Hong Kong Certificate of Education Examination
Mathematics Paper 1

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Makers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

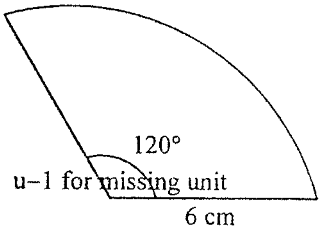
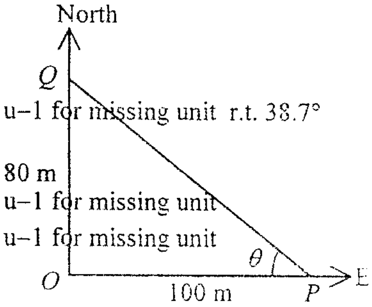
4. Use of notation different from those in the marking scheme should not be penalized.

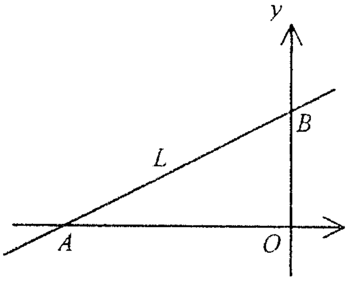
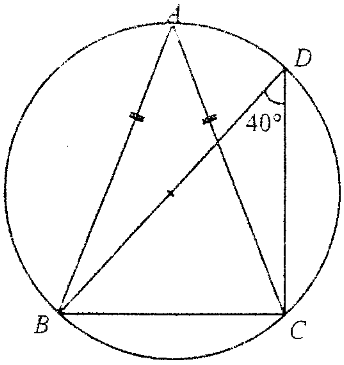
5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

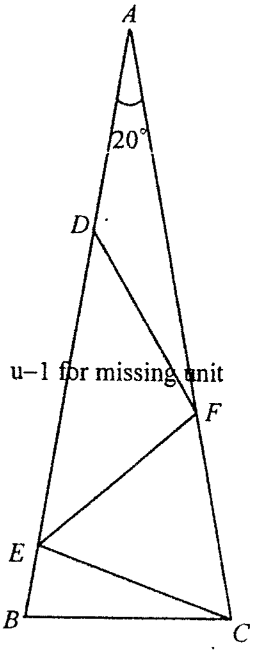
6. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).
   a. The symbol (*u-1*) should be used to denote 1 mark deducted for *u*. At most deduct ***1 mark*** for *u* for the whole paper.
   b. The symbol (*pp-1*) should be used to denote 1 mark deducted for *pp*. At most deduct ***2 marks*** for *pp* for the whole paper. For similar *pp*, deduct 1 mark for the first time that it occurs. Do not penalize candidates twice in the paper for the same *pp*.
   c. At most deduct 1 mark in each question. Deduct the mark for *u* first if both marks for *u* and *pp* may be deducted in the same question.
   d. In any case, do not deduct any marks for *pp* or *u* in those steps where candidates could not score any marks.

7. Marks entered in the Page Total Box should be the NET total scored on that page.

8. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to', 'f.t.' stands for 'follow through' and 'or equivalent' means 'accepting equivalent forms of the equation which has been simplified and without uncollected like terms'. Steps which can be skipped are shaded whereas alternative answers are enclosed with $\boxed{\text{rectangles}}$. All fractional answers must be simplified.

