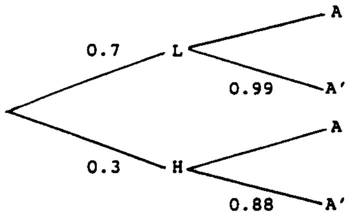
Solution	Marks	Remarks
1. (a) (stem-and-leaf diagram below)	1M + 1A	Award 1M if order or not more than 3 entries are wrong.

Stem (in 10)	Leaf (in 1)
0	5 7 8
1	0 1 2 2 5 8 8 8 8 9
2	0 1 2 3 5 5 5 6 9
3	0 2
4	
5	0

Solution	Marks	Remarks
(b) Mode = 18	1A	
Median = 19	1A	
Interquartile range = 25 − 12	1A	For either
= 13	1A	
	(6)	
2. (a) $e^x+e^y = xy$		
$e^x+e^y\frac{dy}{dx} = y+x\frac{dy}{dx}$	1A + 1A	
$\frac{dy}{dx} = \frac{e^x-y}{x-e^y}$	1A	
(b) $y = \frac{(x-2)^{\frac{1}{2}}(x+3)^{\frac{1}{2}}}{(x+1)^{\frac{3}{2}}}$		
$\ln y = \frac{1}{2}\ln(x-2)+\frac{1}{2}\ln(x+3)-\frac{3}{2}\ln(x+1)$		
$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2(x-2)}+\frac{1}{2(x+3)}-\frac{3}{2(x+1)}$	1M + 1A	1M for taking log. on both sides and applying: $\ln ab = \ln a+\ln b$ $n\ln a = \ln a^n$ $\frac{d}{dx}\ln y = \frac{1}{y}\frac{dy}{dx}$
$\frac{dy}{dx} = \frac{y}{2}\left(\frac{1}{x-2}+\frac{1}{x+3}-\frac{3}{x+1}\right)$	1A	
(or $\frac{1}{2(x+1)}\sqrt{\frac{(x-2)(x+3)}{x+1}}\left(\frac{1}{x-2}+\frac{1}{x+3}-\frac{3}{x+1}\right)$		
or $\frac{y(19-x^2)}{2(x-2)(x+3)(x+1)}$)		
	(6)	

Solution	Marks	Remarks
3. (a) No. of ways of grouping the students		
$= C^{18}_6 \times C^{12}_6$	1A	
= 17153136	1A	
Alternatively,		
No. of ways $= \frac{18!}{6!6!6!}$	1A	
= 17153136	1A	
(b) No. of ways of grouping the students so that each group contains one girl		
$= C^{15}_5 \times C^{10}_5 \times 3!$	1M + 1A	1M for 3!
= 4540536		
Alternatively,		
No. of ways $= \frac{15!}{5!5!5!} \times 3!$	1M + 1A	1M for 3!
= 4540536		
Required probability $= \frac{4540536}{17153136}$	1M	
$= \frac{9}{34}$ (or 0.2647)	1A	
	(6)	

Solution	Marks	Remarks
4. (a) $M(t)=\int\left[\frac{1}{3t+4}+(t+25)^{-\frac{1}{2}}\right]dt$	1A	pp-1 for missing differential
$=\frac{1}{3}\ln(3t+4)+2(t+25)^{\frac{1}{2}}+c$	1A + 1A	1A for integrating 1 term
$\because M(0)=3.1$		
$\therefore c=3.1-\frac{1}{3}\ln4-10$	1M	
$=-\frac{2}{3}\ln2-6.9$ (or -7.3621)		
$M(t)=\frac{1}{3}\ln(3t+4)+2\sqrt{t+25}-\frac{2}{3}\ln2-6.9$	1A	
(or $\frac{1}{3}\ln(3t+4)+2\sqrt{t+25}-7.3621$)		
The value of the house, in million dollars, t years after the end of 1994 is $\frac{1}{3}\ln(3t+4)+2\sqrt{t+25}-\frac{2}{3}\ln2-6.9$.		
(b) The rise in the value of the house, in million dollars, between the end of 1994 and the end of 2000 is		
$M(6)-M(0)$		
$=\frac{1}{3}\ln22+2\sqrt{31}-\frac{2}{3}\ln2-6.9-3.1$	1M	
$=\frac{1}{3}\ln\frac{11}{2}+2\sqrt{31}-10$ (or 1.7038)	1A	
Alternatively,		
$M(6)-M(0)=\left[\frac{1}{3}\ln(3t+4)+2\sqrt{t+25}\right]_0^6$		
$=\frac{1}{3}\ln22+2\sqrt{31}-\frac{2}{3}\ln2-10$	1M	
$=\frac{1}{3}\ln\frac{11}{2}+2\sqrt{31}-10$ (or 1.7038)	1A	
	(7)	

