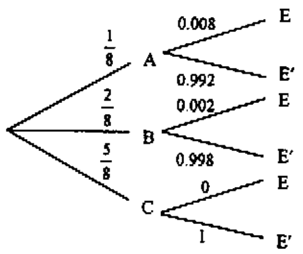
Solution	Marks	Remarks
1. $y = xe^{\frac{1}{x}}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = e^{\frac{1}{x}} + xe^{\frac{1}{x}}(-\frac{1}{x^2})$	1M+1M	1M for product rule 1M for chain rule
$= e^{\frac{1}{x}} - \frac{1}{x}e^{\frac{1}{x}}$	1A	
$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{1}{x^2}e^{\frac{1}{x}} + \frac{1}{x^2}e^{\frac{1}{x}} + \frac{1}{x^3}e^{\frac{1}{x}}$	1A	
$= \frac{1}{x^3}e^{\frac{1}{x}}$ $\therefore \quad x^4\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - y = x^4\left(\frac{1}{x^3}e^{\frac{1}{x}}\right) - xe^{\frac{1}{x}}$ $= 0$	1	accept $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{y}{x^4}$
	(5)	
2. (a) $(1+ax)^{-4} = 1 + (-4)(ax) + \frac{(-4)(-5)}{2!}(ax)^2 + \frac{(-4)(-5)(-6)}{3!}(ax)^3 + \cdots$ $= 1 - 4ax + 10a^2x^2 - 20a^3x^3 + \cdots$ $\therefore \quad 160 = -20a^3$	1M	For any 3 terms or correct coefficients
$a = -2$	1A	
$b = -4a = 8$	1A	
$c = 10a^2 = 40$	1A	
(b) The expansion is valid when $-1 < ax < 1$ (or $\lvert ax\rvert < 1$, $\lvert -ax\rvert < 1$)	1M	can be omitted
$-1 < -2x < 1$ i.e. $-\frac{1}{2} < x < \frac{1}{2}$ (or $\lvert x\rvert < \frac{1}{2}$, $-0.5 < x < 0.5$)	1A	
	(6)	

Solution	Marks	Remarks
3. (a) Median $= \frac{161+162}{2}$ cm $= 161.5$ cm	1A	ignore unit
(b) The probability that a student with a height greater than 170 cm is selected $= \frac{9}{40}$ (0.225)	1A	
(i) Probability required $= (1-\frac{9}{40})^3(\frac{9}{40})$	1M	
≈ 0.1047 (or $\frac{268119}{2560000}$)	1A	a–1 for r.t. 0.105
(ii) Probability required $= C_3^5(\frac{9}{40})^3(1-\frac{9}{40})^2$	1M	
$= 0.0684$ (or $\frac{700569}{10240000}$)	1A	a–1 for r.t. 0.068
	(6)	
4. (a) $x = \int 650e^{-0.004t}\,\mathrm{d}t$	1A	pp–1 for missing $\mathrm{d}t$
$= -162500e^{-0.004t} + c$	1A	pp–1 for missing c pp–1 for missing x
since $x = 57000$ when $t = 0$ $c = 219500$ $\therefore \quad x = 219500 - 162500e^{-0.004t}$	1M+1A	
(b) If $57000 \times 2 = 219500 - 162500e^{-0.004t}$	1M	
then $t = \frac{1}{-0.004}\ln\left(\frac{219500-114000}{162500}\right)$ ≈ 108 (or 107.9918) $\therefore$ the number of customers will be doubled in 108 days after the start of the campaign.	1A	r.t. 108
	(6)	
5. (a) (i) No. of arrangements $= 10!$ $= 3628800$	1A	
(ii) No. of arrangements $= 2!\,9!$ $= 725760$	1A+1A	1A for 9!
(b) (i) No. of arrangements $= 10!$ $= 3628800$	1A	
(ii) No. of arrangements $= 2!(9! - 8!)$ $= 645120$	1A+1A	1A for (9! – 8!)
<u>Alternatively,</u> No. of arrangements $= C_3^8 4!2!5!2!$ (or $8 \cdot 2 \cdot 8!$) $= 645120$	1A+1A	1A for $C_3^8 4!2!5!$
	(6)	