Solution	Marks	Remarks
1. $\frac{(ab^2)^2}{a^5} = \frac{(a^2)(b^2)^2}{a^5}$ $\frac{(ab^2)^2}{a^5} = \frac{a^2b^4}{a^5}$	1M	$(xy)^n = x^n y^n$
$= \frac{b^4}{a^{5-2}}$ $= \frac{b^4}{a^{5-2}}$	1M	$\frac{x^m}{x^n} = x^{m-n}$
$= \frac{b^4}{a^3}$ $= \frac{b^4}{a^3}$	1A	
	----------(3)	
2. Area $= \frac{120}{360}\cdot\pi(6)^2$	1M + 1A	1M for $\frac{120}{360}$, 1A for area of circle
$= 12\pi$ cm^2	1A	u–1 for missing unit
The angle at the centre is $120\times\frac{\pi}{180} = \left(\frac{2\pi}{3}\right)$	1A	
Area $= \frac{1}{2}\cdot\frac{2\pi}{3}\cdot 6^2$	1M	
$= 12\pi$ cm^2	1A	u–1 for missing unit
	----------(3)	
3. (a) $\tan\theta = \frac{80}{100}$	1A	
$\theta \approx 38.66° \approx 38.7°$ (Accept $\theta = 0.675$)	1A	u–1 for missing unit r.t. 38.7°
(b) The bearing of P from Q is $90° + 38.7° = 128.7° \approx 129°$	1M	u–1 for missing unit
S 51.3° E.	1M	u–1 for missing unit
	----------(3)	
4. (a) $f(2) = 2^3 - 2(2)^2 - 9(2) + 18$ $= 0$	1A	
(b) $x-2$ is a factor of $f(x)$. $\therefore$ $f(x) = (x-2)(x^2-9)$	1M	for $f(x) = (x-2)(ax^2+bx+c)$
$= (x-2)(x-3)(x+3)$	1A	
	----------(3)	
5. (a) Mean $= \frac{4+4+5+6+8+12+13+13+13+18}{10} = 9.6$	1A	
(b) Mode = 13	1A	
(c) Median $= \frac{8+12}{2} = 10$	1A	
(d) Standard deviation = 4.59	1A	r.t. 4.59
	----------(4)	

Solution	Marks	Remarks
6. (a) The radius of the new circle is $8(1.1)$ $= 8.8$ cm	1A	
The area of the new circle is $\pi(8.8)^2 = 77.44\pi$ cm^2	1A	u–1 for missing unit
(b) The percentage increase in area is $\frac{77.44\pi - 64\pi}{64\pi} \times 100\%$ $\quad \boxed{\begin{array}{l}\frac{1.1^2-1}{1}\times 100\% \\ = 21\%\end{array}}$	1M	accept without 100%
$= 21\%$	1A	
	----------(4)	
7. (a) $3x + 6 \geq 4 + x$ $2x \geq -2$ $x \geq -1$	1A	
(b) For $2x - 5 < 0$, $x < \frac{5}{2}$.	1A	
Hence $-1 \leq x < \frac{5}{2}$	1A	
The required integers are -1 , 0 , 1 , 2 .	1A	
	----------(4)	
8. (a) The coordinates of A are $(-8, 0)$	1A	
The coordinates of B are $(0, 4)$	1A	
(b) Let the coordinates of the mid-point of AB be (x, y) .		
$x = \frac{-8+0}{2} = -4$ $y = \frac{0+4}{2} = 2$	1M	1M for mid-point formula
∴ The mid-point is $(-4, 2)$.	1A	
	----------(4)	
9. $\angle BAC = 40^\circ$	1A	
∵ $AB = AC$ ∴ $\angle ABC = \frac{180^\circ - 40^\circ}{2}$ $= 70^\circ$ $\quad \boxed{\begin{array}{l}\angle ACB = \frac{180^\circ - 40^\circ}{2} \\ = 70^\circ\end{array}}$	1A	
∵ BD is a diameter ∴ $\angle BCD = 90^\circ$ $\quad \boxed{\begin{array}{l}\angle ACD = 90^\circ - 70^\circ \\ = 20^\circ\end{array}}$	1A	
∴ $\angle CBD = 90^\circ - 40^\circ = 50^\circ$	1A	
∴ $\angle ABD = \angle ABC - \angle CBD$ $= 70^\circ - 50^\circ$ $= 20^\circ$ $\quad \boxed{\begin{array}{l}\angle ABD = \angle ACD \\ = 20^\circ\end{array}}$	1A	u–1 for missing unit
	----------(5)	



