16th) | 15 kg | |
| v. $Q_3 = \frac{r}{4}n$ and round off to the nearest 0.5th term | 75 kg (5th) | 84.5 kg (10.5th) | 90 kg (16th) | 15 kg | |
| (b) (Illustrating method iv only) | | | | | |
| | | | | | |
| (c) No. The diagrams in (b) cannot show individual difference. | | | | | |
| or i. There is no substantial evidence for making the claim. ii. It is possible that the woman weighed 60 kg on completion of the programme is the woman weighed 99 kg when the programme started. (any counterexample) | | | | | |

1A median
1M 1st or 3rd quartile
1A interquartile range

(pp-1 for wrong/missing unit)

1A any correct box-and-whisker diagram with scale

1A all correct, same scale

1

----(6)

| Solution | | Marks |
|--|--|---|
| 6. (a) Probability of having no vanilla flavour ice-cream | | |
| $= \frac{C_3^4}{C_3^6} \quad \left(\text{or } \left(\frac{4}{6}\right)\left(\frac{3}{5}\right)\left(\frac{2}{4}\right), \frac{P_3^4}{P_3^6} \right)$ $= \frac{1}{5} \quad \left(\text{or } 0.2 \right)$ | | {1M denominator 1M numerator 1A |
| (b) Probability of having exactly 1 cup of vanilla flavour ice-cream | | |
| $= \frac{2C_2^4}{C_3^6} \quad \left(\text{or } C_1^1 \left(\frac{2}{6}\right)\left(\frac{4}{5}\right)\left(\frac{3}{4}\right), \frac{2 \times 3 \times P_2^4}{P_3^6} \right)$ $= \frac{3}{5} \quad \left(\text{or } 0.6 \right)$ | | 1M numerator 1A ----(5) |
| 7. The probability that there is no people killed in a traffic accident | | |
| $= e^{-0.1} \quad (p)$ ≈ 0.904837418 | | |
| The required probability | | |
| $= p^5 + 5p^4(1-p)$ ≈ 0.925477591 ≈ 0.9255 | | 2A {1M binomial (at least 2 terms) 1M cases 0 and 1 1A $a-1$ for r.t. 0.925 ----(5) |
| Alternatively, The probability that there is at least 1 people killed in a traffic accident $= 1 - e^{-0.1} \quad (q)$ ≈ 0.095162582 | | |
| The required probability | | |
| $= (1-q)^5 + 5(1-q)^4 q$ ≈ 0.925477591 ≈ 0.9255 | | |
| 8. (a) The probability of a customer winning a prize in 1 trial | | |
| $= 0.3\left(\frac{5}{9}\right) + 0.7\left(\frac{5}{6}\right)$ $= 0.75$ | | 1A 1A |
| (b) Let x and y be the probabilities of generating games A and B respectively. | | |
| Then $\frac{5}{9}x + \frac{5}{6}y = \frac{2}{3}$ | | 1M |
| and $x + y = 1$ | | 1M |
| $\therefore \frac{5}{9}x + \frac{5}{6}(1-x) = \frac{2}{3}$ $\frac{5}{6} - \frac{5}{18}x = \frac{2}{3}$ $x = \frac{3}{5} \quad \left(\text{or } 0.6 \right)$ | | |
| $\therefore \text{The probabilities of generating game } A \text{ and game } B \text{ are}$ $\frac{3}{5} \text{ (or } 0.6) \text{ and } \frac{2}{5} \text{ (or } 0.4) \text{ respectively.}$ | | 1A ----(6) |

