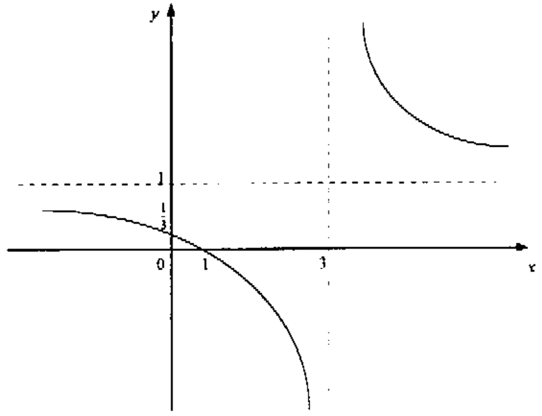
Solution	Marks	Remarks
1. (a) Mean = 59.4	1A	
Mode = 74	1A	
Interquartile range = median of upper half − median of lower half = 72 − 50 = 22	1A	
(b) If 71 is replaced by 11, the mean and interquartile range will be changed.	1A	
New mean = 57.4	1A	
New interquartile range = median of upper half − median of lower half = 72 − 49 = 23	1A	
Alternate methods for finding interquartile ranges:	1A + 1A	

	Interquartile range	Old value	New value
i	$\frac{3}{4}\times 30$ -th term − $\frac{1}{4}\times 30$ -th term	72 − 49.5 = 22.5	72 − 48 5 = 23.5
ii	$\frac{3}{4}(30+1)$ -th term − $\frac{1}{4}(30+1)$ -th term	72 − 49.75 = 22.25	72 − 48.75 = 23.25
iii	$\frac{1}{4}(30\times 3+2)$ -th term − $\frac{1}{4}(30+2)$ -th term	72 − 50 = 22	72 − 49 = 23
iv	$\frac{1}{4}(29\times 3+4)$ -th term − $\frac{1}{4}(29+4)$ -th term	71.75−50.25 = 21.5	71 − 49.25 = 21.75

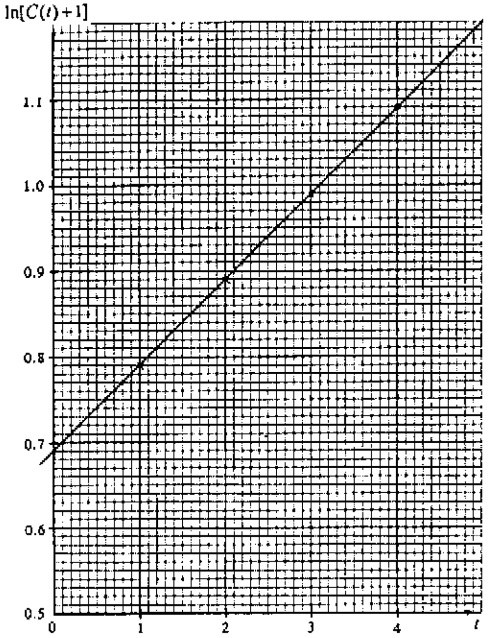
Solution	Marks	Remarks
	(6)	
2. $\frac{dy}{dx} = \frac{1-\ln x}{x^2}$	1M+1A	1M for quotient rule
$\int\left(\frac{1}{x^2}-\frac{\ln x}{x^2}\right)dx = \frac{\ln x}{x}$ $(+c_1)$	1M	For applying anti-differentiation
$\int\frac{\ln x}{x^2}dx = \int\frac{1}{x^2}dx - \frac{\ln x}{x}$ $(+c_1)$	1A	pp-1 for missing dx more than once
$= -\frac{1+\ln x}{x}+c$ (or $-\frac{1}{x}-\frac{\ln x}{x}+c$)	1A	No marks for missing c
	(5)	

Solution	Marks	Remarks
3. ∵ $y=\frac{x-1}{x-3}=1+\frac{2}{x-3}$ ∴ $x=3$ is the vertical asymptote and $y=1$ is the horizontal asymptote.		
When $x=0$, $y=\frac{1}{3}$. When $y=0$, $x=1$.		
	1A+1A 1A+1A 1A+1A	For the asymptotes For the intercepts For the two parts of the curve
	(6)	
4. (a) Area of regions I & III = $\int_0^1\sqrt{x}dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^1$ $\left(=\frac{2}{3}\right)$	1A	Or 0.6667
Area of region III = $\int_0^1 x^3dx = \left[\frac{1}{4}x^4\right]_0^1 = \frac{1}{4}$	1A	Or 0.25
Area of region II = $1-\frac{2}{3} = \frac{1}{3}$	1A	Or 0.3333
Area of region I = $\frac{2}{3}-\frac{1}{4} = \frac{5}{12}$	1A	Or 0.4167
(b) Probability of scoring 40 points= $2\times\frac{5}{12}\times\frac{1}{4}+(\frac{1}{3})^2$	1M+1M	1M for $2\times\frac{5}{12}\times\frac{1}{4}+p$ 1M for $p=(\frac{1}{3})^2$
$=\frac{23}{72}$ (or 0.3194)	1A	
	(7)	