Solution	Marks	Remarks
6. Let E be the event that an ice-cream bar is contaminated.		
	1A	For the tree diagram or all parts in (a) being correct
(a) (i) $P(A)P(E'\|A) = \frac{1}{8}\times 0.992$		
$= 0.124$	1A	(p_1)
(ii) $P(E') = P(A)P(E'\|A) + P(B)P(E'\|B) + P(C)P(E'\|C)$		
$= 0.124 + \frac{2}{8}\times 0.998 + \frac{5}{8}\times 1$	1M	for $p_1 + \frac{2}{8}p_2 + \frac{5}{8}p_3$
$= 0.9985$	1A	a-1 for r.t. 0.999
(b) $P(A\|E) = \frac{P(A)P(E\|A)}{1-P(E')}$		
$= \frac{\frac{1}{8}\times 0.008}{1-0.9985}$ (or $\frac{\frac{1}{8}\times 0.8\%}{\frac{1}{8}\times 0.8\% + \frac{2}{8}\times 0.2\%}$)	1M	
≈ 0.6667	1A (6)	a-1 for r.t. 0.667

Solution	Marks	Remarks
7. (a) Under Poisson (λ), $\frac{100\lambda^3 e^{-\lambda}}{3!} \approx 19.5$ and $\frac{100\lambda^4 e^{-\lambda}}{4!} \approx 19.5$	1A	
Therefore $\frac{100\lambda^3 e^{-\lambda}}{3!} = \frac{100\lambda^4 e^{-\lambda}}{4!}$ $\lambda \approx 4$	1M	can be omitted
Since λ is an integer, $\lambda = 4$.	1A	
<u>Alternatively,</u>		
By calculating the expected frequencies under Po(λ) when $\lambda = 1, 2, 3, \ldots$,	1M	

<u>Number of "over-weight" children</u>	<u>Expected frequency</u> <u>Po(1)</u>	<u>Po(2)</u>	<u>Po(3)</u>	<u>Po(4)</u>
3	6.1	18.0	22.4	19.5
4	1.5	9.0	16.8	19.5
5	0.3	3.6	10.1	15.6

Solution	Marks	Remarks
From the table above, $\lambda = 4$.	2A	1A for just writing $\lambda = 4$
(b) If $\lambda = np$, then $p = \frac{\lambda}{n}$		
$= \frac{4}{50}$	1M	
$= 0.08$	1A (5)	

