94-ASL MATHS & STAT

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG ADVANCED LEVEL EXAMINATION 1994

MATHEMATICS AND STATISTICS AS-LEVEL

9.00 am-12.00 noon (3 hours)

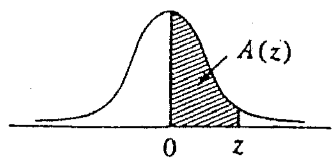
This paper must be answered in English

1. This paper consists of Section A and Section B.
2. Answer ALL questions in Section A, using the AL(C1) answer book.
3. Answer any FOUR questions in Section B, using the AL(C2) answer book.
4. Unless otherwise specified, numerical answers should either be exact or given to 4 decimal places.

Table. Area under the Standard Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the proportion of the area under the entire curve which is between $z = 0$ and a positive value of z. Areas for negative values of z are obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

SECTION A (40 marks)
Answer ALL questions in this section.
Write your answers in the AL(C1) answer book.

1. (a) Write down the sample space of the sex patterns of the children of a 2-child family in the order of their ages.
(You may use B to denote a boy and G to denote a girl.)

(b) Assume that having a boy or having a girl is equally likely. It is known that a family has two children and they are not both girls.

(i) Write down the sample space of the sex patterns of the children in the order of their ages.

(ii) What is the probability that the family has two sons?

(4 marks)

2. The population size x of an endangered species of animals is modelled by the equation

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 3x = 0 ,$$

where t denotes the time.

It is known that $x = 100e^{kt}$ where k is a negative constant. Determine the value of k .

(5 marks)

3. Jack climbs along a cubical framework from a corner A to meet Jill at the opposite corner B . The framework, shown in Figure 1, is formed by joining bars of equal length. Jack chooses randomly a path of the shortest length to meet Jill. An example of such a path, which can be denoted by

Right - Up - Forward - Up - Right - Forward ,

is also shown in Figure 1.

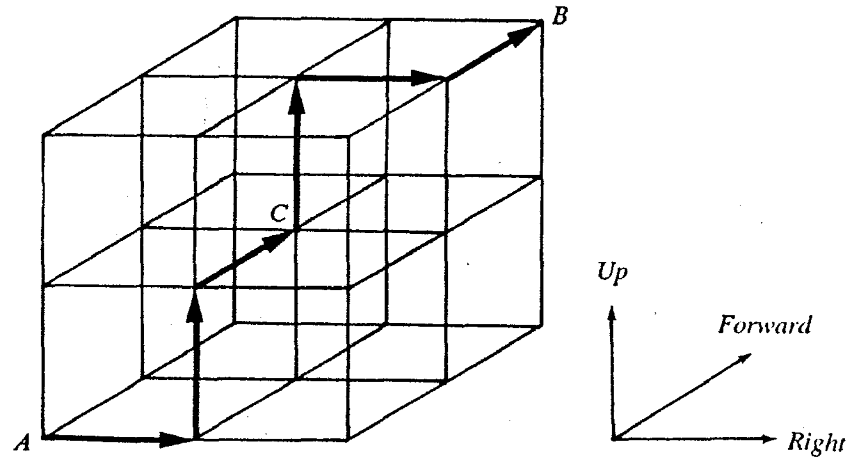


Figure 1

(a) Find the number of shortest paths from A to B .

(b) If there is a trap at the centre C of the framework which catches anyone passing through it,

(i) find the number of shortest paths from A to C ,

(ii) hence find the probability that Jack will be caught by the trap on his way to B .

(5 marks)