Solution	Marks	Remarks
5. (a) Let A' be the event of having no accident within a year.		
 	1A	
Alternatively, $P(L)=0.7,\ \ P(H)=0.3$ $P(A'\|L)=0.99,\ \ P(A'\|H)=0.88$	1A	
$P(H\|A')=\frac{P(A'\cap H)}{P(A')}$		
$=\frac{P(A'\|H)P(H)}{P(A'\|H)P(H)+P(A'\|L)P(L)}$		
$=\frac{0.88\times0.3}{0.88\times0.3+0.99\times0.7}$	1M+1M+1A	1M for the numerator 1M for the denominator
$=0.2759$ (or $\frac{8}{29}$)	1A	
(b) $P(L\|A')=1-P(H\|A')$		
$=1-0.2759$	1M	
$=0.7241$ (or $\frac{21}{29}$)	1A	
Alternatively,		
$P(L\|A')=\frac{P(A'\|L)P(L)}{P(A'\|H)P(H)+P(A'\|L)P(L)}$		
$=\frac{0.99\times0.7}{0.88\times0.3+0.99\times0.7}$	1M	
$=0.7241$ (or $\frac{21}{29}$)	1A	
	(7)	

Solution	Marks	Remarks
6. (a) $2^{2x}+4 = 5(2^x)$		
$(2^x)^2-5(2^x)+4 = 0$		
$(2^x-4)(2^x-1) = 0$	1M	
$2^x = 4$ or 1		
$x = 2$ or 0	1A	
$\therefore$ The intersection points are (0,5) and (2,20)	1A	
(b) If $2^x = e^{ax}$ for all values of x,		
then $a = \ln 2$.	1A	
Area $= \int_0^2 [5(2^x) - 2^{2x} - 4]\,dx$	1A	
$= \int_0^2 [5e^{x\ln 2} - e^{x\ln 4} - 4]\,dx$	1M	
$= \frac{5}{\ln 2}[e^{x\ln 2}]_0^2 - \frac{1}{\ln 4}[e^{x\ln 4}]_0^2 - 4[x]_0^2$	1M	
$= 15\left(\frac{1}{\ln 2} - \frac{1}{\ln 4}\right) - 8$		
$= \frac{15}{2\ln 2} - 8$ (or 2.8202)	1A	
	(8)	

Solution	Marks	Remarks
7. (a) (i) $\begin{array}{\|c\|c\|c\|c\|c\|c\|c\|}\hline x & 0 & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 \\ \hline f(x) & 1 & 1.00504 & 1.02062 & 1.04828 & 1.09109 & 1.15470 \\ \hline\end{array}$	1A	
$I_1 \approx 0.1[\frac{1}{2}(1+1.15470)$		
$+ (1.00504+1.02062+1.04828+1.09109)]$	1M	Using correct formula
≈ 0.5242	1A	
(ii) $f'(x) = \frac{x}{(1-x^2)^{\frac{3}{2}}}$	1A	
$f''(x) = \frac{2x^2+1}{(1-x^2)^{\frac{5}{2}}}$	1A	must be simplified
(iii) By (a)(ii), $f''(x) > 0$ for $0 \le x \le \frac{1}{2}$,		
$\therefore$ $f(x)$ is concave upward (or convex) on $[0, \frac{1}{2}]$	1	argument for convexity
Hence I_1 is an over-estimate of I.	1A	
(b) (i) $f(x) = (1-x^2)^{-\frac{1}{2}}$		
$= 1 + (-\frac{1}{2})(-x^2) + \frac{(-\frac{1}{2})(-\frac{1}{2}-1)}{2!}(-x^2)^2$		
$+ \frac{(-\frac{1}{2})(-\frac{1}{2}-1)(-\frac{1}{2}-2)}{3!}(-x^2)^3 + \ldots$	1M + 1A	1M for binomial series
$= 1+\frac{1}{2}x^2+\frac{3}{8}x^4+\frac{5}{16}x^6+\ldots$ for $0 \le x \le \frac{1}{2}$.		
$\therefore p(x) = 1+\frac{1}{2}x^2+\frac{3}{8}x^4+\frac{5}{16}x^6$	1A	
$I_2 = \int_0^{\frac{1}{2}} (1+\frac{1}{2}x^2+\frac{3}{8}x^4+\frac{5}{16}x^6)\,dx$		
$= \left[x+\frac{1}{6}x^3+\frac{3}{40}x^5+\frac{5}{112}x^7\right]_0^{\frac{1}{2}}$	1A	
$= \frac{1}{2}+\frac{1}{48}+\frac{3}{1280}+\frac{5}{14336}$		
≈ 0.5235	1A	