Solution	Marks	Remarks
8. (a) (i) If $\frac{5000e^{15\lambda}}{15} = \frac{5000e^{95\lambda}}{95}$	1A	
then $e^{80\lambda} = \frac{19}{3}$		
$\lambda = \frac{1}{80}\ln(\frac{19}{3})$	1M+1A	
≈ 0.0231		
(ii) $N = \frac{5000e^{\lambda t}}{t} \approx \frac{5000e^{0.0231t}}{t}$		
$\frac{dN}{dt} = 5000\left(\frac{\lambda te^{\lambda t} - e^{\lambda t}}{t^2}\right)$	1M+1A	
$= \frac{5000e^{\lambda t}(\lambda t - 1)}{t^2}$		
$\begin{cases} < 0 & \text{when} \quad 0 < t < \frac{1}{\lambda} \\ = 0 & \text{when} \quad t = \frac{1}{\lambda} \qquad (\approx 43.3410) \\ > 0 & \text{when} \quad \frac{1}{\lambda} < t < 120 \end{cases}$	1M+1A	
$\therefore$ N attains its minimum when $t \approx 43.3410$ (The number of fish decreased to the minimum in about 43 days after the spread of the disease.)	1A	r.t. 43
(b) $\int_0^{15} \frac{dW}{ds} ds$		
$= \int_0^{15} \frac{3}{50}(e^{-\frac{s}{20}} - e^{-\frac{s}{10}})ds$	1A	
$= \frac{3}{50}\left[-20e^{-\frac{s}{20}} + 10e^{-\frac{s}{10}}\right]_0^{15}$	1A	
≈ 0.1670	1A	
$\therefore$ The increase in the mean weight of fish in the first 15 days is 0.1670 kg.		
If $\int_0^{a} \frac{dW}{ds} ds = 0.5$,		
then $\frac{3}{50}\left[-20e^{-\frac{s}{20}} + 10e^{-\frac{s}{10}}\right]_0^{a} = 0.5$	1M	
$10e^{-\frac{a}{10}} - 20e^{-\frac{a}{20}} = \frac{25}{3} - 10$		
$\left(e^{-\frac{a}{20}}\right)^2 - 2\left(e^{-\frac{a}{20}}\right) + \frac{1}{6} = 0$	1M	
$e^{-\frac{a}{20}} \approx 0.0871$ or 1.9129	1A	
$a \approx 48.8073$ or -12.9721 (rej.)	1A	
$\therefore$ It takes about 49 days for the mean weight of the fish to increase 0.5 kg from the *Recovery Day*.		

Solution	Marks	Remarks
9. (a) $I = \int_{0.5}^{2.5} e^{-x} dx$		
$= \left[-e^{-x}\right]_{0.5}^{2.5}$	1A	
$= e^{-0.5} - e^{-2.5}$	1A	
$\approx 0.5244 \qquad (0.524446)$		
(b) $y = ae^{-x} + bxe^{-x}$		
$\because$ y-intercept is -3		
$\therefore$ $a = -3$	1A	
$y' = -ae^{-x} + be^{-x} - bxe^{-x}$	1A	neglecting the value of a
$= (-a + b - bx)e^{-x}$		
$\because$ y attains its maximum when $x = \frac{3}{2}$		
$\therefore$ $-a + b - \frac{3}{2}b = 0$	1M	
$3 - \frac{1}{2}b = 0$		
$b = 6$	1A	
Hence $y = -3e^{-x} + 6xe^{-x}$		
(c) If $y = 0$, $3e^{-x}(2x - 1) = 0$		
$x = \frac{1}{2}$	1A	
$\therefore$ The x-intercept of the curve is $\frac{1}{2}$.		
$y' = 9e^{-x} - 6xe^{-x}$		
$y'' = -9e^{-x} - 6e^{-x} + 6xe^{-x}$		
$= -15e^{-x} + 6xe^{-x}$	1M	
$= 3(2x - 5)e^{-x}$		
$\therefore$ $y'' \begin{cases} < 0 & \text{if} \quad 0 \le x < \frac{5}{2} \\ = 0 & \text{if} \quad x = \frac{5}{2} \\ > 0 & \text{if} \quad x > \frac{5}{2} \end{cases}$	1M	
The point of inflection is $(\frac{5}{2}, 12e^{\frac{-5}{2}})$ [or $(\frac{5}{2}, 0.9850)$]	1A	

(d) (i)

x	0.5	1	1.5	2	2.5
xe^{-x}	0.303265	0.367879	0.334695	0.270671	0.205212

Solution	Marks	Remarks
$J_0 = \frac{0.5}{2}[0.303265 + 0.205212 + 2(0.367879 + 0.334695 + 0.270671)]$	1M+1A	
$\approx 0.6137 \qquad (0.613742)$		
$A_0 \approx -3 \times 0.524446 + 6 \times 0.613742$		
$\approx 2.1091 \qquad (2.109114)$	1A	
(ii) The argument is not correct because the trapezoidal rule was used to approximate the value of J only. The convexity of the function xe^{-x} should be considered instead of the function $-3e^{-x} + 6xe^{-x}$.	1A+1	1A for either reason 1 for both