4. Figure 2 shows the cumulative frequency polygon of weights (in kg) for a group of 100 students.

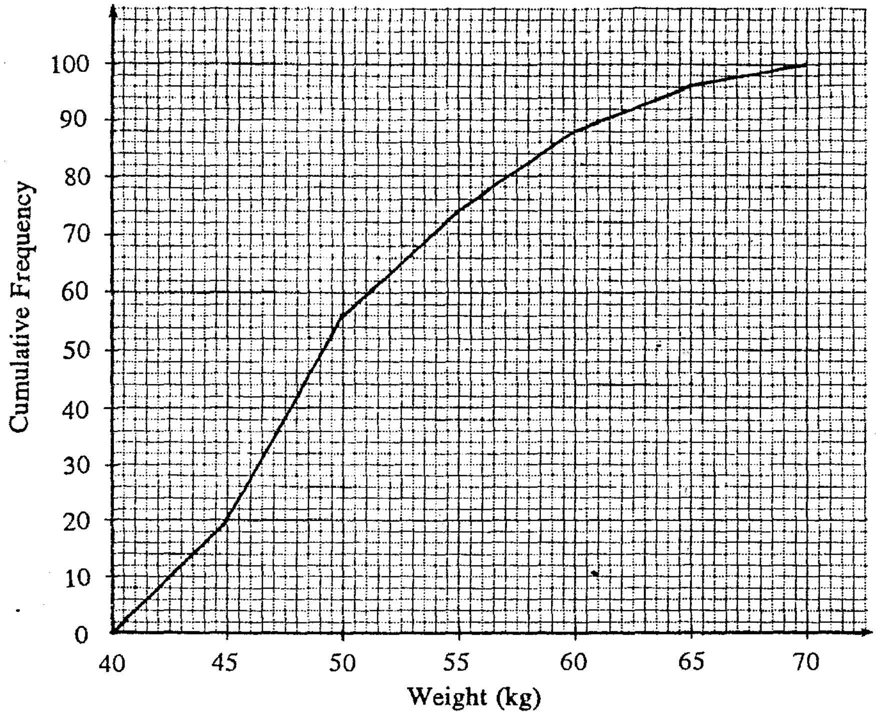


Figure 2 Cumulative frequency polygon of weights for a group of 100 students

(a) Use the graph paper on Page 4 to draw a histogram of the weights.

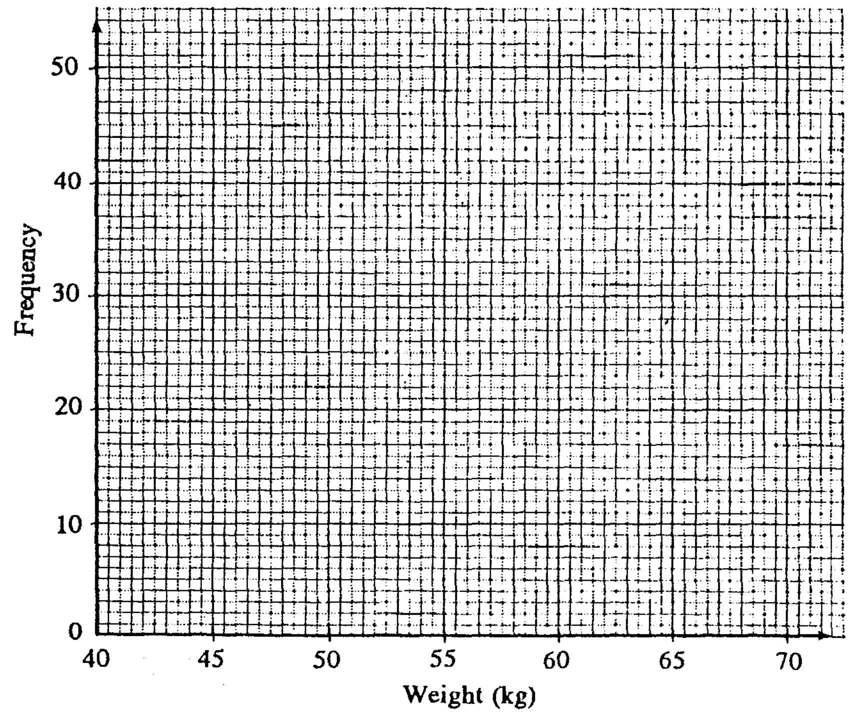
(b) Determine the inter-quartile range of the weights from the cumulative frequency polygon.

(c) Determine the mean weight from the histogram.

(6 marks)

Candidate Number	**Centre Number**	**Seat Number**	
			Page Total

4.(Cont'd) **Fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**



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5. The rate of spread of an epidemic can be modelled by the equation

$$\frac{dx}{dt} = 3t\sqrt{t^2+1} ,$$

where x is the number of people infected by the epidemic and t is the number of days which have elapsed since the outbreak of the epidemic. If $x = 10$ when $t = 0$, express x in terms of t.

(6 marks)

6. (a) Use the exponential series to find a polynomial of degree 6 which approximates $e^{-\frac{x^2}{2}}$ for x close to 0.

Hence estimate the integral $\int_0^1 e^{-\frac{x^2}{2}} dx$.

(b) It is known that the area under the standard normal curve between $z = 0$ and $z = a$ is $\int_0^a \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$. Use the result of (a) and the normal distribution table to estimate, to 3 decimal places, the value of π .

(7 marks)

7. In asking some sensitive questions such as "Are you homosexual?", a *randomized response technique* can be applied: The interviewee will be asked to draw a card at random from a box with one red card and two black cards and then consider the statement 'I am homosexual' if the card is red and the statement 'I am not homosexual' otherwise. He will give the response either 'True' or 'False'. The colour of the card drawn is only known to the interviewee so that nobody knows which statement he has responded to. Suppose in a survey, 790 out of 1 200 interviewees give the response 'True'.

