香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九四年香港高級程度會考
HONG KONG ADVANCED LEVEL EXAMINATION, 1994

高級補充程度數學及統計學(甲部)
AS MATHEMATICS & STATISTICS (A)

評卷參考
MARKING SCHEME

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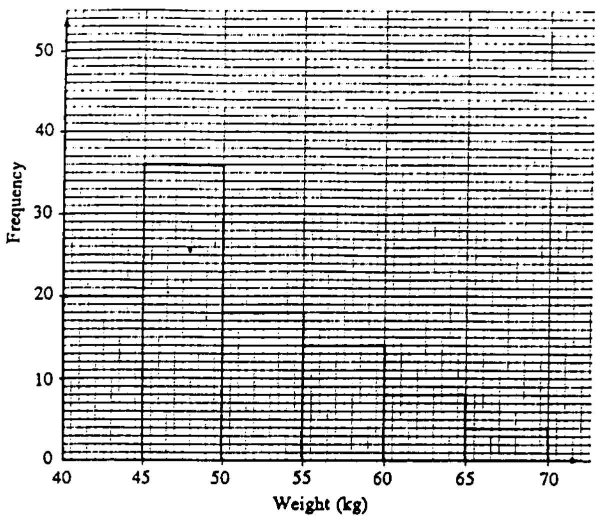
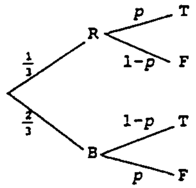
Solution	Marks	Remarks
1. (a) Sample space, S = {BB, BG, GB, GG}	1A + 1A	1A for set notation
(b) (i) Sample space, S = {BB, BG, GB}	1A	
(ii) The event that the family has two sons, E = {BB}		
$P(E) = \frac{1}{3}$	1A	
	4	
2. $\frac{dx}{dt} = 100ke^{kt}$	1A	
$\frac{d^2x}{dt^2} = 100k^2e^{kt}$	1A	
Hence $100k^2e^{kt} - 200ke^{kt} - 300e^{kt} = 0$	1M	
$k^2 - 2k - 3 = 0$	1A	
$(k-3)(k+1) = 0$		
$k = -1$ or 3		
$\because k$ is negative		
$\therefore k = -1$	1A	
	5	
3. (a) Number of shortest paths from A to B		
= Number of permutations of the letters R,R,U,U,F,F		
$= \frac{6!}{2!2!2!}$	1A	
$= 90$	1A	
(b) (i) Number of shortest paths from A to C		
= Number of permutations of the letters R,U,F		
$= 3!$	1A	
$= 6$		
(ii) Number of shortest paths from A to B through C		
$= 6 \times 6$	1M	
$= 36$		
Probability that Jack is caught by the trap		
$= \frac{36}{90}$	1A	
$= \frac{2}{5}$		
	5	

Solution	Marks	Remarks
4. (a) Histogram of the weights		
	1A	For vertical measures
	1A	For horizontal measures
(b) From Figure 2, lower quartile = 46 kg		Accept 45.5-46
upper quartile = 55 kg		Accept 55-55.5
	1A	For either
Interquartile range = (55 − 46) kg = 9 kg	1A	Accept 9-10
(c) From the histogram, mean weight		
$= \frac{42.5\times20+47.5\times36+52.5\times18+57.5\times14+62.5\times8+67.5\times4}{100}$ kg	1M	
= 50.8 kg	1A	
	6	