| Solution | Marks | | | | | | | | | | | | | | | | |
|---|--|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|---|--------|------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|---|
| <p>9. (a) (i) $f(x) = 16 + 4xe^{-0.25x}$</p> <p>$f'(x) = 4e^{-0.25x}(1 - 0.25x)$</p> $\begin{cases} > 0 & \text{if } 0 < x < 4 \\ = 0 & \text{if } x = 4 \\ < 0 & \text{if } x > 4 \end{cases}$ <p>$\therefore f(x) \leq f(4)$ for $x > 0$.</p> | <p>{1M attempting to find f'</p> <p>{1A</p> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 5px auto;"> <p>accept considering $f'(x) = e^{-0.25x}(0.25x - 2)$</p> </div> <p>1 follow through</p> | | | | | | | | | | | | | | | | |
| <p>(ii)</p> <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>$f(x)$</td> <td>16 (16)</td> <td>19.1152 (19.1)</td> <td>20.8522 (20.9)</td> <td>21.6684 (21.7)</td> <td>21.8861 (21.9)</td> <td>21.7301 (21.7)</td> <td>21.3551 (21.4)</td> </tr> </table> <p>$\int_0^6 f(x) dx$</p> <p>$\approx \frac{1}{2}[16 + 21.3551 + 2(19.1152 + 20.8523 + 21.6684 + 21.8861 + 21.7301)]$</p> <p>$\approx 124$</p> <p>$\therefore$ The expected increase in profit is 124 hundred thousand dollars.</p> | x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | $f(x)$ | 16 (16) | 19.1152 (19.1) | 20.8522 (20.9) | 21.6684 (21.7) | 21.8861 (21.9) | 21.7301 (21.7) | 21.3551 (21.4) | <p>1A correct to 1 d.p.</p> <p>1M</p> <p>1A $u-1$ for r.t. 124 $pp-1$ for wrong/missing unit</p> |
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | | | | | | | |
| $f(x)$ | 16 (16) | 19.1152 (19.1) | 20.8522 (20.9) | 21.6684 (21.7) | 21.8861 (21.9) | 21.7301 (21.7) | 21.3551 (21.4) | | | | | | | | | | |
| <p>(b) (i) $g(x) = 16 + \frac{6x}{\sqrt{1+8x}}$</p> $g'(x) = \frac{6\sqrt{1+8x} - \frac{6x \cdot 8}{2\sqrt{1+8x}}}{1+8x}$ $= \frac{6(1+4x)}{(1+8x)^{\frac{3}{2}}}$ <p>> 0 for $x > 0$.</p> <p>$\therefore g(x)$ is strictly increasing for $x > 0$.</p> <p>$\therefore \lim_{x \rightarrow \infty} \left(16 + \frac{6x}{\sqrt{1+8x}} \right) = \lim_{x \rightarrow \infty} \left(16 + \frac{6\sqrt{x}}{\sqrt{\frac{1}{x} + 8}} \right)$</p> <p>$\therefore g(x) \rightarrow \infty$ as $x \rightarrow \infty$</p> | <p>1A</p> <p>1</p> | | | | | | | | | | | | | | | | |

| Solution | Marks |
|---|--|
| <p>(ii) Let $u = \sqrt{1+8x}$, then $u^2 = 1+8x$, $2udu = 8dx$</p> $\int_0^6 g(x) dx = \int_0^6 \left(16 + \frac{6x}{\sqrt{1+8x}} \right) dx \quad \left(\text{or } \int_0^6 16 dx + \int_0^6 \frac{6x}{\sqrt{1+8x}} dx \right)$ $= \int_1^7 \left(16 + \frac{6(u^2-1)}{8u} \right) \frac{1}{4} u du \quad \left(\text{or } [16x]_0^6 + \int_1^7 \frac{6(u^2-1)}{8u} \frac{1}{4} u du \right)$ $= \int_1^7 \left(\frac{3}{16} u^2 + 4u - \frac{3}{16} \right) du \quad \left(\text{or } 96 + \int_1^7 \left(\frac{3}{16} u^2 - \frac{3}{16} \right) du \right)$ $= \left[\frac{1}{16} u^3 + 2u^2 - \frac{3}{16} u \right]_1^7 \quad \left(\text{or } 96 + \left[\frac{1}{16} u^3 - \frac{3}{16} u \right]_1^7 \right)$ $= 116 \frac{1}{4}$ ≈ 116 <p>\therefore The expected increase in profit is 116 hundred thousand dollars.</p> | <p>{1A integrand</p> <p>{1A limits</p> <p>1A ignore limits</p> <p>1A $a-1$ for r.t. 116 $pp-1$ for wrong/missing unit</p> |
| <p>(c) From (a)(i), $f(x) \leq f(4)$ ($= 21.8861$) for $x > 0$.</p> <p>i.e. $f(x)$ is bounded above by $f(4)$.</p> <p>From (b)(i), $g(x)$ increases to infinity as x increases to infinity.</p> <p>$\therefore f(x) > 0$ and $g(x) > 0$ for $x > 0$,</p> <p>the area under the graph of $g(x)$ will be greater than that of $f(x)$ as x increases indefinitely.</p> <p>\therefore Plan G will eventually result in a bigger profit.</p> | <p>1M</p> <p>1A</p> |
| <p>Alternative graph for 10(a)(ii)</p> | |

