1. (a) When \( x = 1 \), \( e^x = \frac{2}{2} = 4 \) 
\( y = \ln(4) \) 
\( (\text{or } y = 2 \ln 2) \) 
\( (\text{or } y = 1.3662) \) 

(b) \( e^x = \frac{x(x+1)^2}{x^2 - 1} \) 
\( xy = \ln(x(x+1)^2) \) 
\( x = \ln x(x+1) - \ln(x+1) \) 
\( \frac{dy}{dx} = \frac{3x^2 + 2x}{x^2 + 1} \) 

When \( x = 1 \), 
\( \frac{dy}{dx} + \ln 4 = 1 + 2 - 1 \) 
\( \frac{dy}{dx} = -\ln 4 \) 
\( (\text{or } 0.137) \) 

Alternatively, 
\( e^x (\frac{dy}{dx} + \ln 4) = \frac{[x(x+1)^2] - x(x+1)^2(2x)^2}{(x^2 - 1)^2} \) 
\( \frac{dy}{dx} = -\ln 4 \) 

2. (a) 
\( e^{2x} = 1 - 2x + \frac{(-2x)^2}{2} + \frac{(-2x)^3}{6} + \cdots \) 
\( = 1 - 2x + 2x^2 - 4x^3 + \cdots \) 
\( (1 + x)^2 = 1 + \frac{1}{2} \left( \frac{1}{3} + \frac{1}{2} + \frac{1}{3} \right) + \frac{1}{2} \left( \frac{1}{3} + \frac{1}{2} + \frac{1}{3} \right)^2 \) 
\( = \frac{1}{2} + \frac{3}{2} \) 
\( = 1 + x + x^2 - \frac{1}{3} \) 

The expansion is valid for \( |x| < 1 \) 

(b) Their chances of having a reaction time shorter than 1.1 seconds are equal. 
This is because (i) the median reaction times for both the boys and girls are 1.1 sec. or (ii) both the probabilities of having a reaction time less than 1.1 sec for the boy and the girl are 0.5.

4. (a) 
\( N = \int_0^1 \left( x^2 + 1 \right) dx \) 
\( = \left[ \frac{x^3}{2} + x \right]_0^1 \) 
\( = 1.333 \) 

For some constant \( c \) 
\( N = 100 \) when \( x = 1 \) 
\( 100 = 6 + 6 + c \Rightarrow c = 88 \) 
\( i.e., N = 60^2 + 60^2 + 88 \) 

(b) When \( x = 64 \), 
\( N = (64)^2 + (64)^2 + 88 \) 
\( = 1552 \)
5. Let \( X \) be the no. of passengers using Octopus in a compartment.

(a) \( P(X = 5) = C_0^5 (0.6)^5 (1 - 0.6)^0 \)
\[ = 0.200658 \]
\[ = 0.2007 \quad (p_1) \]

(b) \( E(X) = \mu = 10 \times 0.6 = 6 \)
The mean number of passengers using Octopus in a compartment is 6.

(c) The probability that the third compartment is the first one to have exactly 5 passengers using Octopus is given by:
\[ = (1 - 0.200658) (0.200658) \]
\[ = 0.1282 \]

6. (a) The number of different groups can be formed is:
\[ = 5 \times 4 \times C_{10}^5 \]
\[ = 160160 \]

(b) The possible numbers of boys are 10, 11, 12, 13, 14, 15, 16.

(c) Using (b) and the method of "try and error":

<table>
<thead>
<tr>
<th>Number of boys among the students</th>
<th>Probability of saving a time keeping group with all the time keepers being boys</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>( C_9^5 ) / ( C_{10}^5 ) = 1 / 8008</td>
</tr>
<tr>
<td>11</td>
<td>( C_{10}^5 ) / ( C_{11}^5 ) = 1 / 720</td>
</tr>
<tr>
<td>12</td>
<td>( C_{10}^5 ) / ( C_{11}^5 ) = 3 / 564</td>
</tr>
</tbody>
</table>

There are 12 boys among the students.