Solution	Marks	Remarks
10. (a) ∵ $AB = AC$		
∴ $\angle B = \frac{180° - 20°}{2} = 80°$	1A	
∵ $BC = CE$		
∴ $\angle CEB = \angle B = 80°$		
∴ $\angle BCE = 180° - 80° - 80° = 20°$	1M	
∴ $\angle ECF = \angle ACB - \angle BCE$		
$= 60°$	1M	
∵ $CE = EF$		
∴ $\angle CEF = 60°$	1A	u−1 for missing unit
	----------(4)	
(b) $\angle DEF = 180° - 60° - 80°$ (adj. ∠s on st. line) [直線上的鄰角]		
$= 40°$		
∵ $EF = FD$		
∴ $\angle FDE = \angle DEF$		
$= 40°$ (base ∠s of isos. Δ) [等腰Δ底角]		
In ΔADF,		
$\angle DFA = 40° - 20°$ (ext ∠ of Δ) [Δ的外角]		
$= 20°$		
$= \angle DAF$		
∴ $\angle DFE = 180° - 40° - 40°$ (∠ sum of Δ)		[Δ內角和]
$= 100°$		
$\angle AFD = 180° - 100° - 60°$ (adj. ∠s on st. line)		[直線上的鄰角]
$= 20°$		
∴ $\angle DFA = \angle DAF$		
$\angle CFE = 60°$ (∠ of equilateral Δ)		[等邊Δ性質]
$\angle AEF = 60° - 20° = 40°$ (ext ∠ of ΔAEF)		[Δ的外角]
∴ $\angle EDF = 40°$ (base ∠s of isos. Δ)		[等腰Δ底角]
∴ $\angle AFD = 40° - 20°$ (ext ∠ of ΔADF)		[Δ的外角]
$= 20°$		
∴ $AD = DF$ (base ∠s of Δ =)		[等角對邊相等] [底角相等] [等邊對等角] [等角對等邊] [等腰Δ底角等的逆定理]

Marking Scheme :		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct angle (e.g. $\angle AEF$, $\angle DFE$) and with correct reason.	1

----------(3)

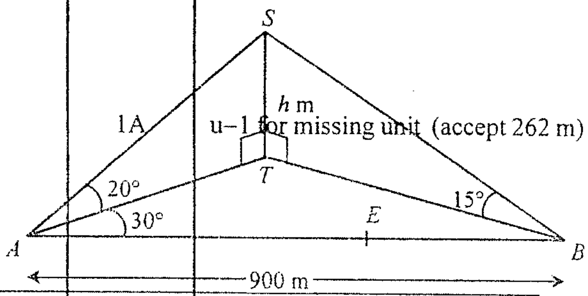
Solution	Marks	Remarks
11. (a) Let $A = aP + bP^2$, where a and b are constants.	1A	
Sub. $P = 24$, $A = 36$, $24a + 576b = 36$ $2a + 48b = 3$ .... (1) Sub. $P = 18$, $A = 9$, $18a + 324b = 9$ $2a + 36b = 1$ .... (2)	1M	for substitution (either)
Solving (1) and (2) $a = -\frac{5}{2}$ $b = \frac{1}{6}$	1A	for both
$\therefore$ $A = -\frac{5}{2}P + \frac{1}{6}P^2$		
	----------(3)	
(b) (i) When $A = 54$, $-\frac{5}{2}P + \frac{1}{6}P^2 = 54$	1M	
$P^2 - 15P - 324 = 0$ $P = 27$ or $P = -12$ (rejected)	1A	
$\therefore$ the required perimeter is 27 cm.		
(ii) Let P' cm be the perimeter of the gold bookmark. $\left(\frac{P'}{27}\right)^2 = \frac{8}{54}$	1M+1A	1M for $\left(\frac{P'}{P}\right)^2 = \frac{8}{54}$
$P' = 6\sqrt{3}$ (≈ 10.4)	1A	r.t. 10.4
The perimeter of the gold bookmark is $6\sqrt{3}$ (≈ 10.4) cm.		
	----------(5)	

Solution	Marks	Remarks
12. (a)		

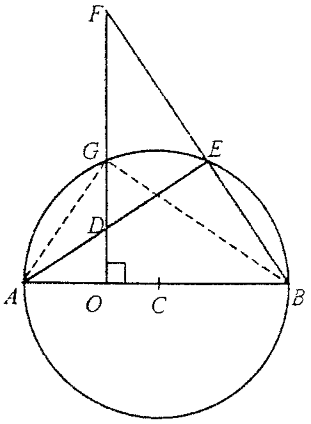
Number of books read (x)	Number of participants	Award
$0 < x \leq 5$	66	Certificate
$5 < x \leq 15$	**34**	Book coupon
$15 < x \leq 25$	64	Bronze medal
$25 < x \leq 35$	**26**	Silver medal
$35 < x \leq 50$	10	Gold medal