(a) Estimate the percentage of persons who are homosexual.

(b) For an interviewee who answered 'True', what is the probability that he is really homosexual?

(7 marks)

SECTION B (60 marks)
Answer any FOUR questions from this section. Each question carries 15 marks.
Write your answers in the AL(C2) answer book.

8. Consider the curve

$$C: \quad y = \frac{x+1}{x-2} \qquad (x \neq 2).$$

(a) Find $\frac{dy}{dx}$.

(2 marks)

(b) Find the equations of the horizontal and vertical asymptotes to the curve C.

(3 marks)

(c) Sketch the curve C and indicate the asymptotes and intercepts.

(5 marks)

(d) Find the area of the region bounded by the curve C, the negative x-axis and the negative y-axis.

(5 marks)

9. A textile factory plans to install a weaving machine on 1st January 1995 to increase its production of cloth. The monthly output x (in km) of the machine, after t months, can be modelled by the function

$$x = 100e^{-0.01t} - 65e^{-0.02t} - 35.$$

(a) (i) In which month and year will the machine cease producing any more cloth ?

(ii) Estimate the total amount of cloth, to the nearest km, produced during the lifespan of the machine.

(5 marks)

(b) Suppose the cost of producing 1 km of cloth is US\$300; the monthly maintenance fee of the machine is US\$300 and the selling price of 1 km of cloth is US\$800. In which month and year will the greatest monthly profit be obtained? Find also the profit, to the nearest US\$, in that month.

(6 marks)

(c) The machine is regarded as 'inefficient' when the monthly profit falls below US\$500 and it should then be discarded. Find the month and year when the machine should be discarded. Explain your answer briefly.

(4 marks)

10. A chemical plant discharges pollutant to a lake at an unknown rate of $r(t)$ units per month, where t is the number of months that the plant has been in operation.

Suppose $r(0) = 0$.

The government measured $r(t)$ once every two months and reported the following figures:

t	2	4	6	8
$r(t)$	11	32	59	90

(a) Use the trapezoidal rule to estimate the total amount of pollutant which entered the lake in the first 8 months of the plant's operation.

(2 marks)

(b) An environmental scientist suggests that

$$r(t) = at^b ,$$

where a and b are constants.

(i) Use the graph paper on Page 10 to estimate graphically the values of a and b correct to 1 decimal place.

(ii) Based on this scientist's model, estimate the total amount of pollutant, correct to 1 decimal place, which entered the lake in the first 8 months of the plant's operation.

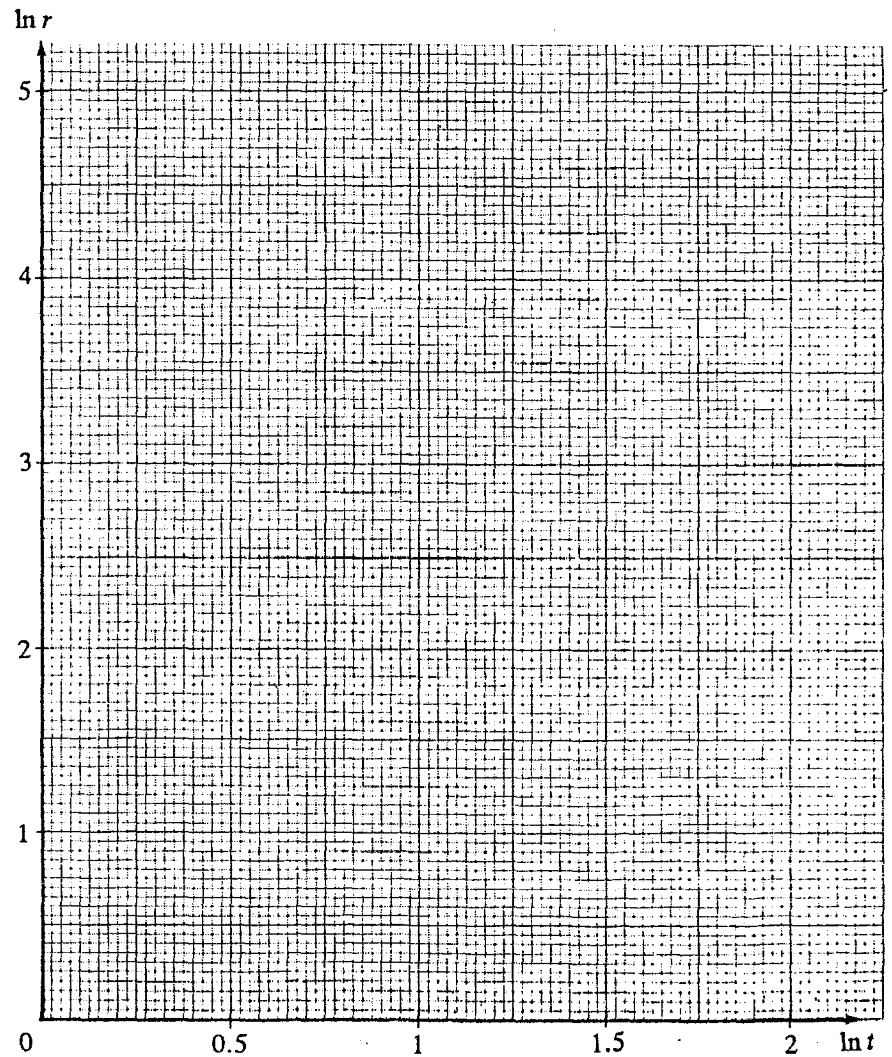
(8 marks)