7. Let \( E_X \) be the event that cable \( X \) is operative,
\( E_Y \) be the event that cable \( Y \) is operative,
\( E' \) be the event that cable \( Z \) is operative, and
\( F \) be the event that \( A \) and \( B \) are not able to make contact.

(a) \( i) \quad P(E'_X \cap E'_Y) = (0.015 \times 0.030) \)
\[ = 0.00045 \quad (p_1) \]

(b) \( ii) \quad P(E_Y \cap E'_Z \cap E'_Y) = (0.015 \times 0.025 \times 0.030) \)
\[ = 0.00000125 \quad (p_2) \]

(c) \( iii) \quad P(F) = P(E_Y \cap E'_Z) - P(E_Y \cap E'_Z \cap E'_Y) \)
\[ = 0.00045 + 0.0000125 \]
\[ = 0.0011875 \quad (p_3) \]

(d) \( b) \quad P(E'_X \cap E'_Y) = P(E'_Y) \quad \frac{p_3}{p_1} \quad (p_4) \]

(e) \( c) \quad P(E_X \cap F) = \frac{P(E'_X) \cdot P(E'_Y \cap F)}{P(F)} \)
\[ = 0.0011875 \]
\[ = 0.0011875 \quad (p_5) \]

\[ = 0.3785 \quad (p_6) \]

\[ = 0.3785 \quad (p_7) \]
Solution

| 8. (a) | \( S_x = \frac{256}{9625} \left( \frac{1}{3} - \frac{47}{2} r + 120 \right) \) | 1A | \( \frac{dS_x}{dr} = \frac{256}{9625} \left( 1 - \frac{47}{2} r + 120 \right) \) | 1M |
| \( \frac{dS_x}{dr} = 128 \left( \frac{1}{3} - \frac{47}{2} (2r - 15) \right) \) | 1M |
| \( > 0 \) when \( 0 \leq r < \frac{15}{2} \) | 1A |
| \( = 0 \) when \( r = \frac{15}{2} \) | 1M |
| \( < 0 \) when \( \frac{15}{2} < r \leq 12.5 \) | 1M |
| \( A \) attains its top speed at \( r = \frac{15}{2} \) (or 7.5) |
| Top speed of \( A = \frac{256}{9625} \left( \frac{1(15)}{3} - \frac{47(15)^2}{4} + 120(\frac{15}{2}) \right) \) m/s | 1A |
| \( = 10.0987 \) m/s |
| (b) | \( S_y = 183 e^{-kt} \) | 1A |
| \( \frac{dS_y}{dr} = 183 e^{-kt} (1 - kr) \) | 1A |
| \( k > 0 \) | 1M |
| \( \frac{dS_y}{dr} = 183 e^{-kt} \) | 1M |
| \( > 0 \) when \( 0 \leq t < \frac{1}{k} \) | 1A |
| \( = 0 \) when \( t = \frac{1}{k} \) | 1M |
| \( < 0 \) when \( t > \frac{1}{k} \) | 1M |
| \( B \) attains its top speed at \( t = \frac{1}{k} \). |
| From (a), \( \frac{1}{k} = \frac{15}{2} \) (or 0.1333) |
| \( k = \frac{2}{15} \) |

### Table

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>2.5</th>
<th>5</th>
<th>7.5</th>
<th>10</th>
<th>12.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_y )</td>
<td>0</td>
<td>6.55626</td>
<td>9.39555</td>
<td>9.09829</td>
<td>9.64766</td>
<td>8.64006</td>
</tr>
</tbody>
</table>

The distance covered by \( B \) in 12.5 seconds

\[ \int_{0}^{12.5} S_y \, dt \, \text{m} \]

\[ = \frac{256}{9625} \left[ 6.55626 + 9.39555 + 9.09829 + 9.64766 + 8.64006 \right] \, \text{m} \]

\[ = 100.0457 \, \text{m} \]

### Alternatives.

\( \int_{0}^{12.5} S_y \, dt = 25 \left[ \frac{1}{2} (12.5)^2 \right] \)

\( = 25 \left[ \frac{1}{2} (12.5)^2 \right] \, \text{m} \]

