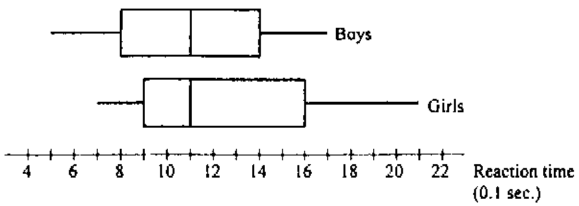
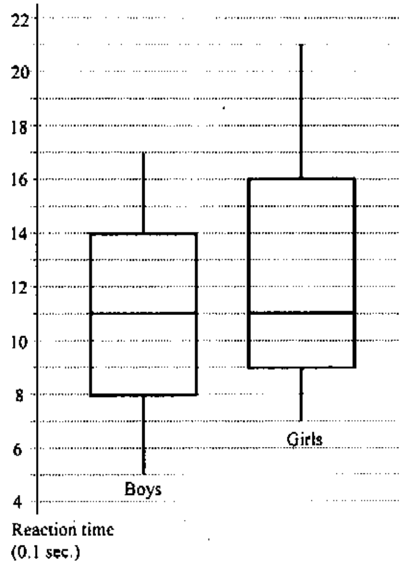
Solution	Marks	Remarks
1. (a) When $x=1$, $e^{y}=\frac{2^{3}}{2}=4$		
$y=\ln 4$ (or $y=2\ln 2$)	1A	
(or $y=1.3863$)		$a-1$ for r.t. 1.386
(b) ∵ $e^{xy}=\frac{x(x+1)^{3}}{x^{2}+1}$		
$xy=\ln\left(\frac{x(x+1)^{3}}{x^{2}+1}\right)$	1A	taking log on both sides (one side must correct)
$xy=\ln x+3\ln(x+1)-\ln(x^{2}+1)$		
$x\frac{\mathrm{d}y}{\mathrm{d}x}+y=\frac{1}{x}+\frac{3}{x+1}-\frac{2x}{x^{2}+1}$	1M+1M	1M for product rule 1M for differentiating log
When $x=1$, $\frac{\mathrm{d}y}{\mathrm{d}x}+\ln 4=1+\frac{3}{2}-1$		
$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2}-\ln 4$ (or 0.1137)	1A	$a-1$ for r.t. 0.114
<u>Alternatively,</u>		
$e^{xy}(x\frac{\mathrm{d}y}{\mathrm{d}x}+y)=\frac{(x^{2}+1)[(x+1)^{3}+3(x+1)^{2}x]-x(x+1)^{3}(2x)}{(x^{2}+1)^{2}}$	1M+1M+1A	1M for differentiating e^{xy} 1M for product/quotient rule
When $x=1$, $e^{xy}(\frac{\mathrm{d}y}{\mathrm{d}x}+\ln 4)=6$		
$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2}-\ln 4$	1A	
	(5)	
2. (a) $e^{-2x}=1-2x+\frac{(-2x)^{2}}{2!}+\frac{(-2x)^{3}}{3!}+\cdots$		
$=1-2x+2x^{2}-\frac{4x^{3}}{3}+\cdots$	1A	
(b) $(1+x)^{\frac{1}{2}}=1+\left(\frac{1}{2}\right)x+\left(\frac{1}{2!}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)x^{2}+\left(\frac{1}{3!}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)x^{3}+\cdots$	1M	for any 3 terms
$=1+\frac{x}{2}-\frac{x^{2}}{8}+\frac{x^{3}}{16}+\cdots$	1A	
$\frac{(1+x)^{\frac{1}{2}}}{e^{2x}}=\left(1+\frac{x}{2}-\frac{x^{2}}{8}+\frac{x^{3}}{16}+\cdots\right)\left(1-2x+2x^{2}-\frac{4x^{3}}{3}+\cdots\right)$	1M	applying the result in (a)
$=1-\frac{3}{2}x+\frac{7}{8}x^{2}-\frac{1}{48}x^{3}+\cdots$	1A	
		pp−1 for missing '+⋯' in all cases
The expansion is valid for $\lvert x\rvert<1$. (or $-1<x<1$)	1A	
	(6)	