Solution	Marks	Remarks
(ii) $\because\ f(x) = p(x) + \sum_{r=4}^{\infty} \frac{(-\frac{1}{2})(-\frac{1}{2}-1)\ldots(-\frac{1}{2}-r+1)}{r!}(-x^2)^r$		
$= p(x) + \sum_{r=4}^{\infty} \frac{(\frac{1}{2})(\frac{1}{2}+1)\ldots(\frac{1}{2}+r-1)}{r!}x^{2r}$	1A	
$> p(x)$ for $0<x\le\frac{1}{2}$.	1	
Hence $I > I_2$		
i.e. I_2 is an under-estimate of I.	1A	

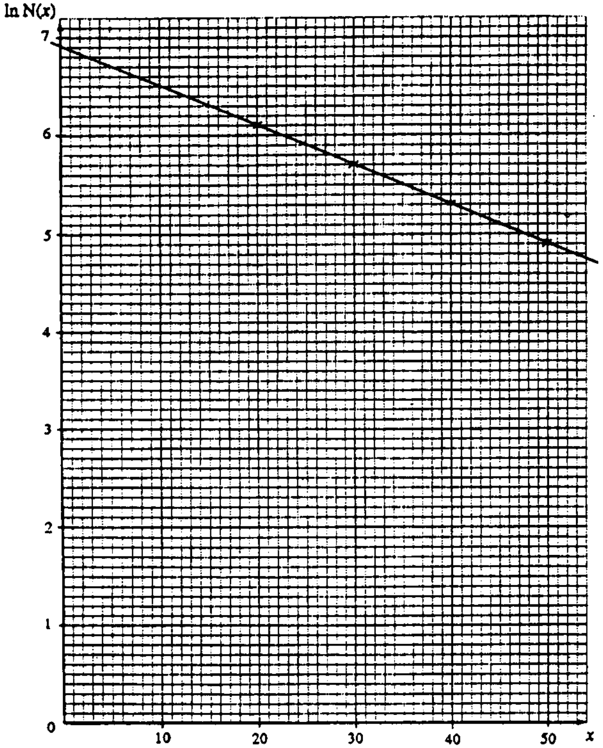
<u>Note</u>:

1. 1 mark for the following argument in b(ii)
   $\because$ Sum to infinity
   $\therefore$ p(x) is just a truncation
   Hence underestimate

2. Withhold 1 mark <u>once</u> for incorrect degree of accuracy.

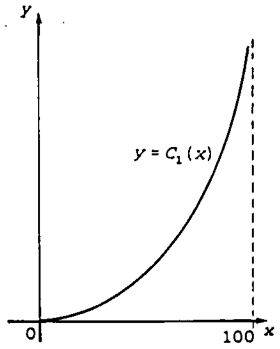
Solution	Marks	Remarks
8. (a) (i) $N(x) = ae^{-bx}$		
$\ln N(x) = \ln a + \ln e^{-bx}$	1A	
$= \ln a - bx$	1A	
(ii)	1A	At least 1 d.p.

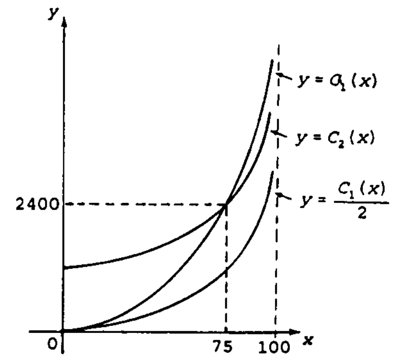
x	20	30	40	50
ln N(x)	6.11	5.71	5.31	4.91



Solution	Marks	Remarks
	1A + 1A	1A for the points 1A for the line
From the graph,		
$\ln a = 6.9$		Accept 6.85 - 6.95
$\therefore\ a = 992.27$	1A	Accept 943.88 - 1043.15
$-b = \frac{4.91-6.11}{50-20}$		
$b = 0.04$	1A	no mark for b = slope or b = -0.04 in any calculation.