Solution	Marks	Remarks
	1A	for both
	----------(1)	
(b) Lower quartile = 3.8 Upper quartile = 22.8		
Inter-quartile range = 22.8 − 3.8	1M	(22→23) − (3→4)
= 19	1A	r.t. 19
	----------(2)	
(c) (i) The number of participants who won medals, $64+26+10=100$ The number of participants who won gold medals is 10. The probability that they both won gold medals		
$=\frac{10}{100}\times\frac{9}{99}$		1M for $\frac{p}{q}\times\frac{p-1}{q-1}$, where $p<q$
$=\frac{1}{110}$		1A 0.00909
(ii) Both won bronze medals $P_1=\frac{64}{100}\times\frac{63}{99}=\frac{112}{275}$ Both won silver medals $P_2=\frac{26}{100}\times\frac{25}{99}=\frac{13}{198}$	1A	0.4073 for both 0.06566
The probability that they won different medals $=1-\frac{1}{110}-\frac{112}{275}-\frac{13}{198}$	2M	for $1-(c)(i)-P_1-P_2$
$=\frac{1282}{2475}$	1A	0.518
P(B and S) = $\frac{64}{100}\times\frac{26}{99}\times 2$ P(B and G) = $\frac{64}{100}\times\frac{10}{99}\times 2$ P(S and G) = $\frac{26}{100}\times\frac{10}{99}\times 2$		
P(different medals) = P(B and S) + P(B and G) + P(S and G)	2M+1A	2M for sum of three different cases $(P_1'\times 2+P_2'\times 2+P_3'\times 2)$
$=\frac{1282}{2475}$	1A	0.518
	----------(6)	

Solution	Marks	Remarks
13. (a) Area of $\Delta C_1C_2C_3 = \frac{1}{2}(1)(1)\sin 60°$	1A	
$= \frac{\sqrt{3}}{4}\text{ m}^2$	1A	u−1 for missing unit
	----------(2)	
(b) Each side of a smaller triangle $= \frac{1}{3}$ m		
Area of each smaller triangle $= \frac{1}{2}(\frac{1}{3})(\frac{1}{3})\sin 60° = \frac{\sqrt{3}}{36}\text{ m}^2$		
Total area $= 4\times\frac{\sqrt{3}}{36}+\frac{\sqrt{3}}{4}$	1M+1M	1M for 4 times, 1M for + (a)
$= \frac{13\sqrt{3}}{36}\text{ m}^2$	1A	u−1 for missing unit
	----------(3)	
(c) The area		
$=\frac{\sqrt{3}}{4}+\frac{4}{9}\times\frac{\sqrt{3}}{4}+\left(\frac{4}{9}\right)^2\times\frac{\sqrt{3}}{4}+\left(\frac{4}{9}\right)^3\frac{\sqrt{3}}{4}+\cdots$	1M + 1A	1M for G. P.
$=\frac{\frac{\sqrt{3}}{4}}{1-\frac{4}{9}}$	1M	for $\frac{a}{1-r}$
$=\frac{9\sqrt{3}}{20}\text{ m}^2$	1A	u−1 for missing unit
The area $=\frac{\frac{\sqrt{3}}{4}}{1-\frac{4}{9}}$	2M+1A	2M for $\frac{(a)}{1-\frac{4}{9}}$
$=\frac{9\sqrt{3}}{20}\text{ m}^2$	1A	u−1 for missing unit
	----------(4)	

