2005 Mathematics 1

SECTION A(1) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

1. Make a the subject of the formula $P = ab + 2bc + 3ac$. (3 marks)

2. Simplify $\dfrac{(x^3y)^2}{y^5}$ and express your answer with positive indices. (3 marks)

3. Factorize
   (a) $4x^2 - 4xy + y^2$,
   (b) $4x^2 - 4xy + y^2 - 2x + y$.
   (3 marks)

4. Solve the inequality $\dfrac{-3x+1}{4} > x - 5$.
   Also write down all integers which satisfy both the inequalities $\dfrac{-3x+1}{4} > x - 5$ and $2x + 1 \geq 0$.
   (3 marks)

5. The ratio of the number of marbles owned by Susan to the number of marbles owned by Teresa is $5:2$. Susan has n marbles. If Susan gives 18 of her own marbles to Teresa, both of them will have the same number of marbles. Find n. (3 marks)

6. The cost of a calculator is \$ 160. If the calculator is sold at its marked price, then the percentage profit is 25%.
   (a) Find the marked price of the calculator.
   (b) If the calculator is sold at a 10% discount on the marked price, find the percentage profit or percentage loss.
   (4 marks)

7. The 1st term and the 2nd term of an arithmetic sequence are 5 and 8 respectively. If the sum of the first n terms of the sequence is 3 925, find n. (4 marks)

8. In Figure 1, $ABCDEF$ is a regular six-sided polygon. AC and BF intersect at G. Find x, y and z.

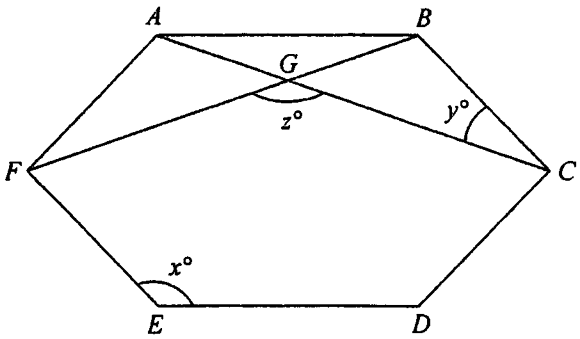

Figure 1

(5 marks)

9. In Figure 2, $OABC$ is a sector with $\overset{\frown}{ABC} = 10\pi$ cm.
   (a) Find OA.
   (b) Find the area of segment ABC.

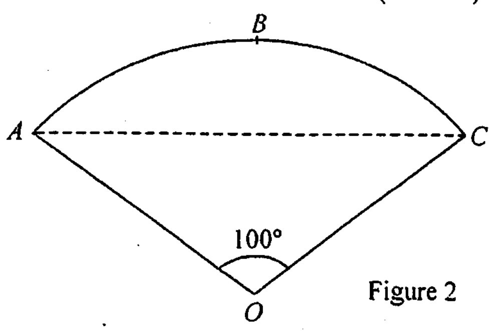

Figure 2

(5 marks)

Section A(2) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

10. It is known that $f(x)$ is the sum of two parts, one part varies as x^3 and the other part varies as x. Suppose $f(2) = -6$ and $f(3) = 6$.

(a) Find $f(x)$. (4 marks)

(b) Let $g(x) = f(x) - 6$.

(i) Prove that $x - 3$ is a factor of $g(x)$.

(ii) Factorize $g(x)$.

(4 marks)

11.

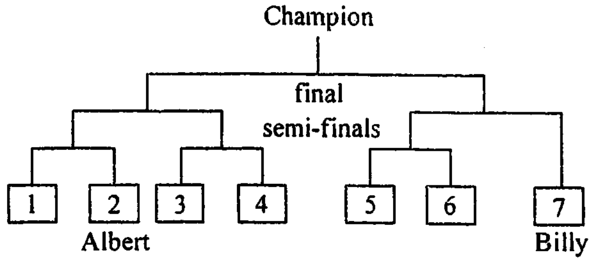

Figure 3

Seven players take part in a men's singles tennis knock-out tournament. They are randomly assigned to the positions 1, 2, 3, 4, 5, 6 and 7. It is known that Albert and Billy are in positions 2 and 7 respectively. The winner of each game proceeds to the next round as shown in Figure 3 and the loser is knocked out. Billy goes straight to the semi-finals. In each game, each player has an equal chance of beating his opponent.