Solution	Marks	Remarks
3. (a) 	1A 1A 1A	for a box-and-whisker diagram for a correct box-and-whisker diagram with scale for all being correct
<u>OR</u> 		
(b) Their chances of having a reaction time shorter than 1.1 seconds are equal. This is because (i) the median reaction times for both the boys and girls are 1.1 sec. or (ii) both the probabilities of having a reaction time being less than 1.1 sec for the boy and the girl are 0.5 .	1A	
	1 (5)	
4. (a) $N=\int(8t^{\frac{1}{3}}+11t^{\frac{5}{6}})\mathrm{d}t$,	1M+1A	1M for integration *pp*−1 for missing N or dt
$=6t^{\frac{4}{3}}+6t^{\frac{11}{6}}+c$ for some constant c .	1A	*pp*−1 for missing c
∵ $N=100$ when $t=1$ ∴ $100=6+6+c \Rightarrow c=88$	1A	
i.e. $N=6t^{\frac{4}{3}}+6t^{\frac{11}{6}}+88$		
(b) When $t=64$, $N=6(64)^{\frac{4}{3}}+6(64)^{\frac{11}{6}}+88$ $=13912$	1A	
	(5)	

Solution	Marks	Remarks
5. Let X be the no. of passengers using Octopus in a compartment.		
(a) $P(X=5) = C_5^{10}(0.6)^5(1-0.6)^5$	1A	
≈ 0.200658		
≈ 0.2007 (p_1)	1A	a-1 for r.t. 0.201
(b) $E(X) = np = 10 \times 0.6 = 6$ The mean number of passengers using Octopus in a compartment is 6 .	1A+1A	
(c) The probability that the third compartment is the first one to have exactly 5 passengers using Octopus		
$= (1-0.200658)^2(0.200658)$	1M	$(1-p_1)^2 p_1$
≈ 0.1282	1A (6)	a–1 for r.t. 0.128
6. (a) The number of different groups can be formed $= 5 \times 4 \times C_{10}^{16}$ (or $P_2^5 \times C_{10}^{16}$)	1M+1A	1M for P_2^5 or C_{10}^{16}
$= 160160$		
(b) The possible numbers of boys are 10, 11, 12, 13, 14, 15, 16.	1A	
(c) Using (b) and the method of "try and error":		

Number of boys among the students	Probability of having a time keeping group with all the time keepers being boys
10	$\frac{C_{10}^{10}}{C_{10}^{16}} = \frac{1}{8008}$
11	$\frac{C_{10}^{11}}{C_{10}^{16}} = \frac{1}{728}$
12	$\frac{C_{10}^{12}}{C_{10}^{16}} = \frac{3}{364}$
⋮	

Solution	Marks	Remarks
	1M	for $\frac{C_{10}^{n}}{C_{10}^{16}}$
	1A	
∴ There are 12 boys among the students.	1A (6)	

Solution	Marks	Remarks
7. Let E_X be the event that cable X is operative, E_Y be the event that cable Y is operative, E_Z be the event that cable Z is operative, and F be the event that A and B are not able to make contact.		
(a) (i) $P(E_X' \cap E_Z') = (0.015)(0.030)$		
$= 0.00045$ (p_1)	1A	a–1 for r.t. 0.0005 (method must be shown)
(ii) $P(E_X' \cap E_Y' \cap E_Z') = (0.015)(0.025)(0.030)$		
$= 0.00001125$ (p_2)	1A	a–1 for r.t. 0.0000 (method must be shown)
(iii) $P(F) = P(E_X' \cap E_Z') + P(E_Y' \cap E_Z') - P(E_X' \cap E_Y' \cap E_Z')$		
$= 0.00045 + (0.025)(0.030) - 0.00001125$	1M	$p_1 + (0.025)(0.030) - p_2$
$= 0.00118875$		
≈ 0.001189 (p_3)	1A	r.t. 0.001189 , a–1 for r.t. 0.0012 (method must be shown)
(b) $P(F \mid E_X') = P(E_Z')$		
$= 0.030$ (p_4)	1A	or 0.03
(c) $P(E_X' \mid F) = \frac{P(E_X')P(F \mid E_X')}{P(F)}$		
$= \frac{(0.015)(0.030)}{0.00118875}$	1M	$\frac{(0.015)p_4}{p_3}$
≈ 0.3785489		
≈ 0.3785	1A (7)	r.t. 0.3785

