<table>
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<th>Solution</th>
<th>Marks</th>
<th>Remarks</th>
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<tbody>
<tr>
<td>(a) ((1 + ax)^2 = 1 + \frac{1}{3} ax + \frac{1}{3} (\frac{1}{3} - D) (ax)^2 + \ldots)</td>
<td>1A+1A</td>
<td>1A for the 1st &amp; 2nd term, 1A for the 3rd term</td>
</tr>
<tr>
<td>(\frac{e^4}{9} = -1)</td>
<td>1A</td>
<td>1A</td>
</tr>
<tr>
<td>(a = 3) or (-3)</td>
<td>1A</td>
<td>(4)</td>
</tr>
</tbody>
</table>

2. (a) Range = \((26 - 18)\, ^\circ\text{C} = 8\, ^\circ\text{C}\) | 1A |

(b) (i) Median = 21.5 \(^{\circ}\text{C}\) = \([\frac{2}{5} (21.5) + 32]^{\circ}\text{F}\) = 70.7 \(^{\circ}\text{F}\) | 1A |

Interquartile range = \([\frac{9}{5}(22.5) + 32] - [\frac{9}{5}(20) + 32]\) \(^{\circ}\text{F}\) = 4.5 \(^{\circ}\text{F}\) | 1A |

(ii) Mean = \((\frac{9}{5}(22) + 32)\, ^{\circ}\text{F} = 71.6\, ^{\circ}\text{F}\) | 1A |

Standard deviation = \(\frac{9}{5} \times 2\, ^{\circ}\text{F} = 3.6\, ^{\circ}\text{F}\) | 1A |

(6) |

3. | 1A | for the 2 vertical asymptotes |

| 1A+1A+1A | for each part of the curves |

| 1A | for the local minimum |

(5) |
4. (a) \( V(t) = \int 2000r - 15dr \)  
\[ = 100t^2 - 30000 + c \]  
\[ V(0) = 20000, \quad c = 20000 \]  
Hence \( V(t) = 100t^2 - 30000 + 20000 \) for \( 0 \leq t \leq 1 \).

(b) \[ V(t) = 0 \]  
\[ 100t^2 - 30000 + 20000 = 0 \]  
\[ t^2 - 300 + 200 = 0 \]  
\[ (t - 20)(t - 10) = 0 \]  
\[ t = 10 \text{ or } 20 \) (rejected)  
\[ t = 10 \]

(c) \( V(5) - V(0) \)  
\[ = 100(5)^2 - 30000 + 20000 - 20000 \]  
\[ = 12500 \]  
\[ \int_0^5 2000 - 15dr \]  
\[ = 100r^2 - 30000r \]  
\[ = 12500 \]  
\( \) The total depreciation in the first 5 years is $12500.

5. (a) Number of ways in which the 10 students can take the seats  
\[ = \frac{10!}{2!4!4!} \]  
\[ = 3150 \]

(b) Number of ways in which the 10 students can take the seats with the 2 students from school A are next to each other  
\[ = \frac{8!}{4!4!} \]  
\[ = 864 \]  
The probability that the 2 students from school A are next to each other  
\[ = \frac{864}{3150} \]  
\[ = 0.28 \]

Alternatively,  
The probability that the 2 students from school A are next to each other  
\[ = 2 \cdot \frac{1}{10} \cdot \frac{1}{9} \cdot \frac{1}{8} \cdot \frac{1}{7} \]  
\[ = \frac{1}{720} \]  
\[ = 0.0014 \]

6. (a) Let \( X \) be the number of cars passing through the auto-toll in a minute.  
\( X \sim \text{Po}(5) \)  
\[ P(X > 5) = \frac{1}{10!} \sum_{r=6}^{10} \frac{5^r}{r!} \]  
\[ = 0.3840 \]

(b) Out of the next 4 minutes, let \( Y \) be the number of minutes in which more than 5 cars will pass through the auto-toll, then \( Y \sim B(4, 0.3840) \)  
\[ P(Y = 3) = C_4(0.3840)^3(1-0.3840)^1 \]  
\[ = 0.1395 \]  
(0.1396)

7. Let \( A_1 \) be the event that the original motor breaks down,  
\( A_2 \) be the event that the backup motor breaks down and  
\( W \) be the event that the machine is working.