Solution	Marks	Remarks
14. (a) $AT=\frac{h}{\tan 20^\circ}$ m and $BT=\frac{h}{\tan 15^\circ}$ m .	1A	for both $AT=2.75h$ m and $BT=3.73h$ m
$\because\ BT^2=AB^2+AT^2-2AB\cdot AT\cos 30^\circ$		
$\therefore\ \left(\frac{h}{\tan 15^\circ}\right)^2=900^2+\left(\frac{h}{\tan 20^\circ}\right)^2-2(900)\left(\frac{h}{\tan 20^\circ}\right)\cos 30^\circ$	1M+1A	
$\left(\frac{1}{\tan^2 15^\circ}-\frac{1}{\tan^2 20^\circ}\right)h^2+\frac{900\sqrt{3}}{\tan 20^\circ}h-810000=0$	1M	in the form of $ah^2+bh+c=0$
$h\approx 153.86\approx 154$	1A	r.t. 154
	----------(5)	
(b) (i) ES is minimum when $SE\perp AB$ (or $TE\perp AB$) .		
When $TE\perp AB$, $ET=AT\sin 30^\circ=\frac{h\sin 30^\circ}{\tan 20^\circ}(\approx 211.36)$	1A	
Shortest distance $=\sqrt{h^2+(AT\sin 30^\circ)^2}$	1M	$\sqrt{153.86^2+211.36^2}$
$=h\sqrt{1+\left(\frac{\sin 30^\circ}{\tan 20^\circ}\right)^2}$		
≈ 261.43 ≈ 261 m .	1A	u−1 for missing unit (accept 262 m)
		
$AS=\frac{h}{\sin 20^\circ}\approx 449.86$ and $SB=\frac{h}{\sin 15^\circ}\approx 594.48$.		
$\cos\angle SAB=\frac{\left(\frac{h}{\sin 20^\circ}\right)^2+(900)^2-\left(\frac{h}{\sin 15^\circ}\right)^2}{2\left(\frac{h}{\sin 20^\circ}\right)(900)}\approx 0.8138$.	1M	
$\angle SAB=35.53^\circ$	1A	r.t. 35.5° (can be absorbed) accept $\angle SBA=26.09^\circ$
Shortest distance $=AS\sin\angle SAB$ $\approx\left(\frac{h}{\sin 20^\circ}\right)\sin 35.53^\circ$		
≈ 261 m	1A	(Accept 262 m)
	----------(3)	
(ii) $\because\ \tan\theta=\frac{h}{ET}$		
$\therefore\ \theta$ is maximum when $TE\perp AB$.	1M	can be omitted
$\tan\theta_{\max}=\frac{h}{AT\sin 30^\circ}$ $=\frac{\tan 20^\circ}{\sin 30^\circ}$		$\tan\theta=\frac{h}{ET}=\frac{153.86}{211.36}$ $\sin\theta=\frac{h}{ES}=\frac{153.86}{261.43}$ $\cos\theta=\frac{ET}{ES}=\frac{211.36}{261.43}$
Maximum value of $\theta\approx 36.1^\circ$	1A	
Hence $15^\circ\le\theta\le 36.1^\circ$.	1A	u−1 for missing unit
Accept using $\cos\theta=\frac{ET}{ES}=\frac{211.4}{261.4}$, $\theta\approx 36.0^\circ$		(Accept $\theta\approx 36.2^\circ$)
	----------(3)	