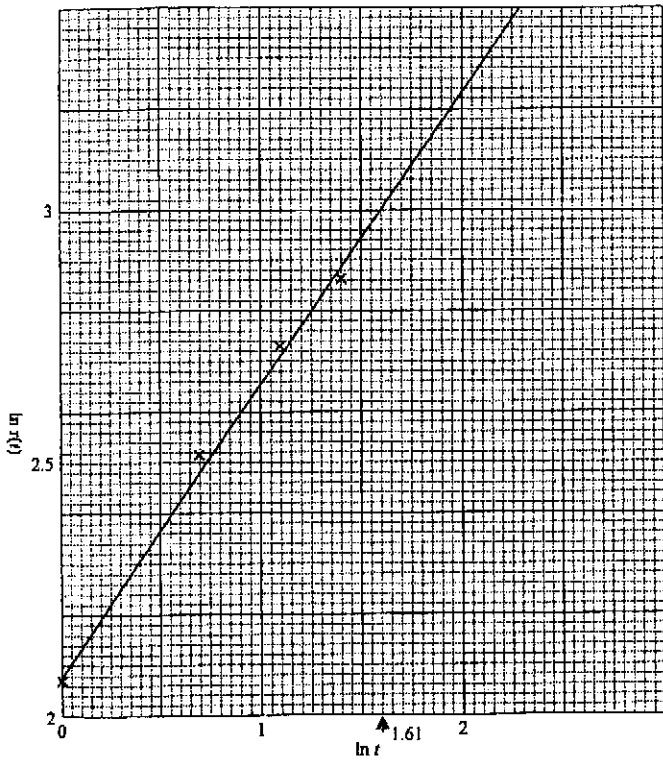
Solution

Marks

10. (a) (i) $r(t) = at^b$
 $\ln r(t) = \ln a + b \ln t$

(ii)

| | | | | |
|------------|------|------|------|------|
| $\ln t$ | 0 | 0.69 | 1.10 | 1.39 |
| $\ln r(t)$ | 2.07 | 2.51 | 2.73 | 2.86 |



When $t = 5$, $\ln t \approx 1.61$
 $\therefore \ln r(5) \approx 3$ from the graph.
 $r(5) \approx 20.1$