(a) \( P(A_1, A_2) = 0.15 \times 0.24 = 0.036 \)

(b) \( P(W) = 1 - P(A_1, A_2) = 1 - 0.036 = 0.964 \)

Alternatively,  
\[ P(W) = P(A_1) + P(A_2) - P(A_1, A_2) \]

\[ = 0.95 \]

The probability that the machine is operated by the original motor  
\[ = \frac{P(A_1)}{P(W)} \]

\[ = 0.95 \]

\[ = 0.9564 \]

\[ = 0.8917 \]

\( a - 1 \) for \( t = 0.982 \)

(c) The prob that the 1st break down of the machine occurs on the 10th day  
\[ = 0.036(1-0.036)^{10-1} \]

\[ = 0.0259 \]

\( a - 1 \) for \( t = 0.026 \)
### Solution

<table>
<thead>
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<tbody>
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<tr>
<td>1A</td>
<td></td>
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</table>

**N(0) = 16**

\[
\begin{align*}
0 &= 40 \quad (1) \\
\frac{1}{4} &= 16 \\
b &= 1.5 \\
N(7a) &= 17.4 \\
\frac{40}{1 + 1.5e^{-0.7a}} &= 17.4 \\
\exp(x) &= \frac{1}{1 + 1.5e^{0.7(174 - 1)}} \\
t &= \frac{1}{1 + 1.5e^{0.7(174 - 1)}} \\
&= 0.02 \\
\end{align*}
\]

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**N(t) = \frac{40}{1 + be^{-rt}}**

\[
\begin{align*}
N'(t) &= \frac{-40(15e^{0.7t})(1 + be^{-0.7t})}{(1 + be^{-0.7t})^2} \\
&= \frac{4000e^{0.7t}}{(1 + be^{-0.7t})^2} \\
&> 0 \\
\end{align*}
\]

\[N(t)\text{ is increasing.}\]

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<td></td>
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\[\lim_{t \to \infty} e^{-rt} = 0\]

\[N_\infty = \lim_{t \to \infty} N(t) = \frac{40}{1 + be^{-rt}} = 40\]

(i) \[N'(t) = \frac{4000e^{0.7t}(15e^{0.7t} - 12e^{-0.7t} - 2)(1 + 1.5e^{0.7t})(15e^{0.7t} - 0.02)e^{0.7t}}{(1 + 1.5e^{0.7t})^2}\]

\[N'(t) = \frac{0.92e^{0.7t}(3e^{0.7t} - 2)}{(1 + 1.5e^{0.7t})^2}\]

(ii) From (i), \[N'(t) = \begin{cases} 
0 & \text{when } t < t_b \\
> 0 & \text{when } t = t_b \\
< 0 & \text{when } t > t_b 
\end{cases}\]

where \[t_b = \frac{\ln 2}{0.02} \approx 20.273\]

\[N'(20) = 0.19999996 \quad \text{and} \quad N'(21) = 0.19998996\]

\[N(20) = 0.19999996 \quad \text{and} \quad N(21) = 0.19998996\]

\[\text{The company should start to advertise on the 20th day after the first week.}\]
10. (a) \( y = x^x \)
   \[ \ln y = x \ln x \]
   \[ \frac{1}{y} \frac{dy}{dx} = 1 + \ln x \]
   \[ \frac{dy}{dx} = x^{x-1}(1 + \ln x) \]

(b) \[ \frac{d^2y}{dx^2} = x^x \left( (1 + \ln x) + (1 + \ln x) \frac{dy}{dx} \right) \]
   \[ = x^x \cdot (1 + \ln x)^2 (1 + \ln x) \]
   \[ = x^{x-2} (1 + \ln x)^2 \]
   \[ > 0 \quad \text{for} \quad 1 \leq x \leq 2 \]
   \( y \) is concave upward (or convex) for \( 1 \leq x \leq 2 \).
   \( \therefore \ J \) would be overestimated if the trapezoidal rule is used to estimate \( J \).

(c) \[ J = \int_1^2 x^x (1 + \ln x) \, dx \]
   \[ = \left[ x^x \right]_1^2 \quad \text{by (a)} \]
   \[ = 3 \]

(d) (i) | \( x \) | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>( y^x \ln y )</td>
<td>0.22691</td>
<td>0.53895</td>
<td>0.99700</td>
<td>1.60321</td>
<td>2.77259</td>
<td></td>
</tr>
</tbody>
</table>

| \( J_0 = \frac{0.22691 + 2.77259 + 2(0.53895 + 0.99700 + 1.60321)}{6} \) |
|---|---|
| 1 |

\( = 0.9685 \)

(ii) \[ y = x^x \ln x \]
   is concave upward (or convex) for \( 1 \leq x \leq 2 \).
   \( \therefore \ J_0 \) is an overestimate of \( J \).

(iii) The estimation can be improved by increasing the number of sub-intervals.

(iv) \( J_0 \) is an underestimate of \( J \) because the value 3 for \( J = J \) is exact and \( J_0 \) is an overestimate of \( J \).
11. (a) Let \( X \) be the number of FICs per day, then \( X \sim Poisson(\lambda) \)
\[
P(X = n) = \frac{e^{-\lambda} \lambda^n}{n!}
\]
\[
\lambda = 0.0013
\]

(b) Let \( Y \) be the number of FICs which are related to house fires in 5 FICs, then \( Y \sim Binomial(n = 5, p) \)
\[
P(Y = k) = \binom{5}{k} p^k (1-p)^{5-k}
\]
\[
p = 0.9135
\]

(c) Let \( U \) and \( I \) be the events of "a FIC is related to a house fire" and "a FIC is large", let \( A \) be the amount of FIC.

