2000-ASL
M & S

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG ADVANCED LEVEL EXAMINATION 2000

MATHEMATICS AND STATISTICS AS-LEVEL

8.30 am – 11.30 am (3 hours)
This paper must be answered in English

1. This paper consists of Section A and Section B.
2. Answer ALL questions in Section A, using the AL(E) answer book.
3. Answer any FOUR questions in Section B, using the AL(C)2 answer book.
4. Unless otherwise specified, numerical answers should either be exact or given to 4 decimal places.

SECTION A (40 marks)
Answer ALL questions in this section.
Write your answers in the AL(E) answer book.

1. Let $\ln(xy) = \frac{x}{y}$ where $x, y > 0$. Show that $\frac{dy}{dx} = \frac{xy - y^2}{xy + x^2}$.

(4 marks)

2. Let $|x| < \frac{1}{2}$.

(a) Expand $(1+2x)^{\frac{1}{2}}$ and $(1+8x^3)^{-\frac{1}{2}}$ respectively in ascending powers of x as far as the term in x^3.

(b) Using (a) and the identity $(1+2x)(1-2x+4x^2) \equiv 1+8x^3$, or otherwise, expand $(1-2x+4x^2)^{-\frac{1}{2}}$ in ascending powers of x as far as the term in x^3.

(5 marks)

3. Figure 1 shows the graphs of the two curves

C_1: $y = e^{\frac{x}{8}}$ and

C_2: $y = 1 + x^{\frac{1}{3}}$.

Find the area of the shaded region.

(5 marks)

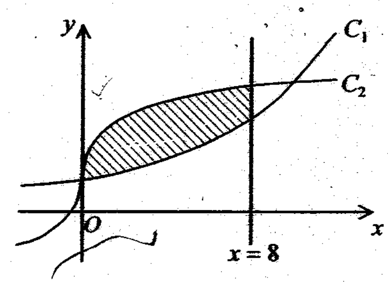

Figure 1

4. A polynomial f(x) has the following properties:

	$x<0$	$x=0$	$0<x<1$	$x=1$	$1<x<3$	$x=3$	$x>3$
f(x)		−27		−16		0	
f′(x)	< 0	0	> 0	> 0	> 0	0	> 0
f″(x)	> 0	> 0	> 0	0	< 0	0	> 0

(a) Find the range of values of x for which the graph of f(x) is concave downward (i.e. convex upward).

(b) Write down all the points of inflexion of the graph of f(x).

(c) Sketch the graph of f(x).

(4 marks)

5. A fitness centre advertised a programme specifically designed for women weighing 70 kg or more, and claimed that their individual weights could be reduced by at least 20 kg on completion of the programme. Twenty-one women joined the programme and their weights in kg when they started are shown below:

Stem (tens)	Leaf (units)
7	0 0 2 3 5 5 7
8	1 1 4 5 6 6 7 8
9	0 2 5 8 9 9

(a) Find the median and the interquartile range of these weights.

(b) On completion of the programme, the median, lower quartile and upper quartile of the weights of these women are 73 kg, 68 kg and 77 kg respectively. The lightest and heaviest women weigh 60 kg and 82 kg respectively. Draw two box-and-whisker diagrams in your answer book comparing the weights of these women before and after the programme.

(c) Referring to the box-and-whisker diagram in (b), someone claimed that none of these women had reduced their individual weights by 20 kg or more on completion of the programme. Determine whether this claim is correct or not. Explain your answer briefly.

(6 marks)

6. Mr. Chan has 6 cups of ice-cream in his refrigerator. There are 5 different flavours as listed:

1 cup of chocolate,
1 cup of mango,
1 cup of peach,
1 cup of strawberry and
2 cups of vanilla.

Mr. Chan randomly chooses 3 cups of the ice-cream. Find the probability that

(a) there is no vanilla flavour ice-cream,

(b) there is exactly 1 cup of vanilla flavour ice-cream.

(5 marks)

7. The number of people killed in a traffic accident follows a Poisson distribution with mean 0.1. There are 5 traffic accidents on a given day, find the probability that there is at most 1 accident in which some people are killed.

(5 marks)

8. A department store uses a machine to offer prizes for customers by playing games A or B. The probability of a customer winning a prize in game A is $\frac{5}{9}$ and that in game B is $\frac{5}{6}$. Suppose each time the machine randomly generates either game A or game B with probabilities 0.3 and 0.7 respectively.