Solution	Marks	Remarks
8. (a) $S_A=\frac{256}{9625}\left(\frac{1}{3}t^3-\frac{47}{4}t^2+120t\right)$		
$\frac{dS_A}{dt}=\frac{256}{9625}\left(t^2-\frac{47}{2}t+120\right)$	1A	
$=\frac{128}{9625}(t-16)(2t-15)$		
$\frac{dS_A}{dt}\begin{cases}>0 & \text{when } 0\le t<\frac{15}{2}\\ =0 & \text{when } t=\frac{15}{2}\\ <0 & \text{when } \frac{15}{2}<t\le 12.5\end{cases}$	1M	
$\therefore$ A attains its top speed at $t=\frac{15}{2}$ (or 7.5)		
Top speed of $A=\frac{256}{9625}\left[\frac{1}{3}\left(\frac{15}{2}\right)^3-\frac{47}{4}\left(\frac{15}{2}\right)^2+120\left(\frac{15}{2}\right)\right]$ m/s		
≈ 10.0987 m/s	1A	*a*−1 for r.t. 10.099 *pp*−1 for missing unit
(b) $S_B=\frac{183}{50}te^{-kt}$		
$\frac{dS_B}{dt}=\frac{183}{50}e^{-kt}(1-kt)$	1A	
$\because$ $k>0$		
$\therefore$ $\frac{dS_B}{dt}\begin{cases}>0 & \text{when } 0\le t<\frac{1}{k}\\ =0 & \text{when } t=\frac{1}{k}\\ <0 & \text{when } t>\frac{1}{k}\end{cases}$	1M	for solving $\frac{dS_B}{dt}=0$ or sub. $t=\frac{15}{2}$ into $\frac{dS_B}{dt}=0$
B attains its top speed at $t=\frac{1}{k}$.		
From (a), $\frac{1}{k}=\frac{15}{2}$		
$k=\frac{2}{15}$ (or 0.1333)	1A	*a*−1 for r.t. 0.133

(c)

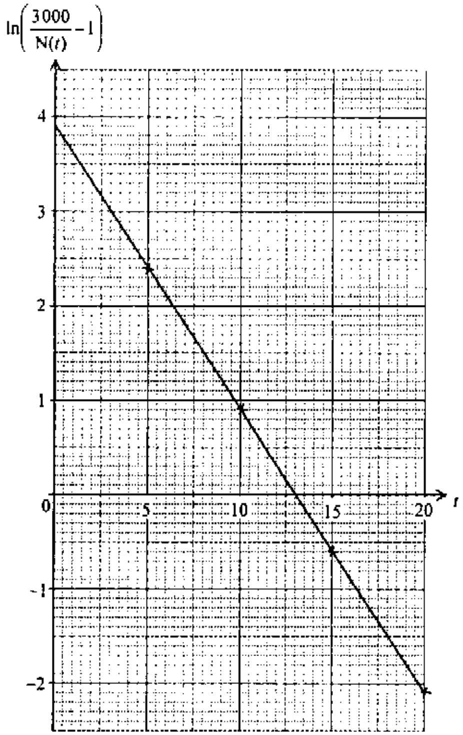
t	0	2.5	5	7.5	10	12.5
S_B	0	6.55626 (6.5563)	9.39553 (9.3955)	10.09829 (10.0983)	9.64766 (9.6477)	8.64106 (8.6411)

Solution	Marks	Remarks
	1M	correct to 4 d.p.
The distance covered by B in 12.5 seconds		
$=\int_0^{12.5} S_B dt$ m		
$\approx\frac{2.5}{2}[0+8.64106+2(6.55626+9.39553+10.09829+9.64766)]$ m	1M	
≈ 100.0457 m	1A	accept 100.0457 to 100.0699 *a*−1 for r.t. 3 d.p.