Solution	Marks	Remarks
5. (a) $\because \quad \frac{dN}{d\theta} = -[\ln(\theta+49)]^2 - \frac{2(\theta+440)\ln(\theta+49)}{\theta+49}$	1M+1A+1A	1M for product rule 1A for diff. of log.
$= -\ln(\theta+49)\left[\ln(\theta+49)+\frac{2(\theta+440)}{\theta+49}\right]$		
$\therefore \quad \frac{dN}{dt} = \frac{dN}{d\theta}\cdot\frac{d\theta}{dt}$		
$= -\ln(\theta+49)\left[\ln(\theta+49)+\frac{2(\theta+440)}{\theta+49}\right]\frac{d\theta}{dt}$	1A	
(b) $\theta=-40, \quad \frac{d\theta}{dt}=-0.5$		
$\frac{dN}{dt} = -\ln(-40+49)\left[\ln(-40+49)+\frac{2(-40+440)}{-40+49}\right](-0.5)$	1M	
≈ 100	1A	
$\therefore$ The rate of increase of the number of tourists is 100 per hour.		
	(6)	
6. (a) The probability that a lot will be accepted		
$= (0.5)[(0.99)^2+(0.96)^2]$	1M+1M	1M for $(0.99)^2+(0.96)^2$ 1M for $0.5p$
$\approx 0.9509 \quad$ (or 0.95085)	1A	
(b) The probability that a lot came from supplier A		
$= \frac{(0.5)(0.96)^2}{0.95085}$	1A+1M	1A for the numerator 1M for the denominator
≈ 0.4846	1A	
	(6)	
7. Her conclusion is not justified because (i) families with no children were not counted. (ii) families with more than one child might be counted more than once; (iii) there might be children in the village that were not in the school such as (1) being absent from school, (2) studying elsewhere, and (3) being not in the age of receiving primary education; (iv) there might be pupils in the school who came from elsewhere.		
Marking scheme		
Saying that the conclusion is not justified with one correct reason.	2A	
Any second correct reason.	1A	
Any third correct reason.	1A	
	(4)	

Solution	Marks	Remarks
8. (a) (i) Coefficient of x^5 in the expansion of $(1+x+x^2+x^3+x^4+x^5)^2 = 6$	1A	
(ii) P (sum = 5) $= \frac{6}{6^2}$	1M+1A	1A for 6^2
$= \frac{1}{6} \quad$ (or 0.1667)	1A	
(b) (i) $(1-x^6)^4 = 1-4x^6+6x^{12}-4x^{18}+x^{24}$	1M+1A	1M for the coefficients
(ii) Coefficient of x^r in the expansion of $(1-x)^{-4}$		
$= \frac{(-4)(-5)\ldots(-4-r+1)}{r!}(-1)^r$	1A	
$= \frac{(r+1)(r+2)(r+3)}{6}$	1A	
(iii) Coefficient of x^8 in the expansion of $\left(\frac{1-x^6}{1-x}\right)^4$		
= Coefficient of x^8 in the expansion of $(1-x^6)^4(1-x)^{-4}$		
$= \frac{9\times10\times11}{6}+(-4)\frac{3\times4\times5}{6}$	1M+1M	
$= 125$	1A	
(c) $\because \quad \frac{1-x^6}{1-x} = 1+x+x^2+x^3+x^4+x^5$	1A	
$\therefore$ Coefficient of x^8 in the expansion of $(1+x+x^2+x^3+x^4+x^5)^4$		
= Coefficient of x^8 in the expansion of $\left(\frac{1-x^6}{1-x}\right)^4$		
$= 125$		
P(Sum = 8) $= \frac{125}{6^4}$	1M+1A	1A for 6^4
$= \frac{125}{1296} \quad$ (or 0.0965)	1A	