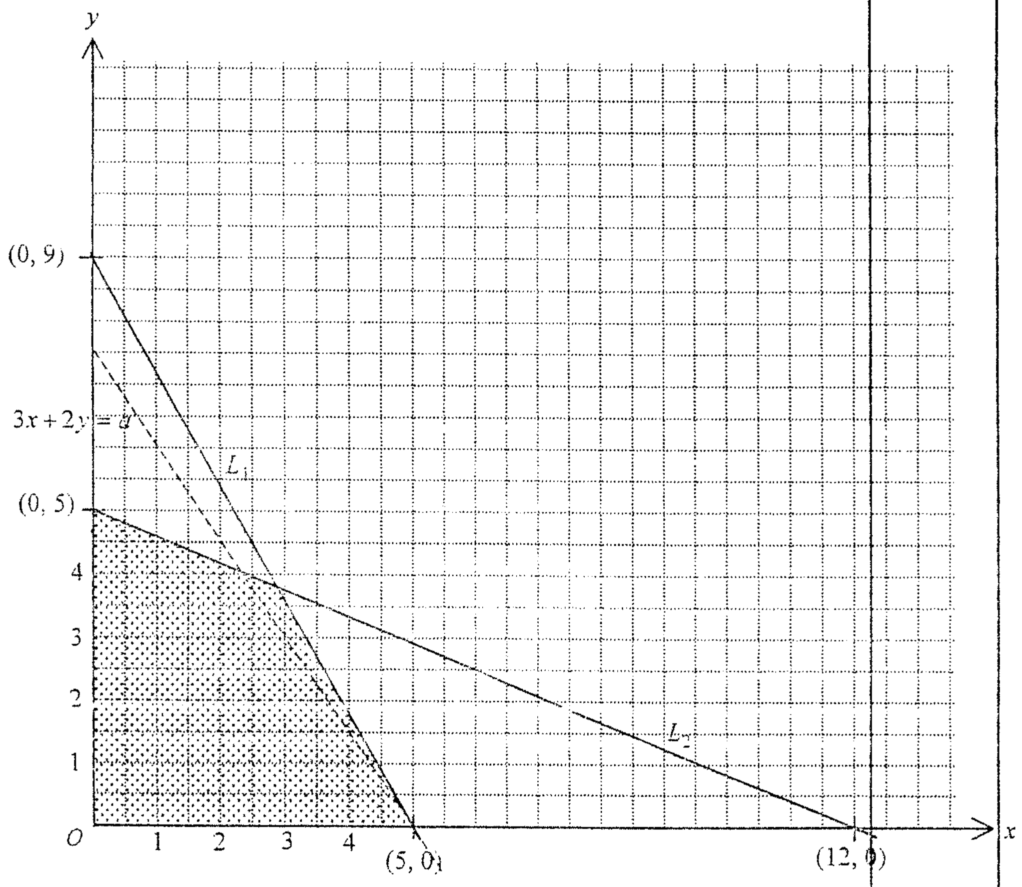
Solution	Marks	Remarks
16. (a) (i) In ΔAOD and ΔFOB ,		
$\angle AOD = \angle FOB = 90°$ (given)		[已知]
$\because \angle AEB = 90°$ ($\angle$ in semicircle)		[半圓上的圓周角]
$\therefore \angle DAO = 90° - \angle ABE$ ($\angle$ sum of Δ)		[Δ內角和]
On the other hand,		
$\angle BFO = 90° - \angle ABE$ ($\angle$ sum of Δ)		[Δ內角和]
$\therefore \angle DAO = \angle BFO$		
Hence, $\Delta AOD \sim \Delta FOB$ (AAA)		[等角] (AA) (equiangular)
Marking Scheme :		
Case 1 Any correct proof with correct reasons.	3	
Case 2 Any correct proof without reasons.	2	
Case 3 Incomplete proof with any one correct angle and correct reason.	1	
(ii) In ΔAOG and ΔGOB ,		
$\angle AOG = \angle GOB = 90°$ (given)		[已知]
$\because \angle AGB = 90°$ ($\angle$ in semicircle)		[半圓上的圓周角]
$\therefore \angle AGO = 90° - \angle BGO$		
$= \angle GBO$ ($\angle$ sum of Δ)		[Δ內角和]
Thus, $\Delta AOG \sim \Delta GOB$ (AAA)		[等角] (AA) (equiangular)
Marking Scheme :		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
(iii) Hence $\dfrac{OD}{OA} = \dfrac{OB}{OF}$		
$OD \cdot OF = OA \cdot OB$	1	either one
Since $\Delta AOG \sim \Delta GOB$		
$\therefore \dfrac{OA}{OG} = \dfrac{OG}{OB}$		
i.e. $OA \cdot OB = OG^2$.		
Thus $OD \cdot OF = OA \cdot OB = OG^2$	1	
	----------(7)	
(b) (i) $A = (c-r, 0)$ and $B = (c+r, 0)$		
$m_{AD} = \dfrac{p}{r-c}$	1A	
$m_{BF} = -\dfrac{q}{r+c}$	1A	
(ii) $\because \angle AEB = 90°$ ($\angle$ in semi circle)		[半圓上的圓周角]
$\therefore m_{AD} \cdot m_{BF} = \dfrac{p}{r-c} \cdot \left(-\dfrac{q}{r+c}\right) = -1$	1M	
$pq = r^2 - c^2$		
Since $pq = OD \cdot OF$		
and $r^2 - c^2 = CG^2 - OC^2 = OG^2$,		
therefore $OD \cdot OF = OG^2$	1	
	----------(4)	