Solution	Marks	Remarks
5. Let $u=t^2+1$, then $du=2t\,dt$.	1M	
$x=\int 3t(t^2+1)^{\frac{1}{2}}dt$	1M	
$=\frac{3}{2}\int u^{\frac{1}{2}}du$	1A	
$=u^{\frac{3}{2}}+c$		
$=(t^2+1)^{\frac{3}{2}}+c$	1A	
Since $x=10$ when $t=0$, $10=(0^2+1)^{\frac{3}{2}}+c$, $c=9$	1M	
$\therefore\ x=(t^2+1)^{\frac{3}{2}}+9$.	1A	
	6	
6. (a) Since $e^x \approx 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}$ for $x \approx 0$,		
$e^{-\frac{x^2}{2}} \approx 1+(-\frac{x^2}{2})+\frac{1}{2}(-\frac{x^2}{2})^2+\frac{1}{6}(-\frac{x^2}{2})^3$	1M	
$=1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}$ for $x \approx 0$.	1A	
$\int_0^1 e^{-\frac{x^2}{2}}dx \approx \int_0^1 (1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48})\,dx$		
$=\left[x-\frac{x^3}{6}+\frac{x^5}{40}-\frac{x^7}{336}\right]_0^1$	1M	
$=1-\frac{1}{6}+\frac{1}{40}-\frac{1}{336}$		
$=0.8554$	1A	
(b) From the normal distribution table, $\frac{1}{\sqrt{2\pi}}\int_0^1 e^{-\frac{x^2}{2}}dx = 0.3413$	1A	
Hence $\frac{1}{\sqrt{2\pi}} \times 0.8554 \approx 0.3413$	1M	
$\therefore\ \pi \approx \frac{0.8554^2}{2\times 0.3413^2} \approx 3.141$	1A	3.140 for using exact value of (a)
	7	

Solution	Marks	Remarks
7. (a) R: red card is drawn T: response 'True' B: black card is drawn F: response 'False' p: percentage of persons who are homosexual		
	1A	
$\frac{790}{1200}=\frac{1}{3}p+\frac{2}{3}(1-p)$	1M + 1A	
$\frac{790}{1200}=\frac{2}{3}-\frac{1}{3}p$		
$p=2.5\%$	1A	
Alternatively Let x, y be the no. of interviewees who are homosexual and not homosexual respectively, then		
$\frac{1}{3}x+\frac{2}{3}y=790$	1A	
$\frac{2}{3}x+\frac{1}{3}y=410$	1A	
Solving the equations, we have $x=30$, $y=1170$	1M	For reducing into one unknown
$\therefore$ The percentage required $=\frac{30}{1200}$ $=2.5\%$	1A	
(b) $P(R\|T)=\frac{P(T\|R)P(R)}{P(T)}$		
$=\frac{(0.025)(\frac{1}{3})}{\frac{79}{120}}$	1M + 1A	
$=0.0127$ (or $\frac{1}{79}$)	1A	
	7	

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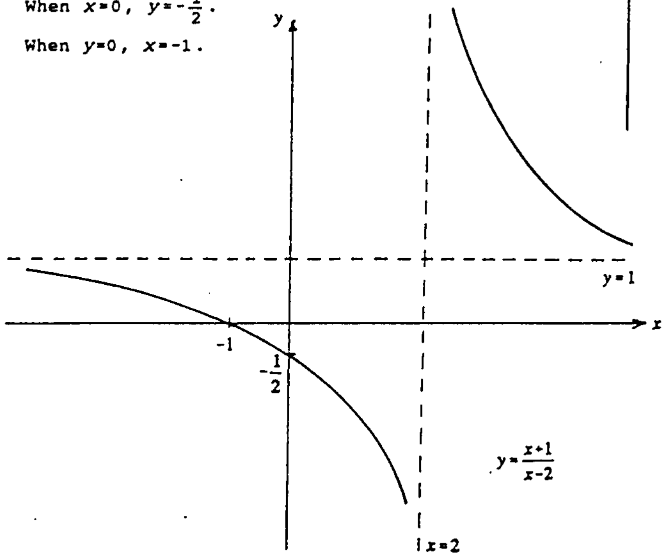
Solution	Marks	Remarks
8. (a) $\frac{dy}{dx}=\frac{(x-2)-(x+1)}{(x-2)^2}$	1M	For applying the quotient rule
$=-\frac{3}{(x-2)^2}$.	1A	
(b) Since $y\to\infty$ as $x\to 2$, $x=2$ is the vertical asymptote.	1A	
Since $y=\frac{1+\frac{1}{x}}{1-\frac{2}{x}}\to\frac{1+0}{1-0}=1$ as $x\to\infty$,	1A	
$y=1$ is the horizontal asymptote.	1A	
(c) $\frac{dy}{dx}<0$ for any $x\neq 2$.		
When $x=0$, $y=-\frac{1}{2}$.		
When $y=0$, $x=-1$.	2A	For the intercepts
	1	For the asymptotes
	2	For the graph