(c) It is known that no life can survive when 1000 units of pollutant have entered the lake. Adopting the scientist's model in (b), how long does it take for the pollutant from the plant to destroy all life in the lake? Give your answer correct to the nearest month.

(5 marks)

Candidate Number	Centre Number	Seat Number	Page Total

10.(Cont'd) **If you attempt Question 10, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**



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11. A day is regarded as humid if the relative humidity is over 80% and is regarded as dry otherwise. In city K, the probability of having a humid day is 0.7 .

(a) Assume that whether a day is dry or humid is independent from day to day.

(i) Find the probability of having exactly three dry days in a week (7 days).

(ii) What is the mean number of dry days before the next humid day? Give your answer correct to 3 decimal places.

(iii) Today is dry. What is the probability of having two or more humid days before the next dry day?

(8 marks)

(b) After some research, it is known that the relative humidity in city K depends solely on that of the previous day. Given a dry day, the probability that the following day is dry is 0.9 and given a humid day, the probability that the following day is humid is 0.8 .

(i) If it is dry on March 19, what is the probability that it will be humid on March 20 and dry on March 21?

(ii) If it is dry on March 19, what is the probability that it will be dry on March 21?

(iii) Suppose it is dry on both March 19 and March 21. What is the probability that it is humid on March 20?

(7 marks)

12. Table 1 shows the number of bankdrafts sold per working day in a certain branch of a bank.

Table 1 Number of bankdrafts sold

Number of bankdrafts sold	Frequency (Days)
1	4
2	7
3	10
4	10
5	8
6	6
7	3
8	2
	50

(a) It is suggested that the number of bankdrafts sold follows a Poisson distribution with mean 4.2. Fill in the expected frequencies, under this distribution, in column 3 of Table 2 (Page 14). (3 marks)

(b) Another suggestion is that the number sold can be approximated by a normal distribution with mean μ and variance σ^2. Using class intervals $0.5 - 1.5$, $1.5 - 2.5$, ..., $7.5 - 8.5$, some expected frequencies are calculated and shown in the fourth column of Table 2. Given that the expected frequency for the class $-\infty - 1.5$ is 3.340, determine μ and σ^2. Fill in the other expected frequencies.

(6 marks)

(c) Compare the observed and the two expected frequency distributions by drawing histograms on the graph paper on Page 15. Hence determine which of the two distributions is more suitable for fitting the observed data. (4 marks)

(d) Find the probability that the branch will sell 4 or more bankdrafts on a working day under

(i) the model of Poisson distribution in (a);

(ii) the model of normal distribution in (b).

(2 marks)