\( = 12.78 \, \text{seconds} \)

\( \therefore C \) needs 12.78 seconds to finish the race but both \( A \) and \( B \) finish the race within 12.5 seconds. \( C \) is the last one to finish the race among the three athletes. | 1A
9. (a) \( N(t) = \frac{3000}{1 + ae^{-bt}} \)

\( \ln \left( \frac{3000}{N(t)} \right) = \ln(3000) - \ln N(t) = -bt \ln a \)

\[ t \quad 5 \quad 10 \quad 15 \quad 20 \]
\[ \ln \left( \frac{3000}{N(t)} \right) \quad 2.40 \quad 0.90 \quad -0.60 \quad -2.09 \]

1A Correct to 1 d.p.

(b) \( N(t) = \frac{3000}{1 + ae^{-bt}} \)

\[ N(t) = \frac{3000(1 + 49.4e^{0.3t})}{1 + 49.4e^{0.3t}} = 44460e^{-0.3t} \]

1A Accept \( \sigma \in [47.0, 51.9] \) and 

3000ab \( c \in [42300, 46710] \)

(ii) \( t = \frac{1}{a(100a - 1)} \)

1A The line must pass through all the 4 points

\[ \frac{44460e^{-0.3t}}{1 + 49.4e^{-0.3t}} = \frac{3000}{1 + 49.4e^{-0.3t}} \]

1A \( t \in [24.0581, 24.3387] \)

(iii) Suppose all the migrants leave Mai Po in \( x \) days.

Then \[ \int_{0}^{x} 600 \sqrt{16 - x^2} \, dx = 2900 \]

1A for integration (including limits)

\[ x = 17.3870 \]

1A The number of days in which we can see the migrants is \( 24.2242 + 17.3870 = 42 \)

1A \( \approx 42 \)

\[ \ln \sigma = 3.9, \]

\[ a = 49.4, \]

\[ b = -2.09 \] to 2 d.p.

1A Accept 3.85 - 3.95

1A Accept 47.0 - 51.9
Let \( X \) be the score on the questionnaire.

(a) (i) \( P(\text{classify as non-PD} \mid \text{PD}) \)
\[
P(X < 75 \mid X \sim N(80, 5^2)) = P(0 < Z < 5)
\]
\[
= P(Z < 5) - P(Z < 0)
\]
\[
= 0.5 - 0.3413
\]
\[
= 0.1587
\]

(i) \( P(\text{classify as PD} \mid \text{non-PD}) \)
\[
P(X > 75 \mid X \sim N(65, 5^2)) = P(Z > 15)
\]
\[
= P(Z > 3)
\]
\[
= 0.5 - 0.4472
\]
\[
= 0.0528
\]

(b) The probability that out of 10 PDs, not more than 2 will be misclassified
\[
= (1 - 0.1587)^{10} + \binom{10}{2} (0.1587)^2 (1 - 0.1587)^8
\]
\[
= 0.7971
\]

(c) Let \( x_0 \) be the required critical level of score.
\[
P(X < x_0 \mid X \sim N(80, 5^2)) = 0.01
\]
\[
P(Z < x_0 - 80 \mid 5) = 0.01
\]
\[
\frac{x_0 - 80}{5} = -2.3267
\]
\[
x_0 = 68.3665
\]

(d) If a teenager is classified by the sociologist, then
\[
P(\text{classify as PD} \mid \text{PD})
\]
\[
= P(X > 68.3665 \mid X \sim N(65, 5^2))
\]
\[
= P(Z > 3.7)
\]
\[
= 0.1678
\]
\[
P(\text{misclassified}) = (0.01)(0.1) + (0.2594)(0.9)
\]
\[
= 0.2264
\]

If a teenager is classified by the criminologist, then
\[
P(\text{misclassified}) = (0.1587)(0.1) + (0.0228)(0.9)
\]
\[
= 0.0364
\]

The probability of teenagers misclassified by the sociologist is greater than that by the criminologist.
12. Let \( N \) be the number of complaints received on a given day and \( X \) be the number of complaints involving the time schedule.