Solution	Marks	Remarks
9. (a) $\begin{array}{c\|c\|c\|c\|c\|c\|c\|c} t & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline e^{\frac{t^2}{10}} & 1 & 1.10517 & 1.49182 & 2.45960 & 4.95303 & 12.18249 & 36.59823 \end{array}$	1A	
$\therefore \int_0^6 e^{\frac{t^2}{10}}\,dt \approx \frac{1}{2}(1+36.59823)+(1.10517+1.49182+2.45960+4.95303+12.18249)$	1M	
$\approx 40\ 9912$	1A	
$\because P\rvert_{t=6}-P\rvert_{t=0}=\int_0^6(5e^{\frac{t^2}{10}}-2t)\,dt$ $\therefore P\rvert_{t=6}=\int_0^6(5e^{\frac{t^2}{10}}-2t)\,dt+10$	2A	
$=5\int_0^6 e^{\frac{t^2}{10}}\,dt-\left[t^2\right]_0^6+10$	1A	
$\approx 5\times 40.9912-36+10$ ≈ 179	1A	
(b) (i) Put $t=6$ and $P=179$ into $P=kte^{-0.04t}-50$. $179=6ke^{-0.24}-50$	1M	
$k\approx 48.5$	1A	
(ii) $P=48.5te^{-0.04t}-50$ $P'\approx 48.5(-0.04te^{-0.04t}+e^{-0.04t})$ $=48.5(1-0.04t)e^{-0.04t}$	1M	
$\because e^{-0.04t}>0$ for all t $\therefore P'=0$ only when $t=25$	1A	
and $P'\begin{cases}>0 & \text{for } t<25\\ <0 & \text{for } t>25\end{cases}$ Hence the population size will attain its max. when $t=25$.	1M	
The maximum population size $\approx 48.5\cdot 25\cdot e^{-0.04\cdot 25}-50$ ≈ 396	1A	
(iii) Substitute $y=e^{0.04t}$ into $48.5te^{-0.04t}-50=0$, we have $y=0.97t$.	1M	
The graphs $y=e^{0.04t}$ and $y=0.97t$ intersect at $t\approx 1$ or 119 $\because t\geq 6$, $\therefore$ The species of reptiles becomes extinct ($48.5te^{-0.04t}-50=0$) when $t\approx 119$.	1A	Accept 118 - 120

Solution	Marks	Remarks
10. (a) (i) $\ln[C(t)+1]=\ln ae^{bt}$		
$=\ln a+\ln e^{bt}$	1M	
$=\ln a+bt$	1A	
(ii) $\begin{array}{c\|c\|c\|c\|c} t & 1 & 2 & 3 & 4 \\ \hline \ln[C(t)+1] & 0.79 & 0.89 & 0.99 & 1.09 \end{array}$	1A	
	1A+1A	For the points & line
From the graph, $\ln a\approx 0.69$, $a\approx 2.0$	1A	
$b\approx\frac{1.09-0.79}{4-1}=0.1$	1A	
(iii) $C(t)=2.0e^{0.1t}-1$ $C(36)\approx 72\ 1965$ When $t=36$, the monthly cost is 72.1965 thousand dollars.	1A	

Solution	Marks	Remarks
(b) (i) Solve $2.0e^{0.1t} - 1 = 439 - e^{0.2t}$	1M	
$e^{0.2t} + 2.0e^{0.1t} - 440 = 0$		
$\left(e^{0.1t}\right)^2 + 2.0\left(e^{0.1t}\right) - 440 = 0$	1M	
$e^{0.1t} = 20$ or -22 (rej.)	1A	
$t = 30$	1A	
(ii) $\int_0^{30} [(439 - e^{0.2t}) - (2.0e^{0.1t} - 1)]\,dt$	1M	
$= \int_0^{30} (440 - e^{0.2t} - 2.0e^{0.1t})\,dt$		
$= \left[440t - 5e^{0.2t} - 20e^{0.1t}\right]_0^{30}$	1A	
$= 10806$		
∴ The total profit is 10806 thousand dollars.	1A	

Solution	Marks	Remarks
11. Let X ml be the amount of soda water in each discharge. $X \sim N(210, 15^2)$.		
(a) $P(200 < X < 220)$		
$= P(\frac{200-210}{15} < Z < \frac{220-210}{15})$	1M	
$= P(-0.6667 < Z < 0.6667)$		
$= 0.4972$	1A	Accept value in [0.494, 0.4972]
(b) (i) $P(X > 240)$		
$= P(Z > \frac{240-210}{15})$	1M	
$= P(Z > 2)$		
≈ 0.0228	1A	
(ii) The probability that there is exactly 1 overflow out of 30 discharges is		
$C_1^{30}(0.0228)(0.9772)^{29}$	1M	
$= 0.3504$	1A	
(iii) The probability that Sam will get the second overflow on 31st July is		
0.3504×0.0228		
≈ 0.0080	1M	
(c) (i) $\because P(X > 205) = 0.8$		
$\because P(Z > \frac{205-\mu}{\sigma}) = 0.8$		
$\frac{205-\mu}{\sigma} = -0.84$ ……….(1)	1M+1A	Accept value in [-0.845, -0.84]
$\because P(X > 220) = 0.01$		
$\therefore P(Z > \frac{220-\mu}{\sigma}) = 0.01$		
$\frac{220-\mu}{\sigma} = 2.33$ ……(2)	1A	Accept value in [2.32, 2.33]
Solving (1) & (2) :		
$\begin{cases} \sigma = 4.7 \\ \mu = 209.0 \end{cases}$	1A+1A	
(ii) $P(X > 225)$		
$= P(Z > \frac{225-209}{4.7})$		
$= P(Z > 3.4042)$		
$= 0.0003$	1A	
Probability required		
$= \frac{0.0003}{0.01}$	1M	
$= 0.03$	1A	