Solution	Marks	Remarks
(d) Area of the bounded region		
$=-\int_{-1}^{0}\frac{x+1}{x-2}dx$	1M + 1A	
$=-\int_{-1}^{0}(1+\frac{3}{x-2})dx$	1M	
$=-[x+3\ln\|x-2\|]_{-1}^{0}$	1M	
$=-[0+3\ln 2-(-1+3\ln 3)]$		
$=-1-3\ln 2+3\ln 3$		
≈ 0.2164	1A	r.t. 0.216

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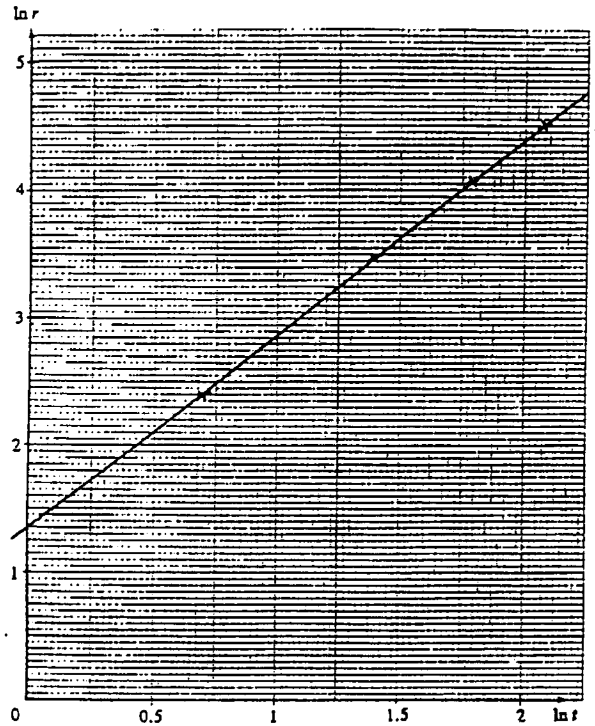
Solution	Marks	Remarks
9. (a) (i) The machine will cease producing cloth when $x=0$,	1M	
$100e^{-0.01t}-65e^{-0.02t}-35=0$		
Put $y=e^{-0.01t}$,		
$100y-65y^2-35=0$		
$13y^2-20y+7=0$		
$(y-1)(13y-7)=0$		
$y=1$ or $\frac{7}{13}$		
$\therefore\ e^{-0.01t}=1$ or $\frac{7}{13}$		
$t=0$ (rej.) or $t=\dfrac{\ln\frac{7}{13}}{-0.01}$		
≈ 61.9039	1A	r.t. 62
It will cease producing cloth in February, 2000		
(ii) The total amount of cloth produced during the lifespan of the machine		
$=\int_0^{61.904}x\,dt$	1M	Accept $\int_0^{62}x\,dt$
$=\int_0^{61.904}(100e^{-0.01t}-65e^{-0.02t}-35)\,dt$		
$=-10000e^{-0.01t}+\frac{65}{0.02}e^{-0.02t}-35t\Big\vert_0^{61.904}$	1M	
≈ 141 (km)	1A	
(b) Let P be the monthly profit, then		
$P=800x-300x-300$	1A	
$=500x-300$		
$=500(100e^{-0.01t}-65e^{-0.02t}-35)-300$		
$=50000e^{-0.01t}-32500e^{-0.02t}-17800$		
$\frac{dP}{dt}=-500e^{-0.01t}+650e^{-0.02t}$	1M	
$\frac{dP}{dt}=0$ when $650e^{-0.02t}=500e^{-0.01t}$		
or $t=t_0$ where $t_0=\frac{1}{0.01}\ln\left(\frac{650}{500}\right)\approx 26.2364$	1A	r.t. 26
$\left.\frac{d^2P}{dt^2}\right\vert_{t=t_0}=(5e^{-0.01t}-13e^{-0.02t})\vert_{t=t_0}\approx -3.85<0$	1M	For proving max.
Hence P is maximum when $t=t_0$		
Alternatively		
$x=100e^{-0.01t}-65e^{-0.02t}-35$		
$\frac{dx}{dt}=-e^{-0.01t}+1.3e^{-0.02t}$	1M	
$\frac{dx}{dt}=0$ when $e^{-0.01t}=1.3e^{-0.02t}$		
or $t=t_0$ where $t_0=\frac{\ln 1.3}{0.01}\approx 26.2364$	1A	
$\because P=800x-300x-300=500x-300$	1A	
$\therefore P$ is maximum when x is maximum		
$\left.\frac{d^2x}{dt^2}\right\vert_{t=t_0}=(0.01e^{-0.01t}-0.026e^{-0.02t})\vert_{t=t_0}\approx -0.0077<0$	1M	
Hence P is maximum when $t=t_0$		

