HONG KONG EXAMINATIONS AUTHORITY

HONG KONG ADVANCED LEVEL EXAMINATION 1995

# MATHEMATICS AND STATISTICS AS-LEVEL

9.00 am-12.00 noon (3 hours)

This paper must be answered in English

1. This paper consists of Section A and Section B.
2. Answer ALL questions in Section A, using the AL(C1) answer book.
3. Answer any FOUR questions in Section B, using the AL(C2) answer book.
4. Unless otherwise specified, numerical answers should either be exact or given to 4 decimal places.



## SECTION A (40 marks)

Answer ALL questions in this section.

**Write your answers in the AL(C1) answer book.**

1. The numbers of hours spent by 25 students in studying for an examination are as follows:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 8 | 25 | 21 | 18 | 25 | 7 | 32 | 29 | 18 |
| 18 | 18 | 22 | 12 | 5 | 30 | 19 | 15 | 20 | 50 |
| 25 | 10 | 26 | 23 | 12 | | | | | |

(a) Copy and complete the following stem-and-leaf diagram for the above data:

| Stem (in 10) | Leaf (in 1) |
|---|---|
| 0 | 5 7 8 |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |

(b) Find the mode, the median and the interquartile range of the numbers of hours spent by the 25 students.

(6 marks)

2. (a) If $e^x + e^y = xy$, find $\frac{dy}{dx}$.

(b) If $y = \frac{1}{x+1}\sqrt{\frac{(x-2)(x+3)}{x+1}}$ where $x > 2$, use logarithmic differentiation to find $\frac{dy}{dx}$.

(6 marks)

3. A teacher wants to divide a class of 18 students into 3 groups, each of 6 students, to do 3 different statistical projects.

(a) In how many ways can the students be grouped?

(b) If there are 3 girls in the class, find the probability that there is one girl in each group.

(6 marks)

4. The value $M$ (in million dollars) of a house is modelled by the equation

$$\frac{dM}{dt} = \frac{1}{3t+4} + \frac{1}{\sqrt{t+25}}$$

where $t$ is the number of years elapsed since the end of 1994. The value of the house was 3.1 million dollars at the end of 1994.

(a) Find, in terms of $t$, the value of the house $t$ years after the end of 1994.

(b) Find the rise in the value of the house between the end of 1994 and the end of 2000.

(7 marks)

5. An insurance company classifies the aeroplanes it insures into class L (low risk) and class H (high risk), and estimates the corresponding proportions of the aeroplanes as 70% and 30% respectively. The company has also found that 99% of class L and 88% of class H aeroplanes have no accident within a year. If an aeroplane insured by the company has no accident within a year, what is the probability that it belongs to

(a) class H ?

(b) class L ?

(7 marks)



6. Figure 1 shows graphs of the two curves

$$C_1 : y = 2^{2x} + 4$$

and $$C_2 : y = 5(2^x) .$$

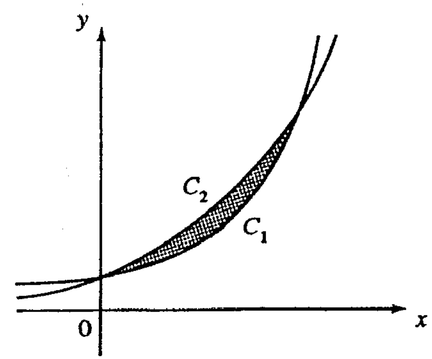


**Figure 1**

(a) Find the coordinates of the points of intersection of $C_1$ and $C_2$.

(b) If $2^x = e^{ax}$ for all $x$, find $a$.

Hence, or otherwise, find the area of the shaded region in Figure 1 bounded by $C_1$ and $C_2$.

(8 marks)

**SECTION B** (60 marks)
Answer any **FOUR** questions from this section. Each question carries 15 marks.
**Write your answers in the AL(C2) answer book.**

7. Let $f(x) = \dfrac{1}{\sqrt{1-x^2}}$ where $0 \le x \le \dfrac{1}{2}$ ,

and $I = \int_0^{\frac{1}{2}} f(x)\,dx$ .

(a) (i) Find the estimate $I_1$ of $I$ using the trapezoidal rule with 5 sub-intervals.

(ii) Find $f'(x)$ and $f''(x)$ .

(iii) Using (a)(ii) or otherwise, state whether $I_1$ in (a)(i) is an over-estimate or under-estimate of $I$. Explain your answer briefly.

(7 marks)

(b) (i) Use the binomial expansion to find a polynomial $p(x)$ of degree 6 which approximates $f(x)$ for $0 \le x \le \dfrac{1}{2}$ .

Let $I_2 = \int_0^{\frac{1}{2}} p(x)\,dx$ . Find $I_2$ .

(ii) State whether $I_2$ in (b)(i) is an over-estimate or under-estimate of $I$. Explain your answer briefly.