Solution	Marks	Remarks
(d) $\frac{d^2S_B}{dt^2}=\frac{183}{50}k^2e^{-kt}(t-\frac{2}{k})$ (or $\frac{183}{50}ke^{-kt}(kt-2)$)	1M	
$=\frac{122}{1875}e^{-\frac{2t}{15}}(t-15)$ (or $\frac{61}{125}e^{-\frac{2t}{15}}(\frac{2}{15}t-2)$)		
<0 for $0\le t\le 12.5$		
$\therefore$ The graph of S_B is concave downward for $0\le t\le 12.5$.		
i.e., The estimated distance covered by B in (c) is underestimated.	1M	
Hence B covers more than 100 m in 12.5 seconds.		
B finishes the race ahead of A .	1	
(e) $\int_0^{12.5}\frac{50[\ln(t+2)-\ln 2]}{t+2}dt$		
$=\int_0^{12.5}\frac{25[\ln\frac{t+2}{2}]}{\frac{t+2}{2}}dt$ (or $50\int_0^{12.5}\left(\frac{\ln(t+2)}{t+2}-\frac{\ln 2}{t+2}\right)dt$)		
$=25\left[\left(\ln\frac{t+2}{2}\right)^2\right]_0^{12.5}$ (or $50\left[\frac{(\ln(t+2))^2}{2}-\ln 2\ln(t+2)\right]_0^{12.5}$)	1A	
≈ 98.1092	1A	
$\therefore$ C covers only 98.1092 m but both A and B finish the race in 12.5 seconds. C is the last one to finish the race among the three athletes	1	
<u>Alternatively,</u>		
$\int_0^x\frac{50[\ln(t+2)-\ln 2]}{t+2}dt=25\left[\left(\ln\frac{t+2}{2}\right)^2\right]_0^x$	1A	
If $25\left(\ln\frac{x+2}{2}\right)^2=100$		
then $\ln\frac{x+2}{2}=2$		
$x\approx 12.78$	1A	
$\therefore$ C needs 12.78 seconds to finish the race but both A and B finish the race within 12.5 seconds. C is the last one to finish the race among the three athletes.	1	

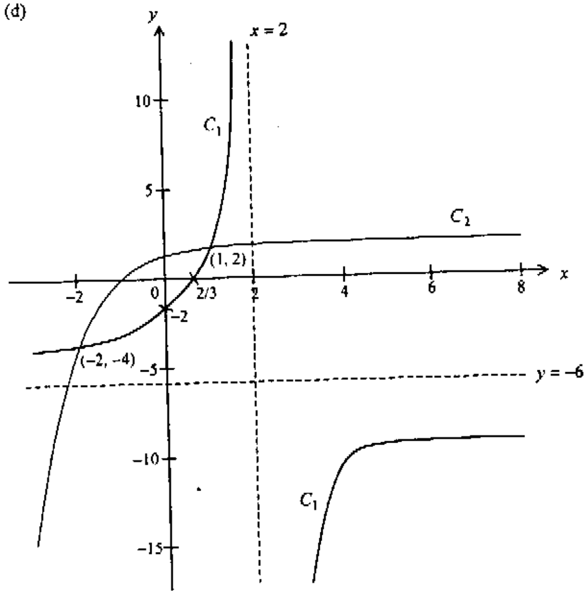
Solution	Marks	Remarks
9. (a) (i) $N(t)=\frac{3000}{1+ae^{-bt}} \Leftrightarrow \frac{3000}{N(t)}-1=ae^{-bt}$		
$\Leftrightarrow \ln\left(\frac{3000}{N(t)}-1\right)=-bt+\ln a$	1A	$pp-1$ for $-bt\ln e+\ln a$
(ii)	1A	Correct to 1 d.p.

t	5	10	15	20
$\ln\left(\frac{3000}{N(t)}-1\right)$	2.40 (2.4)	0.90 (0.9)	−0.60 (−0.6)	−2.09 (−2.1)

Solution	Marks	Remarks
	1A	the line must pass through all the 4 points
From the graph, $\ln a\approx 3.9$,		accept 3.85 – 3.95
$a\approx 49.4$	1A	accept 47.0 – 51.9
$b\approx-\frac{-2.09-2.40}{20-5}\approx 0.3$	1A	