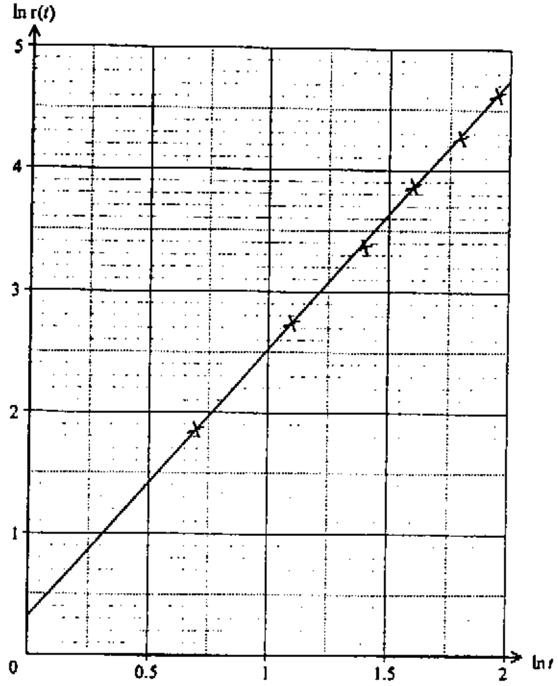
Solution	Marks	Remarks
10. (a) (i) (I) : $\ln r(t) = \ln\alpha + \beta\ln t$	1A	
(II) : $\ln r(t) = \ln\gamma + \lambda t$	1A	

(ii)

t	2	3	4	5	6	7
$r(t)$	6.4	15.7	29.5	48.3	72.2	101.2
$\ln t$	0.69	1.10	1.39	1.61	1.79	1.95
$\ln r(t)$	1.86	2.75	3.38	3.88	4.28	4.62

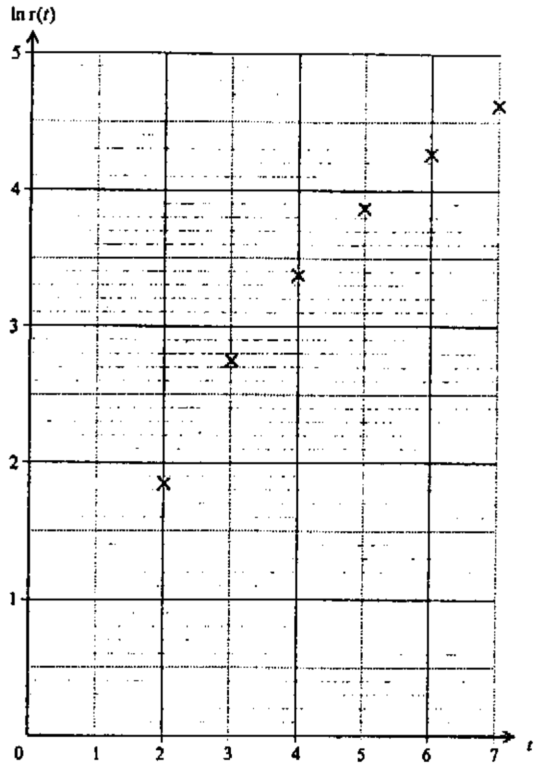
Marks	Remarks
1A	Correct to 1 d.p.
1A	Correct to 1 d.p.

(I)



Marks	Remarks
1A	for any 2 points being correct
1A	for all the 6 points being correct

Solution	Marks	Remarks
(II)		



Marks	Remarks
1A	for any 2 points being correct
1A	for all the 6 points being correct