1A

Accept using common logarithm

1M scale and labelling
 1A points and line

1M for either
 1A $r(5) \in [19.1, 21.1]$
 $\ln r(5) \in [2.95, 3.05]$

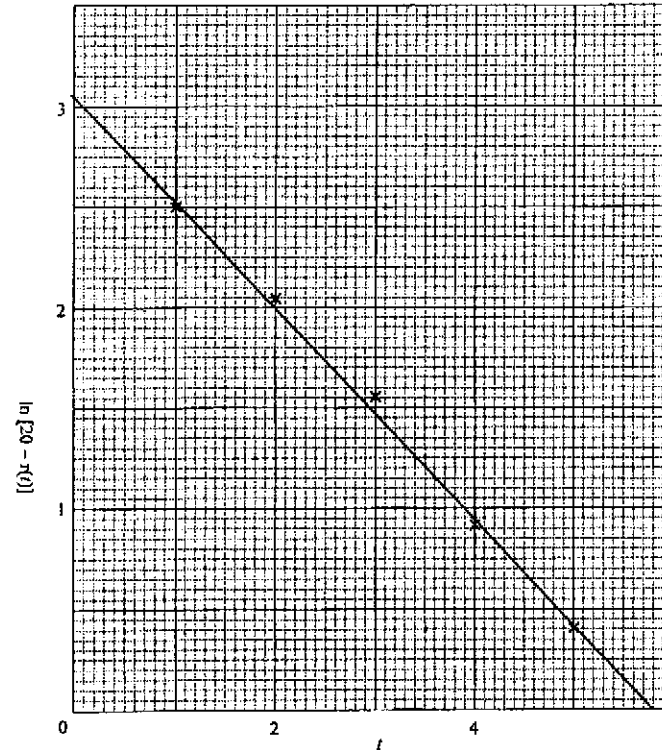
Solution

Marks

(b) (i) $r(t) = 20 - pe^{-qt}$
 $\ln[20 - r(t)] = \ln p - qt$

(ii)

| | | | | | |
|------------------|------|------|------|------|------|
| t | 1 | 2 | 3 | 4 | 5 |
| $\ln[20 - r(t)]$ | 2.49 | 2.04 | 1.55 | 0.92 | 0.41 |



From the graph,
 $\ln p \approx 3.05$
 $p \approx 21.1$
 $-q = \frac{0.41 - 3.05}{5} \approx -0.528$
 $q \approx 0.528$

1A

1M scale and labelling
 1A ignoring the point at $t = 5$

1A $p \in [20.1, 23.3]$
 $\ln p \in [3.00, 3.15]$

1A $q \in [0.518, 0.550]$

No marks for p or q if the graph is not correct.

| Solution | Marks |
|---|--|
| <p>(iii) The total number, in thousands, of bacteria after 15 days of cultivation</p> $= \int_0^{15} [20 - pe^{-qt}] dt + 100$ $= \left[20t + \frac{p}{q} e^{-qt} \right]_0^{15} + 100$ $= 300 + \frac{p}{q} e^{-15q} - \frac{p}{q} + 100$ $\approx 260 + 100$ ≈ 360 | <p>{1M definite integral 1M adding 100</p> <p>1M for integration</p> <p>1A $\int_0^{15} [20 - pe^{-qt}] dt \in [255, 263]$</p> <p>1A Ans. $\in [355, 363]$ pp-1 for wrong/missing unit</p> |
| <p>Alternatively, Let $N(t)$ thousand be the total number of bacteria after t days of cultivation. Then</p> $N(t) = \int [20 - pe^{-qt}] dt$ $= 20t + \frac{p}{q} e^{-qt} + c$ <p>$\therefore N(0) = 100$</p> $\therefore 100 = \frac{p}{q} + c$ $c = 100 - \frac{p}{q} \approx 60.04$ <p>Hence the total number, in thousands, of bacteria after 15 days of cultivation is</p> $N(15) = 20 \times 15 + \frac{p}{q} e^{-15q} + c \approx 360$ | <p>1M</p> <p>1M for integration</p> <p>1A $c \in [55.02, 63.46]$</p> <p>1M+1A $N(15) \in [355, 363]$</p> |