Solution	Marks	Remarks
(b) (i) $N(t)=\frac{3000}{1+ae^{-bt}}$ (or $\frac{3000}{1+49.4e^{-0.3t}}$)		
$N'(t)=\frac{3000abe^{-bt}}{\left(1+ae^{-bt}\right)^2}$	1M	
$=\frac{3000(49.4)(0.3)e^{-0.3t}}{\left(1+49.4e^{-0.3t}\right)^2}$ (or $\frac{44460e^{-0.3t}}{\left(1+49.4e^{-0.3t}\right)^2}$)	1A	accept $a\in[47.0, 51.9]$ and $3000ab\in[42300, 46710]$
$\therefore$ $N'(t)>0$ for all t		
$N(t)$ is increasing	1	
(ii) If $N'(t)=\frac{1}{100}N(t)$		
$\frac{3000abe^{-bt}}{\left(1+ae^{-bt}\right)^2}=\frac{1}{100}\cdot\frac{3000}{1+ae^{-bt}}$	1M	$a\in[47.0, 51.9]$, $b=0.3$
$e^{-bt}=\frac{1}{a(100b-1)}$		$a\in[47.0, 51.9]$, $b=0.3$
$t=\frac{1}{0.3}\ln[a(100b-1)]$		
OR $\frac{44460e^{-0.3t}}{\left(1+49.4e^{-0.3t}\right)^2}=\frac{1}{100}\cdot\frac{3000}{1+49.4e^{-0.3t}}$ $1482e^{-0.3t}=1+49.4e^{-0.3t}$	1M	
$t\approx 24.2242$	1A	$t\in[24.0581, 24.3887]$
$\because$ $N(\frac{1}{0.3}\ln[a(100b-1)])=\frac{3000}{1+ae^{-b(\frac{1}{0.3}\ln[a(100b-1)])}}=2900$	1M	
OR $N(24.2242)=\frac{3000}{1+49.4e^{-0.3(24.2242)}}\approx 2900$	1M	
$\therefore$ The greatest number of migrants found at Mai Po is 2900.	1A	
(iii) Suppose all the migrants leave Mai Po in x days.		
Then $\int_0^x 60\sqrt{s}\,ds=2900$	1M	
$\left[40s^{\frac{3}{2}}\right]_0^x=2900$	1A	for integration (including limits)
$x\approx 17.3870$		
$\therefore$ The number of days in which we can see the migrants is $24.2242+17.3870\approx 42$	1A	r.t. 42

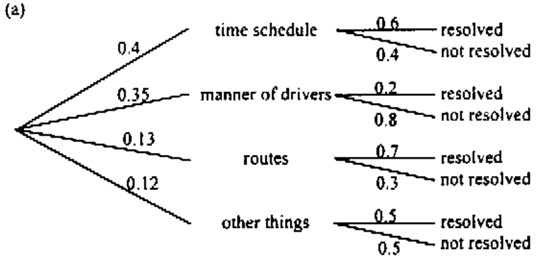
Solution	Marks	Remarks
10. Let X be the score on the questionnaire.		
(a) (i) P(classify as non-PD | PD)		
$= P(X < 75 \mid X \sim N(80, 5^2))$		
$= P(Z < \frac{75-80}{5})$	1A	
$= P(Z < -1)$		
$\approx 0.5 - 0.3413$		
$= 0.1587$	1A	
(ii) P(classify as PD | non-PD)		
$= P(X > 75 \mid X \sim N(65, 5^2))$		
$= P(Z > \frac{75-65}{5})$	1A	
$= P(Z > 2)$		
$\approx 0.5 - 0.4772$		
$= 0.0228$	1A	
(b) The probability that out of 10 PDs, not more than 2 will be misclassified		
$\approx (1-0.1587)^{10} + C_1^{10}(0.1587)(1-0.1587)^9 + C_2^{10}(0.1587)^2(1-0.1587)^8$	1M+1M	1M for 2nd or 3rd term 1M for all
≈ 0.7971	1A	
(c) Let x_0 be the required critical level of score.		
$P(X < x_0 \mid X - N(80, 5^2)) = 0.01$		
$P(Z < \frac{x_0 - 80}{5}) = 0.01$	1A	
$\frac{x_0 - 80}{5} \approx -2.3267$	1M	accept −2.325 to −2.33, for the case '$Z < \ldots$' only
$x_0 \approx 68.3665$	1A	accept 68.35 to 68.375
(d) If a teenager is classified by the sociologist, then		
P(classify as PD | non-PD)		
$= P(X > 68.3665 \mid X \sim N(65, 5^2))$	1M	accept 68.35 to 68.375
$= P(Z > 0.6733)$		accept 0.67 to 0.675
$\approx 0.5 - 0.2496$		
$= 0.2504$		accept 0.2498 to 0.2514
$\therefore$ P(misclassified) $\approx (0.01)(0.1) + (0.2504)(0.9)$	1M	for either
≈ 0.2264	1A	accept 0.2258 to 0.2273
If a teenager is classified by the criminologist, then		
P(misclassified) $\approx (0.1587)(0.1) + (0.0228)(0.9)$		
≈ 0.0364	1A	
$\because$ $0.2264 > 0.0364$		
$\therefore$ The probability of teenagers miscalssified by the sociologist is greater than that by the criminologist.	1	