Solution	Marks	Remarks
From the graphs, equation (I) would be a better model and		
$\ln\alpha \approx 0.3$		Accept 0.3 - 0.4
$\alpha = e^{0.3} \approx 1.3$	1A	Accept 1.3 - 1.5
$\beta \approx \frac{4.62-1.86}{1.95-0.69} \approx 2.2$	1A	Accept 2.0 – 2.4
(b) $\int_0^{14} \alpha t^\beta \mathrm{d}t$ where $\alpha \approx 1.3$, $\beta \approx 2.2$	1M+1A	Accept $\alpha \in [1.3, 1.5]$ $\beta \in [2.0, 2.4]$
$= \frac{\alpha}{\beta+1}\left[t^{\beta+1}\right]_0^{14}$ $\quad \left(\approx \frac{1.3}{3.2}\left[t^{3.2}\right]_0^{14}\right)$		
≈ 1889		
∴ 1889 hundred of trees would be destroyed in the first 14 days.	1A	Accept 1498 - 3015
Consider $\int_0^k \alpha t^\beta \mathrm{d}t = 1889 \times 2$		
$\frac{1.3}{3.2}\left[t^{3.2}\right]_0^k = 3778$	1M	
$k^{3.2} \approx 9299.69$		
$k \approx e^{\frac{\ln 9299.69}{3.2}} \approx 17.3839$		
∴ The total number of trees destroyed will be doubled in 4 days more.	1A	

Solution	Marks	Remarks
11. (a) Let X be the no. of printing mistakes on P.23, then $X \sim \text{Po}(0.2)$.		
$P(X=0) = e^{-0.2}$ ≈ 0.8187	1M+1A	
(b) (i) Let p be the probability that there are printing mistakes on a page, then $p = 1-e^{-0.2}$	1M	
Hence $N \sim$ Geometric (p) and	1M	
$P(N \le 3) = P(N=1) + P(N=2) + P(N=3)$ $= p + p(1-p) + p(1-p)^2$	1M+1A	
$= 1-(1-p)^3$ $= 1-e^{-0.6}$ ≈ 0.4512	1A	
(ii) Mean of $N = \dfrac{1}{p} = \dfrac{1}{1-e^{-0.2}} \approx 5.5167$	1A	
Variance of $N = \dfrac{1-p}{p^2} = \dfrac{e^{-0.2}}{(1-e^{-0.2})^2} \approx 24.9168$	1A	
(c) $M \sim$ Binomial $(200, p)$ where $p = 1-e^{-0.2}$.		
Mean of $M = np = 200(1-e^{-0.2}) \approx 36.2538$	1A	
Variance of $M = np(1-p) = 200e^{-0.2}(1-e^{-0.2}) \approx 29.6821$	1A	
(d) (i) $Y \sim$ Binomial $(40, \dfrac{1}{200})$.	1A+1A	
(ii) $P(Y=0) = \left(1-\dfrac{1}{200}\right)^{40} \approx 0.8183$	1M+1A	

Solution	Marks	Remarks
12. (a) Sample mean ≈ 1.2921	1A	
Sample variance ≈ 1.2924	1A	
Since the sample mean is approximately equal to the sample variance, the results do not point to any objections to the use of a Poisson model.	1	or objection as sample mean and sample variance are not equal
(b)		

Number of cars committed speeding	Observed Frequency (f_O)	Expected Frequency (f_E)*	Absolute Discrepancy $\lvert f_O - f_E \rvert$ *	Marks	Remarks
0	56	55.49	0.51	1A	For any entry being correct in the f_E column
1	71	71.70	0.70		
2	46	46.32	0.32	1A	For any entry being correct in the $\lvert f_O - f_E \rvert$ column
3	20	19.95	0.05		
4	7	6.44	0.56	1A	For all entries being correct
5	2	1.67	0.33		