| Solution | Marks |
|--|---|
| <p>11. (a) $\begin{cases} \ln 55 = a - e^{-2k} \\ \ln 98 = a - e^{-4k} \end{cases}$</p> <p>Eliminating a, we have</p> $e^{-4k} - e^{-2k} + \ln 98 - \ln 55 = 0$ $e^{-4k} - e^{-2k} + \frac{1}{e} \ln \frac{98}{55} = 0$ $(e^{-2k})^2 - e^{-2k} + \frac{1}{e} \ln \frac{98}{55} = 0$ $e^{-2k} = \frac{1 \pm \sqrt{1 - \frac{4}{e} \ln \frac{98}{55}}}{2}$ $\approx 0.30635 \text{ or } 0.69365$ $\approx 0.306 \text{ or } 0.694$ $\begin{cases} k \approx 0.5915 \\ a \approx 4.8401 \end{cases} \text{ or } \begin{cases} k \approx 0.1829 \\ a \approx 5.8929 \end{cases}$ $\begin{cases} k \approx 0.59 \\ a \approx 4.84 \end{cases} \text{ or } \begin{cases} k \approx 0.18 \\ a \approx 5.89 \text{ (or } 5.90) \end{cases} \quad (2 \text{ d.p.})$ | <p>1</p> <p>1M quadratic equation</p> <p>1A r.t. 0.306, 0.694</p> <p>1A $a-1$ for more than 2 d.p.</p> |
| <p>(b) Using $\begin{cases} k \approx 0.59 \\ a \approx 4.84 \end{cases}$, $\ln N(7) = 4.80$, $N(7) \approx 121$.</p> <p>Using $\begin{cases} k \approx 0.18 \\ a \approx 5.89 \end{cases}$, $\ln N(7) = 5.12$, $N(7) \approx 167$. (or comparing $\ln 170 \approx 5.1358$)</p> <p>$\therefore \begin{cases} k \approx 0.18 \\ a \approx 5.89 \end{cases}$ will make the model fit for the known data.</p> <p>$\therefore N(t) = e^{\ln N(t)} \approx e^{5.89 - e^{-0.18t}}$</p> <p>$\therefore N(t) \rightarrow e^{5.89} \approx 361$ as $t \rightarrow \infty$</p> <p>The total possible catch of coral fish in that area since January 1, 1992 is 361 thousand tonnes.</p> | <p>1M r.t. 4.80</p> <p>r.t. 121</p> <p>r.t. 5.12 - 5.14</p> <p>r.t. 167 - 170</p> <p>1A follow through</p> <p>1M</p> <p>1A r.t. 361 - 365 pp-1 for wrong/missing unit</p> |

| Solution | Marks |
|--|--|
| <p>(c) (i) $\therefore \ln N(t) = a - e^{-kt}$ $\therefore \frac{N'(t)}{N(t)} = ke^{-kt}$ $N'(t) = k N(t)e^{-kt}$</p> | 1 |
| <p>Alternatively, $N(t) = e^{a - e^{-kt}}$ $N'(t) = -e^{-kt}(-k)e^{a - e^{-kt}} = ke^{-kt} N(t)$</p> | 1 |
| <p>(ii) $N''(t) = k[N'(t)e^{-kt} - k N(t)e^{-2kt}]$ $= k^2 N(t)e^{-2kt} (e^{-kt} - 1)$ $\begin{cases} > 0 & \text{when } t < \frac{1}{k} \\ = 0 & \text{when } t = \frac{1}{k} \\ < 0 & \text{when } t > \frac{1}{k} \end{cases}$</p> | 1A 1M |
| <p>$\therefore N'(t)$ is maximum at $t = \frac{1}{k}$ ≈ 5.56</p> | 1A $t \in [5.47, 5.56]$ |
| <p>The maximum rate of change of the total catch of coral fish in that area since January 1, 1992 occurred in 1997.</p> | 1A |
| <p>$\ln N(6) \approx 4.97, N(6) \approx 143.6$</p> | $\ln N(6) \in [4.97, 4.99]$ $N(6) \in [143.6, 146.3]$ |
| <p>$\ln N(5) \approx 4.78, N(5) \approx 119.7$</p> | $\ln N(5) \in [4.78, 4.80]$ $N(5) \in [119.7, 122.0]$ |
| <p>\therefore The volume of fish caught in 1997 $= [N(6) - N(5)]$ thousand tonnes ≈ 24 thousand tonnes</p> | 1M 1A $pp-1$ for wrong/missing unit |