Solution	Marks	Remarks
11. (a) $f'(x) = \frac{6(2-x)+(6x-4)}{(2-x)^2} = \frac{8}{(2-x)^2} > 0$ for $x \neq 2$	1	
(b) $\because$ $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{6x-4}{2-x} = \infty$ and $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{6x-4}{2-x} = -\infty$		
$\therefore$ $x = 2$ is a vertical asymptote to C_1 .	1A	
$\because$ $\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{6 - \frac{4}{x}}{\frac{2}{x} - 1} = -6$		
$\therefore$ $y = -6$ is a horizontal asymptote to C_1 .	1A	
(c) $\because$ $f(-2) = g(-2)$ and $f(1) = g(1)$		
$\therefore$ $\begin{cases} -4 = a\left(\frac{e^0 - 1}{e^{-2}}\right) + b \\ 2 = a\left(\frac{e^3 - 1}{e}\right) + b \end{cases}$		
$\begin{cases} a = \frac{6e}{e^3 - 1} \\ b = -4 \end{cases}$	1A+1A	
(d) 	1A 1A 1A	points of intersection intercepts shape and asymptotes

Solution	Marks	Remarks
(e) $g(x)=\frac{6e}{e^3-1}\left(\frac{e^{x+2}-1}{e^x}\right)-4$		
$g'(x)=\frac{6e}{e^3-1}\left(\frac{e^xe^{x+2}-(e^{x+2}-1)e^x}{e^{2x}}\right)=\frac{6e}{e^3-1}e^{-x}$	1M	must be simplified
$\therefore$ $g'(x)>0$ and hence $g(x)$ is (strictly) increasing for all values of x. For $x<-3$, $f(x)>-6$ but $g(x)<-6$. For $x>8$, $f(x)<-6$ but $g(x)>-6$. Thus C_1 and C_2 has no point of intersection	1M	The 2 method marks can be awarded only when all calculations and arguments are correct except the constant a in $g(x)$.
beyond the range $-3\le x\le 8$.	1	
(f) Area of the region bounded by C_1 and C_2		
$=\int_{-2}^{1}(g(x)-f(x))\,dx$	1M	
$=\int_{-2}^{1}\left[\frac{6e}{e^3-1}\left(\frac{e^{x+2}-1}{e^x}\right)-4-\frac{6x-4}{2-x}\right]dx$		
$=\frac{6e}{e^3-1}\int_{-2}^{1}\left(e^2-e^{-x}\right)dx-\int_{-2}^{1}4dx-\int_{-2}^{1}\left(-6+\frac{8}{2-x}\right)dx$		1A for $\int(e^2-e^{-x})dx$
$=\frac{6e}{e^3-1}\left[e^2x+e^{-x}\right]_{-2}^{1}+\left[2x\right]_{-2}^{1}+8\left[\ln(2-x)\right]_{-2}^{1}$	1A+1A	1A for $\int\frac{6x-4}{2-x}dx$
$\approx 12.94312254+6-11.09035489$		
≈ 7.8528	1A	$a-1$ for r.t. 7.853

12\. Let N be the number of complaints received on a given day and X be the number of complaints involving the time schedule.