(a) Find the probability of a customer winning a prize in 1 trial.

(b) The department store wants to adjust the probabilities of generating game A and game B so that the probability of a customer winning a prize in 1 trial is $\frac{2}{3}$. Find the probabilities of generating game A and game B respectively.

(6 marks)

SECTION B (60 marks)
Answer any FOUR questions in this section. Each question carries 15 marks.
Write your answers in the AL(C)2 answer book.

9. A department store has two promotion plans, F and G, designed to increase its profit, from which only one will be chosen. A marketing agent forecasts that if x hundred thousand dollars is spent on a promotion plan, the respective rates of change of its profit with respect to x can be modelled by $f(x) = 16 + 4xe^{-0.25x}$ and $g(x) = 16 + \frac{6x}{\sqrt{1+8x}}$.

(a) Suppose that promotion plan F is adopted.

(i) Show that $f(x) \le f(4)$ for $x > 0$.

(ii) If six hundred thousand dollars is spent on the plan, use the trapezoidal rule with 6 sub-intervals to estimate the expected increase in profit to the nearest hundred thousand dollars.

(6 marks)

(b) Suppose that promotion plan G is adopted.

(i) Show that $g(x)$ is strictly increasing for $x > 0$.

As x tends to infinity, what value would $g(x)$ tend to?

(ii) If six hundred thousand dollars is spent on the plan, use the substitution $u = \sqrt{1+8x}$, or otherwise, to find the expected increase in profit to the nearest hundred thousand dollars.

(7 marks)

(c) The manager of the department store notices that if six hundred thousand dollars is spent on promotion, plan F will result in a bigger profit than G. Determine which plan will eventually result in a bigger profit if the amount spent on promotion increases indefinitely. Explain your answer briefly.

(2 marks)

10. A researcher studied the growth of a certain kind of bacteria. 100 000 such bacteria were put into a beaker for cultivation. Let t be the number of days elapsed after the cultivation has started and $r(t)$, in thousands per day, be the growth rate of the bacteria. The researcher obtained the following data:

t	1	2	3	4
$r(t)$	7.9	12.3	15.3	17.5

(a) The researcher suggested that $r(t)$ can be modelled by $r(t) = at^b$, where a and b are positive constants.

(i) Express $\ln r(t)$ as a linear function of $\ln t$.

(ii) Using the graph paper on Page 6, estimate graphically the value of $r(5)$ to 1 decimal place without finding the values of a and b.

(5 marks)

(b) The researcher later observed that $r(5)$ was 18.5 and considered the model in (a) unsuitable. After reviewing some literature, he used the model $r(t) = 20 - pe^{-qt}$, where p and q are positive constants.

(i) Express $\ln[20 - r(t)]$ as a linear function of t.

(ii) Using the graph paper on Page 7, estimate graphically the values of p and q to 3 significant figures.

(iii) Estimate the total number of bacteria, to the nearest thousand, after 15 days of cultivation.

(10 marks)

Candidate Number | Centre Number | Seat Number | Page Total

10. (Cont'd) **If you attempt Question 10, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**

Graph paper for part (a)(ii), Question 10

Page Total

Graph paper for part (b)(ii), Question 10

11. A researcher studied the commercial fishing situation in a certain fishing zone. Denoting the total catch of coral fish in that zone in t years time from January 1, 1992 by $N(t)$ (in thousand tonnes), he obtained the following data:

t	2	4
$N(t)$	55	98

The researcher modelled $N(t)$ by $\ln N(t) = a - e^{1-kt}$ where a and k are constants.

(a) Show that $e^{-4k} - e^{-2k} + \frac{1}{e}\ln\frac{98}{55} = 0$.

Hence find, to 2 decimal places, two sets of values of a and k.

(4 marks)

(b) The researcher later found out that $N(7) = 170$. Determine which set of values of a and k obtained in (a) will make the model fit for the known data.

Hence estimate, to the nearest thousand tonnes, the total possible catch of coral fish in that zone since January 1, 1992.

(4 marks)

(c) The rate of change of the total catch of coral fish in that zone since January 1, 1992 at time t is given by $\frac{dN(t)}{dt}$.

(i) Show that $\frac{dN(t)}{dt} = kN(t)e^{1-kt}$.

(ii) Using the values of a and k chosen in (b), determine in which year the maximum rate of change occurred.

Hence find, to the nearest integer, the volume of fish caught in that year.