| Solution | Marks |
|---|---|
| <p>12. (a) Let $X \sim N(20, 5^2)$ and $Z \sim N(0, 1)$.</p> | |
| <p>(i) $P(\text{risky but not hazardous} A)$ $= P(12 < X < 27)$ $= P\left(\frac{12-20}{5} < Z < \frac{27-20}{5}\right)$ $= P(-1.6 < Z < 1.4)$ $\approx 0.4452 + 0.4192$ ≈ 0.8644</p> | 1A 1M 1A $a-1$ for r.t. 0.864 |
| <p>(ii) $P(\text{risky} A) = P(X > 12)$ $= P(Z > -1.6)$ $\approx 0.4452 + 0.5$ ≈ 0.9452</p> | 1A |
| <p>$P(\text{hazardous} A) = P(X > 27)$ $= P(Z > 1.4)$ $\approx 0.5 - 0.4192$ ≈ 0.0808</p> | 1A |
| <p>$\therefore P(\text{a risky bottle is hazardous} A) = \frac{0.0808}{0.9452} \approx 0.0855$</p> | 1M |
| <p>(b) (i) $P(\text{risky}) = 0.6 P(\text{risky} A) + 0.4 P(\text{risky} B)$ $\approx 0.6(0.9452) + 0.4(0.058)$ ≈ 0.59032 $\approx 0.5903 \quad (p)$</p> | 1M 1A $a-1$ for r.t. 0.590 |
| <p>$P(B \text{ and risky} \text{risky}) = \frac{P(\text{risky} B)P(B)}{P(\text{risky})}$ $\approx \frac{(0.058)(0.4)}{0.59032}$ ≈ 0.0393</p> | { 1A numerator 1M Bayes' theorem |
| <p>(ii) $P(B \text{ and hazardous} \text{risky}) = \frac{P(\text{hazardous} B)P(B)}{P(\text{risky})}$ $\approx \frac{(0.004)(0.4)}{0.59032}$ ≈ 0.00271 ≈ 0.0027</p> | { 1A numerator 1M Bayes' theorem |
| <p>(iii) $P(\text{license suspended}) = 1 - (1-p)^5 - 5p(1-p)^4$ $\approx 1 - (1-0.59032)^5 - 5(0.59032)(1-0.59032)^4$ ≈ 0.9053</p> | { 1M binomial 1M complement of cases 0 & 1 1M p from b(i) |