(a)



Solution	Marks	Remarks
P(manner of drivers | not resolved)		1A for p_1, 1A for p_2
$=\frac{0.35\times0.8}{0.4\times0.4+0.35\times0.8+0.13\times0.3+0.12\times0.5}$ $(\frac{p_1}{p_2})$	1M+1A+1A	1M for $\frac{p_1}{p_2}$
≈ 0.5195	1A	$a-1$ for r.t. 0.519
(b) (i) $P(N=5)=\frac{10^5e^{-10}}{5!}$	1A	
≈ 0.0378 (p_3)	1A	
(ii) $P(N=5 \text{ and } X=3)=\frac{10^5e^{-10}}{5!}\left(C_3^5(0.4)^3(0.6)^2\right)$	1M	$p_3\left(C_3^5(0.4)^3(0.6)^2\right)$
≈ 0.0087	1A	$a-1$ for r.t. 0.009
(c) $n\ge 9$. (or $P(N=n \text{ and } X=9)=0$ for $n<9$)	1M	
$P(N=n \text{ and } X=9)=\frac{10^ne^{-10}}{n!}C_9^n(0.4)^9(0.6)^{n-9}$	1A	
(d) (i) $\sum_{k=9}^{\infty}\frac{x^k}{(k-9)!}=x^9+\frac{x^{10}}{1!}+\frac{x^{11}}{2!}+\frac{x^{12}}{3!}+\cdots$		
$=x^9(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots)$	1A	
$=x^9e^x$	1	
(ii) $P(X=9)=\sum_{n=9}^{\infty}P(N=n \text{ and } X=9)$		
$=\sum_{n=9}^{\infty}\frac{10^ne^{-10}}{n!}C_9^n(0.4)^9(0.6)^{n-9}$	1M	
$=\sum_{n=9}^{\infty}\frac{10^ne^{-10}}{n!}\cdot\frac{n!}{(n-9)!9!}(0.4)^9(0.6)^{n-9}$		
$=\frac{e^{-10}(0.4)^9}{9!(0.6)^9}\sum_{n=9}^{\infty}\frac{6^n}{(n-9)!}$	1A	
$=\frac{e^{-10}(0.4)^9}{9!(0.6)^9}6^9e^6$ (by (b)(i))		
$=\frac{4^9e^{-4}}{9!}$ (or 0.0132)	1A	$a-1$ for r.t. 0.013

Solution	Marks	Remarks
13. (a) Sample mean = 3	1A	
(b)		marks in (b) can be awarded independent of (a)

Number of medicinal herbs	Expected frequency *				Marks	Remarks
	Po(3)	B(7, 3/7)	Normal			
3		29.38				
4		22.03			1A+1A	for 29.38 and 22.03
5	10.08		11.73	(or 11.87)	1A+1A	for 10.08 and 5.04
6	5.04		5.32	(or 5.23)	1A+1A	for 11.73 and 5.32
7						
8		0			1A	

Solution	Marks	Remarks
(c) The maximum error for Po(3) is less than 1 ($\lvert 14.94-14\rvert = 0.94$) while the maximum errors for B(7, 3/7) and the normal distributions are all greater than 1.		
$\therefore$ The Poisson distribution is the best.	1	provided that entries in the Poisson column are correct
(d) (i) Let p be the probability that there is no medicinal herb in the tea,		
then $p = e^{-3}$ (≈ 0.0498)	1M	for Po(3) only
The required probability $= (p)^3(1-p)$	1M	
$= (e^{-3})^3(1-e^{-3})$		
≈ 0.0001	1A	$a-1$ for r.t. 0.0001
(ii) Let q be the probability that a cup of tea contains exactly 3 kinds of medicinal herbs,		
then $q = \dfrac{3^3 e^{-3}}{3!}$	1M	
≈ 0.22404 (or 0.2240)		
The required probability $= 1-[(1-q)^{10} + C_1^{10} q(1-q)^9]$	1M	
≈ 0.6924 (or 0.6923)	1A	$a-1$ for r.t. 0.692