(8 marks)



8. A merchant sells compact discs (CDs). A market researcher suggests that if each CD is sold for $\$x$ , the number $N(x)$ of CDs sold per week can be modelled by

$$N(x) = ae^{-bx}$$

where $a$ and $b$ are constants.

The merchant wants to determine the values of $a$ and $b$ based on the following results obtained from a survey:

| $x$ | 20 | 30 | 40 | 50 |
|---|---|---|---|---|
| $N(x)$ | 450 | 301 | 202 | 136 |

(a) (i) Express $\ln N(x)$ as a linear function of $x$.

(ii) Use the graph paper on Page 6 to estimate graphically the values of $a$ and $b$ correct to 2 decimal places.

(7 marks)

(b) Suppose the merchant wishes to sell 400 CDs in the next week. Use the values of $a$ and $b$ estimated in (a) to determine the price of each CD. Give your answer correct to 1 decimal place.

(2 marks)

(c) It is known that the merchant obtains CDs at a cost of $\$10$ each. Let $G(x)$ dollars denote the weekly profit. Using the values of $a$ and $b$ estimated in (a),

(i) express $G(x)$ in terms of $x$ ;

(ii) find $G'(x)$ and hence determine the selling price for each CD in order to maximize the profit.

(6 marks)

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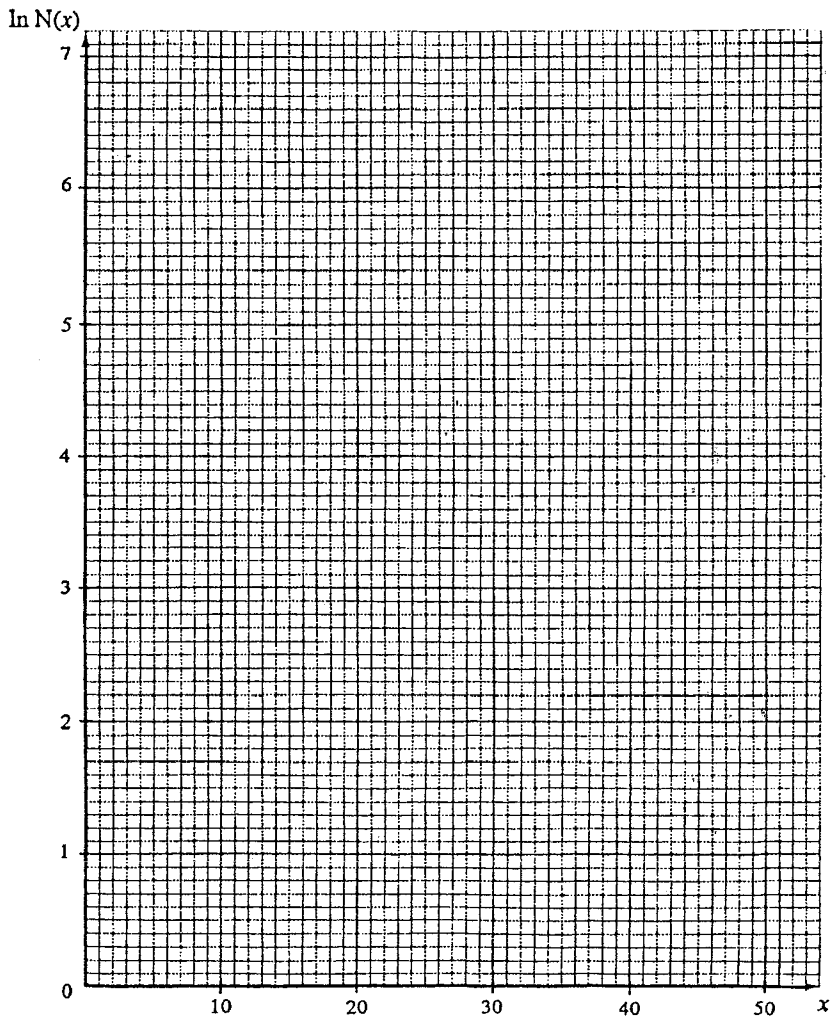
8.(Cont'd) **If you attempt Question 8, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**



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9. A local engineering company estimates that if the loudness of the noise released from the Hong Kong Stadium is to be reduced by $x\%$ , the cost $C_1(x)$ , in thousand dollars, will be

$$C_1(x) = \frac{800x}{100-x}, \qquad 0 \le x < 100 .$$

(a) (i) If the loudness of the noise released from the Stadium is to be reduced by $80\%$ , find the cost required.

(ii) If the Urban Council spends 2 million dollars for the work, by what percentage can the loudness of the noise released from the Stadium be reduced?

(3 marks)

(b) (i) Find $C_1'(x)$ and $C_1''(x)$ .