| Solution | Marks |
|--|--|
| 13. (a) Possible teams: $B_1 G_1, B_1 G_2, B_2 G_1$ and $B_2 G_2$. | 1A |
| (b) The probability that $B_1 G_1$ can enter the second round of the contest = 0.9×0.8 = 0.72 | 1A |
| (c) Probability required = $\frac{1}{4}(0.9 \times 0.8 + 0.9 \times 0.6 + 0.7 \times 0.8 + 0.7 \times 0.6)$ = 0.56 | 1M 4 cases 1A |
| (d) Suppose $B_1 G_1$ and $B_2 G_2$ are formed to represent the school. | |
| (i) The probability that exactly one team can enter the second round = $(0.9 \times 0.8)(1 - 0.7 \times 0.6) + (0.7 \times 0.6)(1 - 0.9 \times 0.8)$ <u>or $1 - (0.9 \times 0.8)(0.7 \times 0.6) - (1 - 0.9 \times 0.8)(1 - 0.7 \times 0.6)$</u> = 0.5352 | { 1A probability of 1 case 1M summation of 2 cases |
| (ii) The probability that at least one team can enter the second round = $0.5352 + 0.9 \times 0.8 \times 0.7 \times 0.6$ <u>or $1 - (1 - 0.9 \times 0.8)(1 - 0.7 \times 0.6)$</u> <u>or $0.9 \times 0.8 + 0.7 \times 0.6 - 0.9 \times 0.8 \times 0.7 \times 0.6$</u> = 0.8376 | 1A 1M |
| (e) (i) If the two teams are formed randomly, the probability that exactly one team can enter the second round = $\frac{1}{2} \times 0.5352 + \frac{1}{2} [(0.9 \times 0.6)(1 - 0.7 \times 0.8) + (0.7 \times 0.8)(1 - 0.9 \times 0.6)]$ = $\frac{1}{2}(0.5352 + 0.4952)$ = 0.5152 | { 1M the combination $B_1 G_2, B_2 G_1$ 1M multiplying by $\frac{1}{2}$ |
| (ii) If $B_1 G_2$ and $B_2 G_1$ are formed to represent the school, the probability that at least one team can enter the second round = $0.4952 + 0.9 \times 0.8 \times 0.7 \times 0.6$ <u>or $1 - (1 - 0.9 \times 0.6)(1 - 0.7 \times 0.8)$</u> <u>or $0.9 \times 0.6 + 0.7 \times 0.8 - 0.9 \times 0.6 \times 0.7 \times 0.8$</u> = 0.7976 | 1A 1M |
| From (d)(ii), the combination $B_1 G_1$ and $B_2 G_2$ will have a better chance of having at least one team that can enter the second round of the contest. | 1M |

| Solution | Marks |
|--|---|
| 14. (a) <u>Poisson distribution</u> <u>Binomial distribution</u> frequency: $100 \cdot \frac{e^{-1}}{x!}$ $100 \cdot C_x^6 \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{6-x}$ missing values: 36.79 20.09 (or 20.09 20.10 6.13 0.80 0.81 0.80) | 1A+1A 1A+1A $\alpha-1$ for more than 2 d.p. |
| (b) The maximum error for Po(1) is 2.79 while that for $B(6, \frac{1}{6})$ is 1.19. The binomial distribution is better. | 1A awarded only if (a) is correct |
| (c) (i) The probability of getting at least 1 stamp in a box of the Chips = $1 - \left(1 - \frac{1}{6}\right)^6$ <u>or $1 - 0.3349$</u> = 0.6651020233 = 0.6651 (p_1) <u>or 0.6650</u> | 1M read from table 1A $\alpha-1$ for r.t. 0.665 |
| (ii) The probability of getting at least 1 stamp in buying not more than 3 boxes = $p_1 + (1 - p_1)p_1 + (1 - p_1)^2 p_1$ = $(0.665102)[1 + (1 - 0.665102) + (1 - 0.665102)^2]$ = 0.9624389632 = 0.9624 | { 1M cases 1, 2 and 3 1M Geometric (p) 1A $\alpha-1$ for r.t. 0.962 |
| (iii) The probability of getting exactly 5 stamps in 2 boxes with stamps = $2 \left[C_1^6 \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^5 C_4^6 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^2 + C_2^6 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^4 C_3^6 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^3 \right]$ <u>or $2[0.4019 \times 0.0080 + 0.2009 \times 0.0536]$</u> = $2 \left[C_1^6 C_4^6 \left(\frac{1}{6}\right)^5 \left(\frac{5}{6}\right)^7 + C_2^6 C_3^6 \left(\frac{1}{6}\right)^5 \left(\frac{5}{6}\right)^7 \right]$ = $\frac{2 \times 5^7 (6 \times 15 + 15 \times 20)}{6^{12}}$ = 0.027994301 (p_2) The probability of getting stamps in both boxes in buying two boxes = p_1^2 = 0.442360701 (p_3) The required conditional probability = $\frac{p_2}{p_3}$ = $\frac{0.027994301}{0.442360701}$ = 0.0633 | { 1M combinations (1, 4) and (2, 3) 1M multiplying by 2 1M binomial distribution read from table 1M 1M 0.0632 if using figures in table |