Candidate Number	Centre Number	Seat Number	Page Total

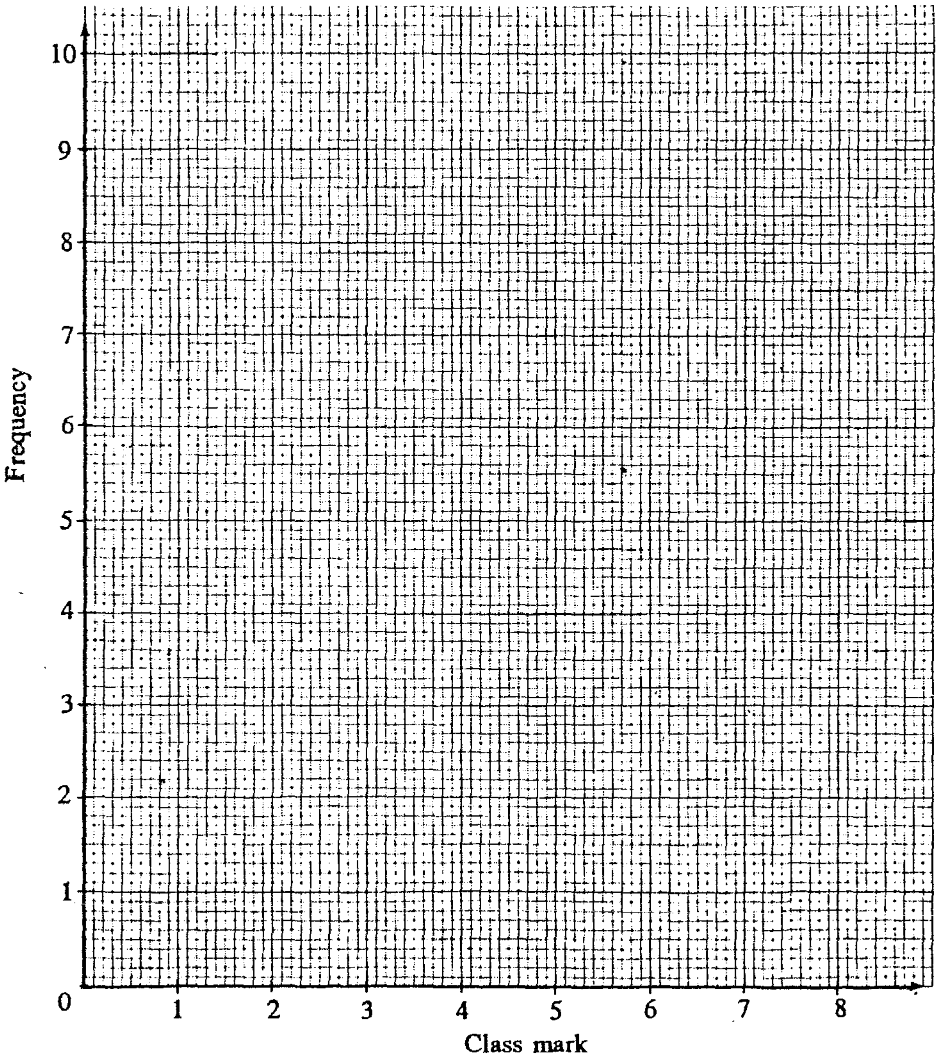
12.(Cont'd) **If you attempt Question 12, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**

Table 2 Observed and expected frequencies of bankdrafts sold

Number of Bankdrafts Sold	Observed Frequency	Expected Frequency	
		Poisson	Normal
1	4	3.149	2.200
2	7		4.595
3	10	9.258	
4	10		
5	8	8.166	9.575
6	6		7.490
7	3	3.430	
8	2		

12.(Cont'd)

Page Total



13. Batches of screws are produced by a manufacturer under two different sets of conditions, favourable and unfavourable. If screws are produced under favourable conditions, the diameters of the screws will follow a normal distribution with mean 10 mm and standard deviation 0.4 mm . If screws are produced under unfavourable conditions, the diameters of the screws will follow a normal distribution with mean 12.3 mm and standard deviation 0.6 mm . A batch of screws is examined by measuring the diameter X mm of a screw randomly selected from the batch.

(a) The batch is classified as acceptable by the manufacturer if $X < c_1$ and as unacceptable if otherwise. The value c_1 satisfies $P(X < c_1) = 0.95$ under favourable conditions. Determine the value of c_1 . (3 marks)

(b) The buyer uses a different criterion instead. He classifies the batch as acceptable if $X < c_2$ and as unacceptable if otherwise. The value c_2 satisfies $P(X < c_2) = 0.01$ under unfavourable conditions. Determine the value of c_2 . (3 marks)

(c) For a batch of screws produced under favourable conditions and based on the same measurement of a screw, find the probability that the batch will be classified as unacceptable by the manufacturer, but acceptable by the buyer. (4 marks)

(d) After some negotiation, the manufacturer and the buyer agree to use a common cut-off point c_3 such that $P(X < c_3)$ under favourable conditions is equal to $P(X \geq c_3)$ under unfavourable conditions. Determine the value of c_3 . (3 marks)

(e) The manufacturer and the buyer later agree that a batch will be rejected in the future if $X > 10.8$ (too thick) or $X < 9.4$ (too thin). If the population mean μ mm of the diameters of the screws produced can be modified by adjusting the machine, find μ so that the probability of rejection, $P(X < 9.4 \text{ or } X > 10.8)$, is minimized. (2 marks)

END OF PAPER