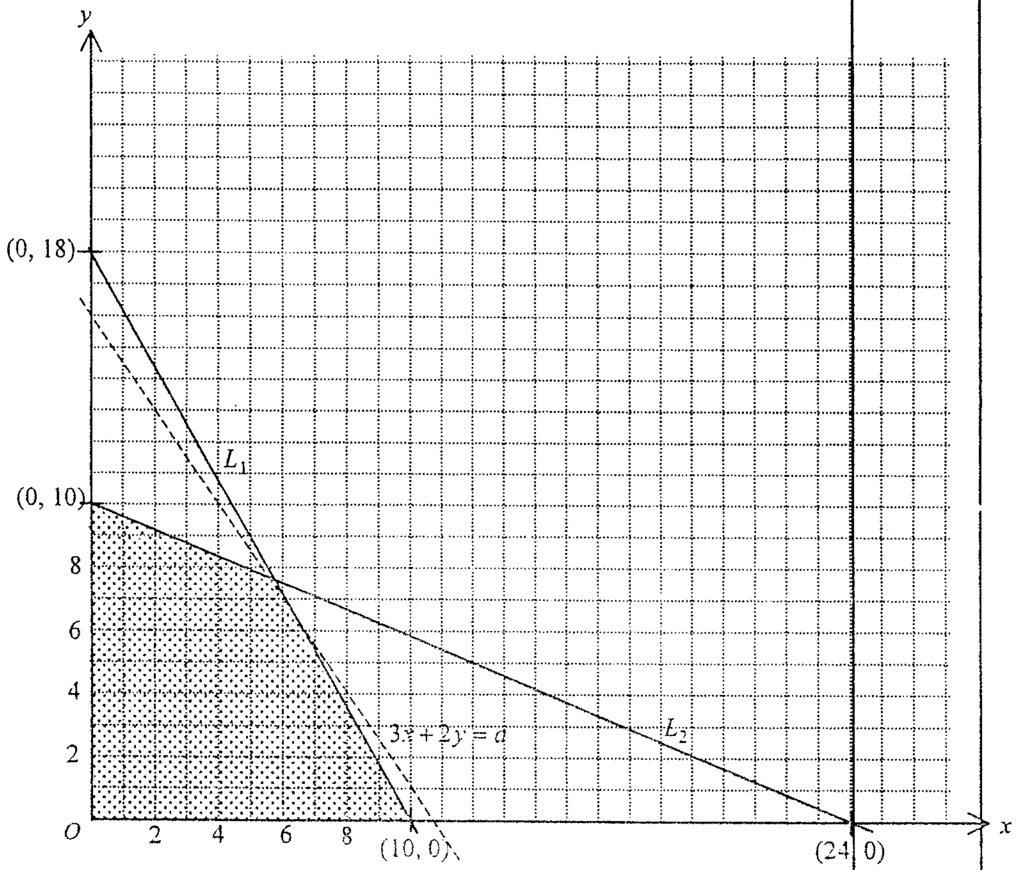


Solution	Marks	Remarks
15. (a) (i) Total amount of water = $\frac{1}{3}\pi\cdot9^2\cdot24 = 648\pi$ cm^3		
Volume of water in the cylinder = $\pi\cdot6^2h = 36\pi h$ cm^3		
Volume of water in the cone = $\frac{1}{3}\pi\cdot9^2\cdot24\cdot\left(\frac{h+5}{24}\right)^3$ cm^3	1M+1A	1M for $V = V'\cdot\left(\frac{h+5}{24}\right)^3$
Let r cm be the radius of the water surface in the cone when water is being poured into the cylinder.		
Then $\frac{r}{h+5} = \frac{9}{24}$.	1A	
Volume of water remains in the cone		
$= \frac{\pi}{3}\left[\frac{3}{8}(h+5)\right]^2(h+5) = \frac{3\pi}{64}(h+5)^3$ cm^3.	1M	
∴ $\frac{3\pi}{64}(h+5)^3 + 36\pi h = 648\pi$	1M	$\frac{1}{3}\pi\cdot9^2\cdot24\cdot\left[1-\left(\frac{h+5}{24}\right)^3\right] = \pi\cdot6^2h$
$1-\left(\frac{h+5}{24}\right)^3 = \frac{h}{18}$		
$h^3 + 15h^2 + 75h + 125 = 768(18-h)$	1A	for expanding $(h+5)^3$
$h^3 + 15h^2 + 75h + 125 + 768h = 13824$		
$h^3 + 15h^2 + 843h - 13699 = 0$	1	
(ii) Let $f(h) = h^3 + 15h^2 + 843h - 13699$		
∵ $f(11) = -1280 < 0$ and $f(12) = 305 > 0$	1M	
∴ The value of h lies between 11 and 12 .		

a $[f(a) < 0]$	b $[f(b) > 0]$	$m = \frac{a+b}{2}$	$f(m)$
11	12	11.5	−500
11.5	12	11.75	−101
11.75	12	11.875	+101
11.75	11.875	11.8125	+0.224
11.75	11.8125		

	1M	Testing sign of mid-value
	1M	Choosing the correct interval
∴ $11.75 < h < 11.8125$		
$h \approx 11.8$ (correct to 1 decimal place)	1A	f.t.
	----------(9)	
(b) The situation in Figure 9(b) is the same as the situation in Figure 9(a) if the lower part (5 cm height) of the water of the cone is ignored.		
Thus the depth of water in the frustum is		
h cm		
≈ 11.8 cm	2M	2M for the answer in (a)(ii) u−1 for missing unit
	----------(2)	

Solution	Marks	Remarks