Solution	Marks	Remarks
(b) $992.27e^{-0.04x} = 400$ $-0.04x = \ln \frac{400}{992.27}$ $x = 22.7$	1M	
In general, accept $ae^{-0.04x} = 400$ where $a \in (943.88, 1043.15)$ $-0.04x = \ln \frac{400}{a}$ $\Rightarrow x \in (21.5, 24.0)$	1M	
$\therefore$ The price of each CD should be \$22.7.	1A	Accept \$21.5 - \$ 24.0
(c) (i) $G(x) = 992.27(x-10)e^{-0.04x}$	1A	
In general, accept $G(x) = a(x-10)e^{-0.04x}$ where $a \in (943.88, 1043.15)$	1A	
(ii) $G'(x) = 992.27[e^{-0.04x}+(-0.04)(x-10)e^{-0.04x}]$	1M	
$= 992.27e^{-0.04x}[1.4-0.04x]$	1A	
In general, accept		
$G'(x) = a[e^{-0.04x}+(-0.04)(x-10)e^{-0.04x}]$	1M	
$= ae^{-0.04x}[1.4-0.04x]$ where $a \in (943.88, 1043.15)$	1A	
$G'(x) = 0$ when $x = 35$ and	1M	for $G'(x) = 0$ and solving
$G'(x) \begin{cases} >0 & \text{if } x<35 \\ <0 & \text{if } x>35 \end{cases}$	1M	
Alternatively $G''(x) = 1.59xe^{-0.04x} - 95.18e^{-0.04x}$ $G''(35) = -9.750$	1M	
Therefore $G(x)$ is maximum when $x=35$. For maximum profit, the selling price for each CD should be \$35.	1A	

Solution	Marks	Remarks
9. (a) (i) $C_1(80) = \frac{800(80)}{100-80} = 3{,}200$ $\therefore$ The cost required is 3,200 thousand dollars.	1A	
(ii) $\frac{800x}{100-x} = 2000$ $x = 71\frac{3}{7}$ (or 71.4286)	1M	
Hence $71\frac{3}{7}\%$ (or 71.4286%) of the loudness of the noise released from the Stadium can be reduced.	1A	
(b) (i) $C_1'(x) = \frac{800(100-x)+800x}{(100-x)^2}$	1M	
$= \frac{80000}{(100-x)^2}$	1A	
$C_1''(x) = \frac{160\,000}{(100-x)^3}$	1A	
(ii) $\because$ $C_1''(x) > 0$ for $0 \leq x < 100$ $\therefore$ The curve $y = C_1(x)$ is concave upward (or convex).	1A	
The vertical asymptote is $x=100$.	1A	
	1A	For not crossing the asymptote
	1A	For the curve

Solution	Marks	Remarks
(c) (i) $\frac{800x}{100-x} = \frac{400x}{100-x} + 1200$		
$x = 75$	1A	
(ii) 	2A	For the graph of $y = C_2(x)$ (award 1A if a candidate correctly sketche the graph of $y = \frac{C_1(x)}{2}$ only).
(iii) $y = C_1(x)$ and $y = C_2(x)$ intersect at (75, 2400).	1A	
The local engineering company is more cost effective.	1A	

Solution	Marks	Remarks

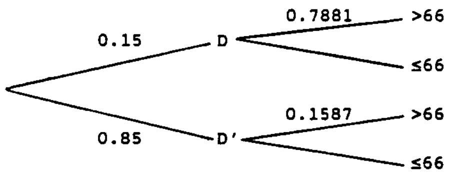
10. (a) (i)

Number of Bicycles Sold	Observed Frequency	Po(2)		Po(3)	
		Probability	Expected * Frequency	Probability	Expected * Frequency
0	10	0.1353	27.1	0.0498	10.0
1	28	0.2707	54.1	0.1494	29.9
2	49	0.2707	54.1	0.2240	44.8
3	44	0.1804	36.1	0.2240	44.8
4	38	0.0902	18.0	0.1680	33.6
5 or more	31	0.0527	10.5	0.1847 (or 0.1848)	37.0 (or 36.9)

* Correct to 1 decimal place.