(ii) Determine the convexity and the vertical asymptote of the curve $y = C_1(x)$ . Sketch this curve.

(7 marks)

(c) An overseas engineering company estimates that if the loudness of the noise released from the Hong Kong Stadium is to be reduced by $x\%$ , the cost $C_2(x)$ , in thousand dollars, will be

$$C_2(x) = \frac{400x}{100-x} + 1200, \qquad 0 \le x < 100 .$$

(i) Find the value of $x$ such that $C_1(x) = C_2(x)$ .

(ii) On the same graph sketched in (b)(ii), sketch the curve $y = C_2(x)$ .

[Hint: You may sketch the curve $y = \frac{C_1(x)}{2}$ first.]

(iii) If the Urban Council has a budget of 2 million dollars for reducing the loudness of the noise released from the Stadium, use c(ii) to determine which company is more cost effective.

(5 marks)



10. A shop special. .g in bicycles recently opened up a new branch selling bicycles only. Experience from other branches showed that the number of bicycles sold in a day could be modelled by a Poisson distribution with mean either 2 or 3. The branch manager recorded the number of bicycles sold in a day for the first 200 days as follows:

| Number of bicycles sold | 0 | 1 | 2 | 3 | 4 | 5 or more |
|---|---|---|---|---|---|---|
| Observed frequency (days) | 10 | 28 | 49 | 44 | 38 | 31 |

(a) (i) Using the Poisson distributions with means 2 and 3 respectively, fill in the missing probabilities and expected frequencies in Table 1 (Page 10).

(ii) State which of the two Poisson distributions fits the data better.

(6 marks)

(b) Adopting the model you have chosen in (a)(ii), what is the probability that

(i) no bicycle will be sold in a day?

(ii) no bicycle will be sold in exactly 3 out of the next 7 days?

(5 marks)

(c) The branch manager decides to sell tricycles as well. He knows that the number of tricycles sold in a day can be modelled by a Poisson distribution with mean 2. Adopting the model chosen in (a)(ii) for the number of bicycles sold and assuming that the numbers of bicycles and tricycles sold are independent of each other, what is the probability that exactly three items are sold in a day?

(4 marks)

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10.(Cont'd) **If you attempt Question 10, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**

**Table 1** Observed and expected frequencies of the number of bicycles sold

| Number of Bicycles Sold | Observed Frequency | Po(2) | | Po(3) | |
|---|---|---|---|---|---|
| | | Probability | Expected * Frequency | Probability | Expected * Frequency |
| 0 | 10 | 0.1353 | | 0.0498 | |
| 1 | 28 | | | 0.1494 | |
| 2 | 49 | | 54.1 | | 44.8 |
| 3 | 44 | | 36.1 | 0.2240 | |
| 4 | 38 | 0.0902 | | | 33.6 |
| 5 or more | 31 | 0.0527 | | | |

* Correct to 1 decimal place.



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11. Madam Wong purchases cartons of oranges from a supplier every day. Her buying policy is to randomly select five oranges from a carton and accept the carton if all five are not rotten. Under usual circumstances, $2\%$ of the oranges are rotten.

(a) Find the probability that a carton of oranges will be rejected by Madam Wong.

(3 marks)

(b) Every day, Madam Wong keeps on buying all the accepted cartons of oranges and stops the buying exercise when she has to reject a carton. What is the mean, correct to 1 decimal place, of the number of cartons inspected by Madam Wong in a day?

(3 marks)

(c) Today, Madam Wong has a target of buying 20 acceptable cartons of oranges from the supplier. Instead of applying the stopping rule in (b), she will keep on inspecting the cartons until her target is achieved. Unfortunately, the supplier has a stock of 22 cartons only.

(i) Find the probability that she can achieve her target.

(ii) Assuming she can achieve her target, find the probability that she needs to inspect 20 cartons only?

(7 marks)

(d) The supplier would like to import oranges of better quality so that each carton will have at least a $95\%$ probability of being accepted by Madam Wong. If $r\%$ of these oranges are rotten, find the greatest acceptable value of $r$.

(2 marks)



12. A test is used to diagnose a disease. For people *with* the disease, it is known that the test scores follow a normal distribution with mean 70 and standard deviation 5. For people *without* the disease, the test scores follow another normal distribution with mean $\mu$ and the same standard deviation 5. It is known that $33\%$ of those people *without* the disease will achieve a test score over 63.2.

(a) Find $\mu$.

(3 marks)

(b) It is estimated that $15\%$ of the population of a city has the disease. A doctor has proposed that a person be classified as having the disease if the person's test score exceeds 66, otherwise the person will be classified as not having the disease.

If a person is randomly selected from the population to take the test,

(i) what is the probability that this person will be classified as having the disease?

(ii) find the probability that this person will be misclassified.

(12 marks)

**END OF PAPER**