Solution	Marks	Remarks
$\because P\vert_{t=26}\approx 1431$		
$P\vert_{t=27}\approx 1430$		
$\therefore$ The greatest monthly profit will be obtained when $t=26$,	1	For checking P when $t=26, 27$
i.e. in February, 1997.		
The greatest monthly profit is US$1431.	1A	Accept $P\vert_{t=t_0}\approx 1431$. r.t. 1431
(c) If $P=500$, then		
$500=50000e^{-0.01t}+32500e^{-0.02t}-17800$	1A	
$5=500e^{-0.01t}-325e^{-0.02t}-178$		
Alternatively		
$500x-300=500$		
$x=1.6$		
$100e^{-0.01t}-65e^{-0.02t}-35=1.6$	1A	
$500e^{-0.01t}-325e^{-0.02t}-183=0$		
Put $y=e^{-0.01t}$,		
$325y^2-500y+183=0$		
$(65y-61)(5y-3)=0$		
$y=\frac{61}{65}$ or $\frac{3}{5}$		
$e^{-0.01t}=\frac{61}{65}$ or $\frac{3}{5}$		
$t=\frac{1}{-0.01}\ln\frac{61}{65}$ or $\frac{1}{-0.01}\ln\frac{3}{5}$		
≈ 6.35 or 51.08	1A	
$\because$ P is increasing when $t\approx 6.35$	1	
(OR The machine has not yet reached its production climax when $t\approx 6.35$)		
$\therefore$ The machine should be replaced when $t\approx 51.08$, i.e. in April, 1999.	1A	

Solution	Marks	Remarks
(a) Amount of pollutant		
$= \int_0^8 r(t)\,dt$		
$\approx \frac{2}{2}[0+2(11+32+59)+90]$	1M	
$= 294$ (units)	1A	
(b) (i) $r = at^b$		
$\ln r = \ln a + b\ln t$	1M	For taking logarithm

$\ln t$	0.69	1.39	1.79	2.08
$\ln r$	2.40	3.47	4.08	4.50

1A — For entries in the table



Solution	Marks	Remarks
	1	For the graph
From the graph,		
$\ln a = 1.35$		Accept 1.3 - 1.4
$a = 3.9$	1A	or 3.7 - 4.1 respectively
$b = \frac{4.50-2.40}{2.08-0.69} = 1.5$	1A	Accept 1.4 - 1.6

Solution	Marks	Remarks
(b) (ii) Amount of pollutant		
$= \int_0^8 3.9t^{1.5}\,dt$	1M	
$= \left[\frac{3.9}{2.5}t^{2.5}\right]_0^8$	1M	
$= \frac{3.9}{2.5}\times 8^{2.5}$		
$= 282.4$ (units)	1A	

<u>In general, accept</u>

Amount of pollutant

$= \int_0^8 at^b\,dt$

$= \left[\frac{a}{b+1}t^{b+1}\right]_0^8$

$= \frac{a}{b+1}8^{b+1}$ where $a \in (3.7, 4.1)$, $b \in (1.4, 1.6)$

$\in (226.7, 351.4)$

Solution	Marks	Remarks
(c) Amount of pollutant after x months		
$= \int_0^x 3.9t^{1.5}\,dt$	1M	
$= \left[\frac{3.9}{2.5}t^{2.5}\right]_0^x$	1M	
$= \frac{3.9}{2.5}x^{2.5}$	1A	

<u>In general, accept</u>

...