(i) \( P(U | I) = P(A > 20000) \)
\[
P[A > 20000] = P[Z > \frac{20000 - 50000}{100000}]
\]
\[
P[Z > 2] = 0.0228
\]

(ii) \( P(U | I, \overline{I}) = P(A > 20000) \)
\[
P[Z > 2.5] = 0.0062
\]

(iii) \( P(U | I, \overline{I}) = P(A > 20000) \)
\[
P[Z > 2] = 0.0228
\]
\[
P[Z > 2.5] = 0.0062
\]
\[
P[Z > 3] = 0.0814
\]

12. (a) & (b) Note: Under Binomial distribution, the expected mean \( \mu = 60 \times C^6_1 (0.4)^1 (0.6)^5 \)

<table>
<thead>
<tr>
<th>Merit Points</th>
<th>Observed Frequency</th>
<th>Binomial Frequency</th>
<th>Normal Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>4.67</td>
<td>4.00</td>
</tr>
<tr>
<td>1</td>
<td>14</td>
<td>15.55</td>
<td>14.50</td>
</tr>
<tr>
<td>2</td>
<td>23</td>
<td>20.74</td>
<td>22.98</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>13.82</td>
<td>14.50</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4.61</td>
<td>3.64</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0.61</td>
<td>0.37</td>
</tr>
</tbody>
</table>

* Correct to 2 decimal places.

(b) \( X \sim N(\mu, \sigma^2) \)
\[
P(X < 9000) = \frac{4.00}{60}
\]
\[
P(Z < \frac{9000 - \mu}{\sigma}) = \frac{4.00}{60} = 0.0667
\]
\[
9000 - \mu = 1.50
\]
\[
\sigma = \frac{1.50}{0.0667}
\]

(c) \( N(15500, 3000^2) \) is used to model the sales volumes.

(i) The probability that a salesman will get a warning
\[
= P(V < 10) + 3P(V = 10) + 3P(V = 20) + P(V = 20)
\]
\[
= \left( 0.40 \right) + 3 \left( 0.40 \right) \left( 0.30 \right) + 3 \left( 0.40 \right) \left( 0.10 \right) + \left( 0.10 \right) = 0.203
\]

(ii) The probability that a salesman got no merit points in at least 2 of the previous 3 months
\[
= 3P(V = 0) P(V = 1)
\]
\[
= 0.0036
\]

\[
= 0.0426
\]

The number of salesmen who are expected to get no merit points in at least 2 of the previous 3 months
\[
= 0.0426
\]

For the 3 columns 1A for any one being correct
1A for the remaining two

For the 4th column 1A for any one being correct
1A for the remaining two
13. Let \( \lambda \) cm be the length of the from portion of Mr. Wong’s necklace.

(a) 
\[
\begin{align*}
P(44 < \lambda < 45) &= P\left( \frac{44 - 44.6}{0.12} < Z < \frac{45 - 44.6}{0.12} \right) \\&= P(-0.5 < Z < 0.3333) \\&= 0.1915 + 0.1293 \\&= 0.3208 \\
\end{align*}
\]
IM (or 0.3211) (or 0.3212)

(b) Let \( X \) be the number of trials that Mr. Wong gets the first perfect tying, then \( X \sim \text{Geometric}(p) \), where 
\[
E(X) = \frac{1}{p} \\
\]
IM \( \approx 3.1172 \) (or 3.1046)

(c) \( (\text{not more than 3 trials}) \)
\[
\begin{align*}
P(X) &= P(1 \text{ trial}) + P(2 \text{ trials}) + P(3 \text{ trials}) \\&= p + p(1-p) + p(1-p)^2 \\&= 0.6867 \\
\end{align*}
\]
IM or \( 1 - (1-p)^3 \)

(d) Let \( X \) be the event that Mr. Wong has to go to work by taxi.

(i) 
\[
P(7) = 1 - 0.6867 = 0.3133 \\
\]
IM \( \approx 0.3118 \) (or 0.3115)

(ii) \( P(X \leq 7) \)
\[
\begin{align*}
P(X \leq 7) &= 1 - (1-p)^7 \\&= 0.2179 \\
\end{align*}
\]
IM \( \approx 0.2184 \)

(iii) \( \text{Probability required} \)
\[
\begin{align*}
&= 0.3139^2 \times 0.6867^8 \\
&= 0.1091 \\
\end{align*}
\]
IM=1A (or 0.1090)