(a) Write down the probability that Albert will reach the semi-finals. (1 mark)

(b) Find the probability that Albert will be the champion. (2 marks)

(c) Find the probability that Albert will fail to reach the final. (3 marks)

(d) Find the probability that Albert will play against Billy in the final. (2 marks)

12. Figure 4 shows a solid consisting of a right circular cone and a hemisphere with a common base. The height and the base radius of the cone are h cm and $(h - 4)$ cm respectively. It is known that the volume of the cone is equal to the volume of the hemisphere.

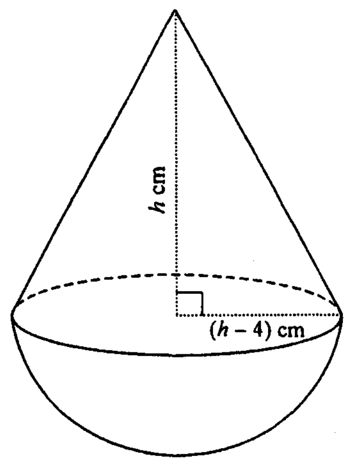

Figure 4

(a) Find h. (3 marks)

(b) Find the total surface area of the solid correct to the nearest cm^2. (3 marks)

(c) If the solid is cut into two identical parts, find the increase in the total surface area correct to the nearest cm^2. (2 marks)

13. In Figure 5 , the straight line L_1 : $2x - y + 4 = 0$ cuts the x-axis and the y-axis at A and B respectively. The straight line L_2 , passing through B and perpendicular to L_1 , cuts the x-axis at C . From the origin O , a straight line perpendicular to L_2 is drawn to meet L_2 at D .

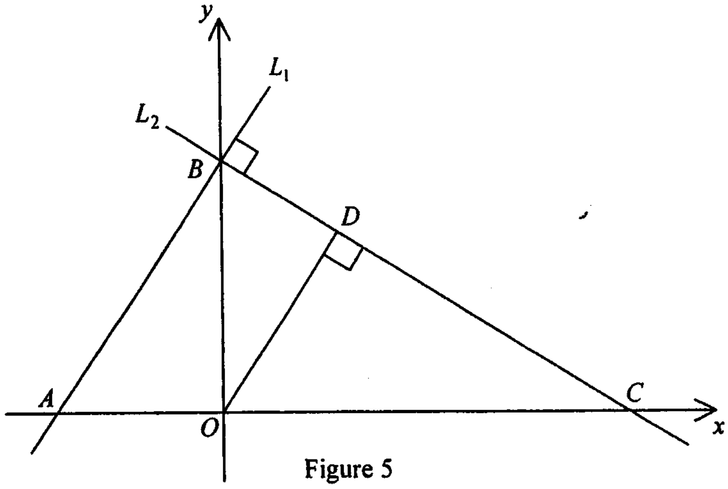

Figure 5

(a) Write down the coordinates of A and B . (2 marks)

(b) Find the equation of L_2 . (3 marks)

(c) Find the ratio of the area of ΔODC to the area of quadrilateral $OABD$. (4 marks)

SECTION B (33 marks)
Answer any THREE questions in this section and write your answers in the spaces provided.
Each question carries 11 marks.

14.

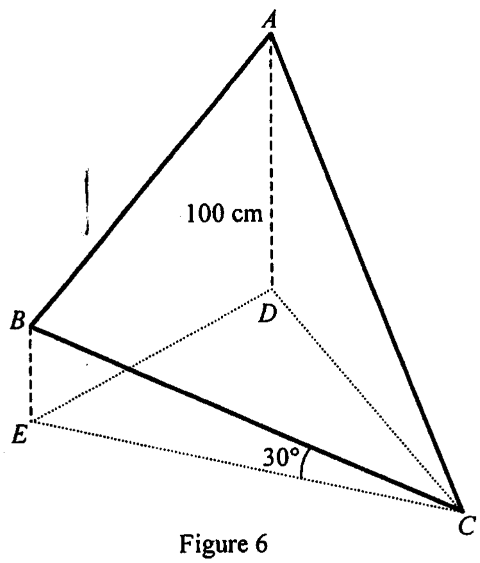

Figure 6

In Figure 6 , a thin triangular board ABC is held with the vertex C on the horizontal ground. D and E are points on the ground vertically below A and B respectively. BC is inclined at an angle of $30°$ with the horizontal. It is known that $AD = 100$ cm , $BC = 120$ cm , $\angle CAB = 60°$ and $\angle ABC = 80°$.