Solution	Marks	Remarks
12. (a) & (b) (ii)		

Number of Defective Chips	Observed Frequency	Expected Frequency *		Marks	Remarks
		Binomial	Poisson		
0	33	42.5	32.5		
1	29	28.3	29.3		
2	13	7.9	13.2		
3	4	1.2	4.0		
4	1	0.1	0.9		
5	0	0.0	0.2	1M+1A	For the freq. under Binomial distribution
6	0	0.0	0.0	1M+1A	For the freq. under Poisson distribution

(b) (i) $P(X=0)=e^{-\lambda}=\frac{32.5}{80}$ $\lambda=0.9$	1A	
(c) The Possion distribution Po(0.9) in (b) is adopted since it fits the data better.	1A	
(i) Let p be the probability that a batch is good. $p = P(X=0)$ $= e^{-0.9}$ or $\frac{32.5}{80}$ ≈ 0.4063	1A	Accept 0.4066
The probability that at least 3 out of the 4 batches are good $= p^4 + C^4_3 p^3(1-p)$	1M+1A	1M for applying the binomial distribution
≈ 0.1865	1A	Accept 0.1869
(ii)		

	The original 4 batches	The 6 more batches
No. of good batches	4 3	4 5

(table above)	1M	
The required probability $= \frac{p^4 \cdot C^6_4 p^4(1-p)^2 + C^4_3 p^3(1-p)\cdot C^6_5 p^5(1-p)}{0.1865}$	1M+1M+1A	
$= 0.0547$	1A	Accept 0.0549

Solution	Marks	Remarks
13. (a) Let X be the number of rainstorms in a year. $X \sim Po(2)$		
$P(X=x)=\frac{e^{-2}2^x}{x!}$, $x=0, 1, 2, \ldots$		
$P(X \ge 3) = 1-[P(X=0)+P(X=1)+P(X=2)]$	1M	
$=1-e^{-2}\left[1+2+\frac{4}{2}\right]$	1A	
$=1-5e^{-2}$ ≈ 0.3233	1A	
(b) Let Y be the number of years which will elapse before the next occurence of more than two rainstorms in a year. $Y \sim$ Geometric (p=0.3233).	1M	
Number of years which will elapse $=\frac{1}{p}-1$	1M	For $\frac{1}{p}$
$= 2.0929$ ≈ 2	1A	
(c) Let A be the event of having at least one serious landslide in city A. $P(A \mid X=0)=0.2$ $P(A \mid X=1, 2)=0.3$ $P(A \mid X \ge 3)=0.5$		
(i) $P(\bar{A})$ $= P(\bar{A} \mid X=0)P(X=0)+P(\bar{A} \mid X=1.2)P(X=1.2)+P(\bar{A} \mid X\ge 3)P(X \ge 3)$		
$= 0.8(e^{-2})+0.7(4e^{-2})+0.5(1-5e^{-2})$	1M+1A	
≈ 0.6489	1A	
Alternatively, $P(\bar{A}) = 1-P(A)$ $= 1-[0.2(e^{-2})+0.3(4e^{-2})+0.5(1-5e^{-2})]$	1M+1A	
≈ 0.6489	1A	
(ii) $P(X=0 \mid \bar{A}) = \frac{P(\bar{A} \mid X=0)P(X=0)}{P(\bar{A})}$		
$= \frac{0.8(e^{-2})}{0.6489}$	1M+1M	1A for the numerator 1M for the denominator
≈ 0.1669	1A	
(iii) The probability that there is no serious landslide for at most 2 out of 5 years $= C^5_0(1-0.6489)^5+C^5_1(0.6489)(1-0.6489)^4+C^5_2(0.6489)^2(1-0.6489)^3$	1M+1M	
≈ 0.2369	1A	
Alternatively, $1-[C^5_3(0.6489)^3(1-0.6489)^2+C^5_4(0.6489)^4(1-0.6489)+C^5_5(0.6489)^5]$	1M+1M	
≈ 0.2369	1A	