(7 marks)

12. The milk produced by Farm A has been contaminated by dioxin. The amount of dioxin presented in each bottle of milk follows a normal distribution with mean 20 ng (1 ng = 10^{-6} g) and standard deviation 5 ng. Bottles which contain more than 12 ng of dioxin are classified as *risky*, and those which contain more than 27 ng are *hazardous*.

(a) Suppose a bottle of milk from Farm A is randomly chosen.

(i) Find the probability that it is *risky* but not *hazardous*.

(ii) If it is *risky*, find the probability that it is *hazardous*.

(6 marks)

(b) A distributor purchases bottles of milk from both Farm A and Farm B and sells them under the same brand name 'Healthy'. It is known that 60% of the milk is from Farm A and the rest from Farm B. A bottle of milk from Farm B has a probability of 0.058 of being *risky* and 0.004 of being *hazardous*.

(i) If a randomly chosen bottle of 'Healthy' milk is *risky*, find the probability that it is from Farm B.

(ii) If a randomly chosen bottle of 'Healthy' milk is *risky*, find the probability that it is a *hazardous* bottle from Farm B.

(iii) The Health Department inspects 5 randomly chosen bottles of 'Healthy' milk. If 2 or more bottles of milk in the batch are *risky*, the distributor's license will be suspended immediately. Find the probability that the license will be suspended.

(9 marks)

13. Boys B_1, B_2 and girls G_1, G_2 are students who have qualified to represent their school in a singing contest. One boy and one girl will form one team. The team formed by B_i and G_j is denoted by $B_i G_j$, where $i = 1, 2$ and $j = 1, 2$. A team can enter the second round of the contest if both team members do not make any mistakes during their performance. Suppose that a student making mistakes in a performance is an independent event, and the probabilities that B_1, B_2, G_1 and G_2 do not make any mistakes in a performance are 0.9, 0.7, 0.8 and 0.6 respectively.

(a) List all the possible teams that can be formed.

(1 mark)

(b) Find the probability that $B_1 G_1$ can enter the second round of the contest.

(1 mark)

(c) If a team is selected randomly to represent the school, find the probability that the team can enter the second round of the contest.

(2 marks)

(d) If two teams $B_1 G_1$ and $B_2 G_2$ are formed to represent the school, find the probability that

(i) exactly one team can enter the second round of the contest,

(ii) at least one team can enter the second round of the contest.

(5 marks)

(e) Suppose that two teams are allowed to represent the school and each student can only join one team.

(i) If the two teams are formed randomly, find the probability that exactly one team can enter the second round of the contest.

(ii) How should the teams be formed so that the school has a better chance of having at least one team that can enter the second round of the contest?

(6 marks)

14. The company which sells boxes of Energy Chips wanted to increase its sales volume by issuing stamps which could be exchanged for toys. Stamps were put into some of the boxes and a collection of 5 stamps could be exchanged for one toy. Some students conducted a survey to study the chance of getting stamps. They randomly selected 100 boxes of the Chips and counted the number of stamps in each box. The result is shown in the first two columns of Table 1.

(a) The students tried to fit the data by a binomial ($n = 6$) and a Poisson distribution. They took 1 as the mean of each of the distributions. Fill in the missing values in Table 1.

(4 marks)

(b) Suppose the absolute values of the differences between observed and expected frequencies are regarded as errors. The distribution with a smaller maximum error will fit the data better. Which distribution in (a) is better?

(1 mark)

(c) Assume the distribution chosen in (b) is adopted.

(i) Find the probability of getting at least 1 stamp in a box of the Chips.

(ii) Mrs. Wong keeps on buying the Chips until she gets at least 1 stamp. Find the probability that she needs not buy more than 3 boxes.

(iii) Suppose Mr. Cheung buys 2 boxes of the Chips. If he gets stamps in both boxes, find the probability that he gets just enough stamps to exchange for a toy.

(10 marks)

Candidate Number	Centre Number	Seat Number	Page Total

14. (Cont'd) **If you attempt Question 14, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**

Table 1 Observed and expected frequencies of the number of boxes by the number of stamps in 100 boxes of Energy Chips

Number of stamps	Observed frequency	Expected frequency *	
		Poisson	Binomial
0	34	36.79	33.49
1	39		40.19
2	21	18.39	
3	5		5.36
4	1	1.53	
5	0	0.31	0.06
6	0	0.05	0.00
Total	100		

* Correct to 2 decimal places.

END OF PAPER

This is a blank page.