(a) Find BE and CE . (2 marks)

(b) Find AB and AC . (3 marks)

(c) Find $\angle CDE$ and the shortest distance from C to DE . (6 marks)

15. The scores (in marks) obtained by a class of 20 students in a music test are shown below:

84	86	90	93	100
103	120	120	120	121
122	134	134	136	137
144	146	146	146	158

(a) Find the mean, the mean deviation and the standard deviation of the above scores. (4 marks)

(b) Mary is one of the students in the class and her standard score in the music test is 1 . Is Mary one of the top 20% students of the class in the music test? Explain your answer. (3 marks)

(c) (i) If one student in the class withdraws, find the probability that the mean of the scores obtained by the remaining 19 students in the music test is 122 marks.

(ii) If two students in the class withdraw, find the probability that the mean of the scores obtained by the remaining 18 students in the music test is 122 marks.

(4 marks)

16. Peter borrows a loan of \$200 000 from a bank at an interest rate of 6% per annum, compounded monthly. For each successive month after the day when the loan is taken, loan interest is calculated and then a monthly instalment of $\$x$ is immediately paid to the bank until the loan is fully repaid (the last instalment may be less than $\$x$), where $x < 200\,000$.

(a) (i) Find the loan interest for the 1st month.

(ii) Express, in terms of x , the amount that Peter still owes the bank after paying the 1st instalment.

(iii) Prove that if Peter has not yet fully repaid the loan after paying the nth instalment, he still owes the bank $\$\left\{200\,000(1.005)^n - 200x[(1.005)^n - 1]\right\}$. (6 marks)

(b) Suppose that Peter's monthly instalment is \$1 800 (the last instalment may be less than \$1 800).

(i) Find the number of months for Peter to fully repay the loan.

(ii) Peter wants to fully repay the loan with a smaller monthly instalment. He requests to pay a monthly instalment of \$900 . However, the bank refuses his request. Why?

(5 marks)

17.

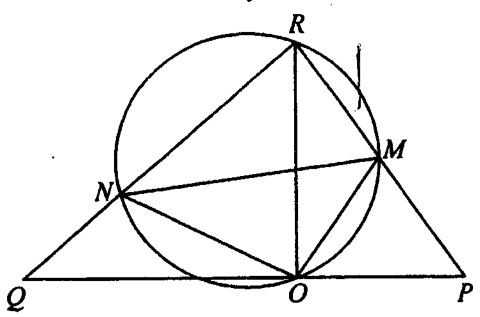

Figure 7(a)

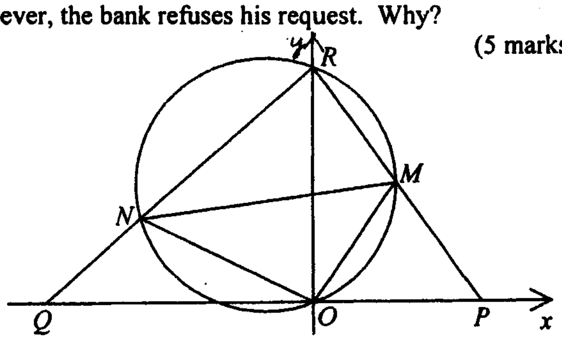

Figure 7(b)

(a) In Figure 7(a), MN is a diameter of the circle $MONR$. The chord RO is perpendicular to the straight line POQ . RNQ and RMP are straight lines.

(i) By considering triangles OQR and ORP , prove that $OR^2 = OP \cdot OQ$.

(ii) Prove that $\Delta MON \sim \Delta POR$.

(5 marks)

(b) A rectangular coordinate system, with O as the origin, is introduced to Figure 7(a) so that R lies on the positive y-axis and the coordinates of P and Q are (4, 0) and (–9, 0) respectively (see Figure 7(b)).

(i) Find the coordinates of R .

(ii) If the centre of the circle $MONR$ lies in the second quadrant and $ON = \frac{3\sqrt{13}}{2}$, find the radius and the coordinates of the centre of the circle $MONR$.

(6 marks)