Solution	Marks	Remarks
As the maximum absolute discrepancy is 0.70 which is less than 1, the Poisson model is acceptable.	1	accept contrary conclusion due to wrong entries in the table
(c) (i) P(2 cars speeding and both are private cars) $= (0.4)^2$ $= 0.16$	1A	
(ii) P(3 cars speeding and 2 of them are private cars) $= C_2^3 (0.4)^2 (1-0.4)$	1A	
$= 0.288$	1A	
(d) Let X be the number of private cars speeding and Y be the total number of cars speeding in an hour.		
(i) $P(X=2 \text{ and } Y=2)$ $= P(X=2 \mid Y=2)P(Y=2)$	1M	
$\approx 0.16 \times 0.229301$ $\approx 0.0367 \quad (0.036688)$	1A	
(ii) $P(X=2 \text{ and } Y=3)$ $= P(X=2 \mid Y=3)P(Y=3)$ $\approx 0.288 \times 0.098758$ $\approx 0.0284 \quad (0.028442)$	1A	accept 0.0285
(iii) $P(X=2 \mid Y<4)$ $= \dfrac{P(X=2 \text{ and } Y<4)}{P(Y<4)}$ $= \dfrac{P(X=2 \text{ and } Y=2) + P(X=2 \text{ and } Y=3)}{P(Y<4)}$	1M	
$\approx \dfrac{0.036688+0.028442}{0.957691}$ ≈ 0.0680	1A	accept 0.0678 – 0.0681

Solution	Marks	Remarks
13. Let X, Y be the weights of the randomly selected boxes in parts 1 and 2 of a test respectively.		
(a) $P(X < 490 \text{ or } X > 510)$		
$= 1 - P(\frac{490-500}{5} \le Z \le \frac{510-500}{5})$	1A	deduct 1 mark once for the whole question for any wrong inequality sign
$= 1 - P(-2 \le Z \le 2)$		
$\approx 1 - 2 \times 0.4772$		
≈ 0.0456	1A	
(b) $P(490 \le X < 492) + P(508 < X \le 510)$	1A	
$= P(\frac{490-500}{5} \le Z < \frac{492-500}{5}) + P(\frac{508-500}{5} < Z \le \frac{510-500}{5})$	1A	
$= P(-2 \le Z < -1.6) + P(1.6 < Z \le 2)$		
$\approx (0.4772 - 0.4452) \times 2$		
≈ 0.0640	1A	
Alternatively, $P(X < 492) + P(X > 508) - P(\text{a black signal is generated in the first part})$	1A	
$= P(Z < \frac{492-500}{5}) + P(Z > \frac{508-500}{5}) - 0.0456$	1A	
$\approx 0.0548 + 0.0548 - 0.0456$		
≈ 0.0640	1A	
(c) P(black)		
= P(black in part 1) + P(black in part 2)		
$\approx 0.0456 + 0.0640 \times 0.0456$	1M+1M	
≈ 0.0485	1A	
(d) $P(508 < X \le 510 \text{ and } 508 < Y \le 510 \mid 490 \le X < 492 \text{ or } 508 < X \le 510)$		
$= \frac{P(508 < X \le 510)\,P(508 < Y \le 510)}{P(490 \le X < 492) + P(508 < X \le 510)}$		
$\approx \frac{0.0320 \times 0.0320}{0.0320 + 0.0320}$	1M+1M	
≈ 0.0160	1A	
(e) $P(\text{red} \mid \text{part 2})$		
$= P(508 < X \le 510 \text{ and } 508 < Y \le 510 \mid 490 \le X < 492 \text{ or } 508 < X \le 510)$		
$+ P(490 \le X < 492 \text{ and } 490 \le X < 492 \mid 490 \le X < 492 \text{ or } 508 < X \le 510)$		
$\approx 2 \times 0.0160$	1M	
≈ 0.0320	1A	
(f) $P(\text{red}) = P(\text{red} \mid \text{part 2})\,P(\text{part 2})$		
$\approx 0.0320 \times 0.0640$	1M	
≈ 0.0020	1A	
Alternatively, $P(\text{red}) = P(508 < X \le 510 \text{ and } 508 < Y \le 510)$		
$+ P(490 \le X < 492 \text{ and } 490 \le Y < 492)$		
$\approx 0.0320^2 \times 2$	1M	
≈ 0.0020	1A	