$= \frac{a}{b+1}x^{b+1}$ where $a \in (3.7, 4.1)$, $b \in (1.4, 1.6)$

Solution	Marks	Remarks
The lake will "die" after x months if		
$\frac{3.9}{2.5}x^{2.5} = 1000$ (or $\frac{3.9}{2.5}x^{2.5} \geq 1000$)	1M	
$x = \left(\frac{2.5\times 1000}{3.9}\right)^{\frac{1}{2.5}}$		
≈ 13	1A	

<u>In general, accept</u>

...

$x = \left(\frac{1000(b+1)}{a}\right)^{\frac{1}{b+1}}$ where $a \in (3.7, 4.1)$, $b \in (1.4, 1.6)$

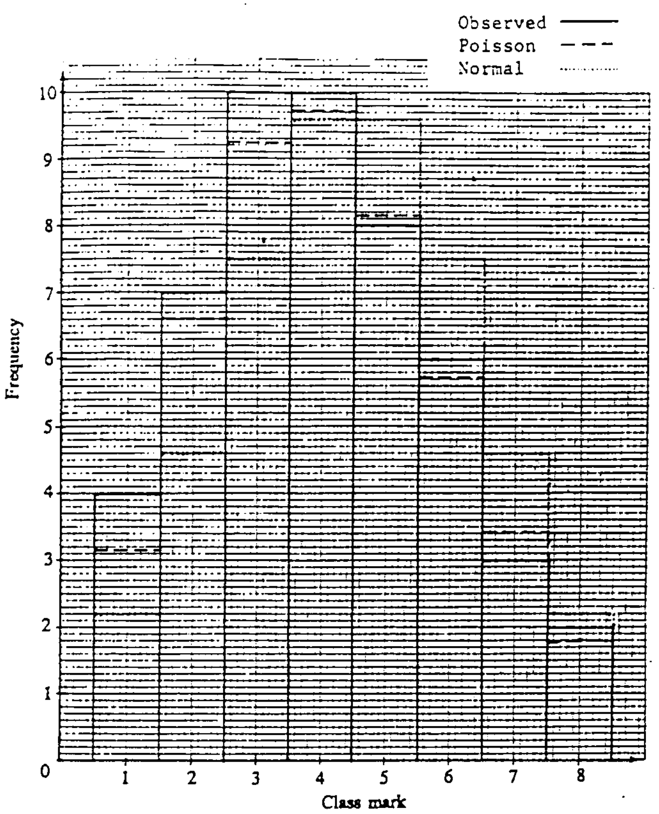
$x \in (12, 15)$

Solution	Marks	Remarks
11. (a) (i) Let X be the number of dry days in a week.		
$X \sim Bin(7,\ 0.3)$	1M	
$f_X(x) = \binom{7}{x}(0.3)^x(0.7)^{7-x}$ for $x=0,1,2,\ldots,7$	1A	
The prob. of having exactly 3 dry days in a week is		
$f_X(3) = \binom{7}{3}(0.3)^3(0.7)^4 = 0.2269$	1A	
(ii) Let Y be the no. of days elapsed until the 1st humid day.		
$Y \sim Geom(0.7)$	1M	
$E(Y) = \frac{1}{0.7}$	1A	
Hence the mean no. of dry days before the next humid day is		
$E(Y) - 1 = \frac{1}{0.7} - 1 = 0.429$	1A	
(iii) The prob. of having 2 or more humid days before the next dry day is		
$1 - 0.3 - (0.7)(0.3)$	1A	
$= 1 - 0.51$		
$= 0.49$	1A	
Alternatively		
$\sum_{k=2}^{\infty}(0.3)(0.7)^k$	1A	
$= (0.3)(0.7)^2[1+0.7+(0.7)^2+\ldots]$		
$= (0.3)\frac{(0.7)^2}{1-0.7}$		
$= 0.49$	1A	
(b) Let a dry day and a humid day be denoted by D and H respectively.		
(i) 19th-20th-21st : D-H-D		
P(H on 20th, D on 21st | D on 19th)		
$= (1-0.9)(1-0.8)$	1M	
$= 0.02$	1A	
(ii) 19th-20th-21st : D-H-D or D-D-D		
P(D on 21st | D on 19th)		
$= 0.02 + (0.9)(0.9)$	1M	
$= 0.83$	1A	
(iii) P(H on 20th | D on 19th and 21st)		
$= \frac{0.02}{0.83}$	2M	1 for nominator, 1 for denominator
$= 0.02410$	1A	