(a)\[ \begin{array}{c|c|c}
\text{time schedule} & 0.4 & 0.6 \\
\hline
\text{resolved} & 0.4 & 0.6 \\
\text{not resolved} & & 0.35 \\
\end{array} \]

(b)\[ \begin{array}{c|c|c}
\text{manner of drivers} & 0.2 & 0.8 \\
\hline
\text{resolved} & 0.2 & 0.8 \\
\text{not resolved} & & 0.13 \\
\end{array} \]

(c)\[ \begin{array}{c|c|c}
\text{routes} & 0.7 & 0.3 \\
\hline
\text{resolved} & 0.7 & 0.3 \\
\text{not resolved} & & 0.12 \\
\end{array} \]

(d)\[ \begin{array}{c|c|c}
\text{other things} & 0.4 & 0.6 \\
\hline
\text{resolved} & 0.4 & 0.6 \\
\text{not resolved} & & 0.5 \\
\end{array} \]

\[ P_{1}(\text{manner of drivers}) = 0.4 \times 0.4 + 0.33 \times 0.8 + 0.13 \times 0.3 + 0.12 \times 0.5 = 0.5195 \]

\[ P_{1}(\text{routes}) = 0.4 \times 0.2 + 0.33 \times 0.8 + 0.13 \times 0.7 + 0.12 \times 0.3 = 0.4067 \]

\[ P_{1}(\text{other things}) = 0.4 \times 0.4 + 0.33 \times 0.2 + 0.13 \times 0.7 + 0.12 \times 0.3 = 0.3432 \]

\[ P_{1}(\text{time schedule}) = 0.4 \times 0.1 + 0.33 \times 0.6 + 0.13 \times 0.2 + 0.12 \times 0.7 = 0.4067 \]

\[ P_{1}(\text{manner of drivers}) = 0.4 \times 0.4 + 0.33 \times 0.8 + 0.13 \times 0.3 + 0.12 \times 0.5 = 0.5195 \]

\[ P_{1}(\text{routes}) = 0.4 \times 0.2 + 0.33 \times 0.8 + 0.13 \times 0.7 + 0.12 \times 0.3 = 0.4067 \]

\[ P_{1}(\text{other things}) = 0.4 \times 0.4 + 0.33 \times 0.2 + 0.13 \times 0.7 + 0.12 \times 0.3 = 0.3432 \]
13. (a) Sample mean = 3

(b) Number of medicinal herbs | Expected Frequency *
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Po(3)</td>
<td>29.38</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>22.05</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10.08</td>
<td>11.73</td>
<td>(or 11.87)</td>
</tr>
<tr>
<td>6</td>
<td>5.04</td>
<td>5.32</td>
<td>(or 5.23)</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

(c) The maximum error for Po(3) is less than 1 (|14.94 - 14| = 0.94) while the maximum errors for Bi(7, 3/7) and the normal distributions are all greater than 1.

- The Poisson distribution is the best.

(d) (i) Let \( p \) be the probability that there is no medicinal herb in the tea, then

\[
p = e^{-3} \approx 0.0498
\]

The required probability = \((p)^3(1-p)\)

\[
= e^{-6}(1-e^{-3})
\]

\[= 0.0001
\]

- Let \( q \) be the probability that a cup of tea contains exactly 3 kinds of medicinal herbs, then

\[
q = 3^3e^{-3}
\]

\[= 0.22404 \quad \text{(or 0.2240)}
\]

The required probability = \[1 - (1-q)^9 - C_{9}^1q(1-q)^8\]

\[= 0.6924 \quad \text{(or 0.6923)}
\]

- Provided that entries in the Poisson column are correct

\[(a) + (b) \text{ can be awarded independently of (a)}
\]

\[1A + 1A \text{ for 29.38 and 22.03}
\]

\[1A + 1A \text{ for 10.08 and 5.04}
\]

\[1A + 1A \text{ for 11.73 and 5.32}
\]

\[1A
\]

\[1A
\]

\[1A
\]

\[a-1 \text{ for r.t. 0.0001}
\]

\[a-1 \text{ for r.t. 0.692}
\]