Solution	Marks	Remarks
	2M + 3A	1M for Expected Frequency 1M for Probability 1A+1A for the 2 pairs of data 1A for all being correct
(ii) Po(3) fits the data better.	1A	
(b) (i) Let X be the no. of bicycles sold, then $X \sim Po(3)$. $P(X=0) = 0.0498$	1M + 1A	
(ii) Let Y be the no. of days that no bicycles will be sold, then $Y \sim B(7,0.0498)$	1M	
$P(Y=3) = C_3^7(0.0498)^3(1-0.0498)^4$	1M	
≈ 0.0035	1A	
(c) P(3 items sold) = P(3 bicycles sold) + P(2 bicycles and 1 tricycle sold) + P(1 bicycles and 2 tricycles sold) + P(3 tricycles sold)	1M	For addition
$= 0.2240 \times 0.1353 + 0.2240 \times 0.2707 + 0.1494 \times 0.2707 + 0.0498 \times 0.1804$	1M + 1M	1M for the 1st & 4th terms 1M for the 2nd & 3rd terms
≈ 0.1404	1A	

Solution	Marks	Remarks
11. (a) Probability of acceptance, $p_a=(1-0.02)^5$	1A	
$=0.9039$		
Probability of rejection, $p_r=1-p_a$	1M	
$=0.0961$	1A	
(b) Let X be the number of cartons inspected by Madam Wong in a day, then $X \sim \text{Geom}(p_r)$.	1M	
$\therefore$ mean $= \frac{1}{p_r}$	1A	
$= 10.4$	1A	
(c) (i) Prob. that Madam Wong can achieve her target,		
p_t = P(All cartons are acceptable) + P(exactly 1 carton is not acceptable) + P(exactly 2 cartons are not acceptable)	1M	
$= (p_a)^{22}+\binom{22}{1}p_r(p_a)^{21}+\binom{22}{2}(p_r)^2(p_a)^{20}$	1M + 1A	
$= 0.6445$	1A	Accept 0.6444 - 0.6445
<u>Alternatively</u>,		
p_t = P(the 1st 20 cartons are accepted) + P(1 is rejected in the 1st 20 cartons and the 21st carton is accepted) + P(2 is rejected in the 1st 21 cartons and the 22nd carton is accepted)	1M	
$= (p_a)^{20}+\binom{20}{1}p_r(p_a)^{20}+\binom{21}{2}(p_r)^2(p_a)^{20}$	1M + 1A	
≈ 0.6445	1A	Accept 0.6444 - 0.6445
(ii) If Madam Wong can achieve her target, the prob. that she needs to inspect 20 cartons only		
$= \frac{(p_a)^{20}}{p_t}$	1M + 1A	
$= 0.2058$	1A	Accept 0.2057
(d) $(1-r\%)^5 \geq 0.95$	1M	
$r\% \leq 0.010206$		
$r \leq 1.0206$		
$\therefore$ The greatest acceptable value of r is 1.0206 .	1A	

Solution	Marks	Remarks
12. Let X denote the test score and D the event that a person has the disease.		
(a) $P(X>63.2 \mid D') = 0.33$		
$P(Z>\frac{63.2-\mu}{5}) = 0.33$	1A	
From the normal distribution table,		
$\frac{63.2-\mu}{5} = 0.44$	1A	
$\therefore \quad \mu = 61$	1A	
(b) (i) $P(X>66 \mid D) = P(Z>\frac{66-70}{5})$	1A	
$= P(Z>-0.8)$		
$= 0.7881$	1A	
and $P(X>66 \mid D') = P(Z>\frac{66-61}{5})$		
$= P(Z>1)$		
$= 0.1587$	1A	
$\therefore$ P(the person will be classified as having the disease)		
$= 0.15\times0.7881+(1-0.15)\times0.1587$	1M + 1A	
$= 0.2531$	1A	
(ii) 	1M	
$P(X\leq66 \mid D) = 1 - 0.7881$	1M	
$= 0.2119$	1A	
$\therefore$ P(the person will be misclassified)		
$= 0.15\times0.2119+(1-0.15)\times0.1587$	1M + 1A	
$= 0.1667$	1A	