17. (a) Equation of L_1: $\frac{y-9k}{x}=-\frac{9}{5}$	1M	$\frac{x}{5k}+\frac{y}{9k}=1$
$9x+5y=45k$		
Equation of L_2: $\frac{y-5k}{x}=-\frac{5}{12}$		$\frac{x}{12k}+\frac{y}{5k}=1$
$5x+12y=60k$	1A	for both equations
	----------(2)	
(b) (i) Let x and y be respectively the number of articles produced by lines A and B. The constraints are		
$45x+25y \le 225$ (or $9x+5y \le 45$),	1A	withhold 1 mark for strict inequality
$50x+120y \le 600$ (or $5x+12y \le 60$),	1A	
x and y are non-negative integers.		
The profit is \$ 1 000$(3x+2y)$.	1A	
Using the graph in Figure 11 with $k=1$, the feasible solutions are represented by the lattice points in the shaded region below.		
		
From the graph, the most profitable combinations are (3, 3) and (5,0)		
At (3, 3), the profit is \$ 1 000 (9 + 6) = \$ 15 000 At (5, 0), the profit is \$ 1 000 (15 + 0) = \$ 15 000 At (0, 5), the profit is \$ 1 000 (10) = \$ 10 000 At (2, 4), the profit is \$ 1 000 (6 + 3) = \$ 14 000	1M	Testing
The greatest possible profit is \$ 15 000 .	1A	u−1 for missing unit

Solution	Marks	Remarks
(ii) Let x and y be respectively the number of articles produced by production lines A and B. The constraints are $\begin{cases} 45x+25y \le 450 & (\text{ or } 9x+5y \le 90), \\ 50x+120y \le 1200 & (\text{ or } 5x+12y \le 120), \end{cases}$ x and y are non-negative integers.	1A	
		
Using the same graph as in (i) and taking $k=2$, the feasible solutions are represented by the lattice points in the shaded region.	1M	can be absorbed
From the graph, the most profitable combinations is (6, 7).	1A	can be absorbed
The greatest possible profit is \$ 1 000 (18 + 14) = \$ 32 000	1A	u−1 for missing unit (accept drawing 2 lines on Figure 11 with correct labels)
	----------(9)	