Solution	Marks	Remarks
12. Table 2 Observed and expected frequencies	3A	Col.3: 1A for the 1st correct ans., 1A for the 2nd correct ans, 1A for last 2 correct ans.
	2A	Col.4: 1A for any 2 correct ar.

Number of Bankdrafts Sold	Observed Frequency	Expected Frequency	
		Poisson	Normal
1	4	3.149	2.200
2	7	[redacted]	4.595
3	10	9.258	[redacted]
4	10	[redacted]	[redacted]
5	8	8.166	9.575
6	6	[redacted]	7.490
7	3	3.430	[redacted]
8	2	[redacted]	[redacted]

Solution	Marks	Remarks
(a) Note: Under Po(4.2), $f(x) = \frac{e^{-4.2}4.2^x}{x!}$,		
expected frequency $= 50f(x)$.		
(b) (1) $P(X<1.5) = \frac{3.340}{50}$		
$P(Z<\frac{1.5-\mu}{\sigma}) = 0.0668$		
$\Rightarrow \frac{1.5-\mu}{\sigma} = -1.5$	1A	
(2) $P(X<2.5) = \frac{4.595+3.340}{50}$		
$P(Z<\frac{2.5-\mu}{\sigma}) = 0.1587$		
$\Rightarrow \frac{2.5-\mu}{\sigma} = -1$	1A	
Solving the equations, we have		
$\mu = 4.5$,	1A	
$\sigma = 2$, $\sigma^2 = 4$.	1A	
(c) With reference to the histograms (P.T.O.), the Poisson distribution, Po(4.2), is more suitable for fitting the observed data.	1	
(d) (i) $1 - \sum_{x=0}^{3}\frac{e^{-4.2}4.2^x}{x!} = 0.6046$	1M	
(ii) $P(X>3.5) = P(Z>-0.5) = 0.6915$	1M	

Solution	Marks	Remarks
(c)		
	1	For labels
	1M	For using histogram
	1	For accuracy and appropriateness

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Solution	Marks	Remarks
13. (a) $\because \quad P\left(Z<\frac{c_1-10}{0.4}\right)=0.95$	1M	
$\therefore \quad \frac{c_1-10}{0.4}=1.645$	1A	Accept 1.64-1.65
$c_1=10.658$	1A	Accept 10.656-10.65
(b) $\because \quad P\left(Z<\frac{c_2-12.3}{0.6}\right)=0.01$	1M	
$\therefore \quad \frac{c_2-12.3}{0.6}=-2.327$	1A	-2.33 to -2.32
$c_2=10.9038$	1A	10.902 to 10.908
(c) Given the batch is produced under the favourable condition, the required probability is $P(c_1<X \text{ and } X<c_2)$		
$=P(10.658<X<10.9038)$	1M	
$=P\left(\frac{10.658-10}{0.4}<Z<\frac{10.9038-10}{0.4}\right)$	1M	
$=P(1.645<Z<2.2595)$		
$=0.4881-0.45$	1M	For subtraction
$=0.0381$	1A	
(d) $P(X<c_3 \text{ where } \sigma=0.4,\ \mu=10)=P(X\geq c_3 \text{ where } \sigma=0.6,\ \mu=12.3)$		
i.e. $-\left(\frac{c_3-10}{0.4}\right)=\frac{c_3-12.3}{0.6}$	1M + 1A	
$c_3=10.92$	1A	
(e) The probability would be minimized if μ is in the middle of the 2 limits,		
i.e. $\mu=\frac{10.8+9.4}{2}$	1M	
$=10.1$	1A	

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