2002-CE
MATH
PAPER 1

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2002

MATHEMATICS PAPER 1

Question-Answer Book

8.30 am – 10.30 am (2 hours)
This paper must be answered in English

1. Write your Candidate Number, Centre Number and Seat Number in the spaces provided on this cover.
2. This paper consists of THREE sections, A(1), A(2) and B. Each section carries 33 marks.
3. Attempt ALL questions in Sections A(1) and A(2), and any THREE questions in Section B. Write your answers in the spaces provided in this Question-Answer Book. Supplementary answer sheets will be supplied on request. Write your Candidate Number on each sheet and fasten them with string inside this book.
4. Write the question numbers of the questions you have attempted in Section B in the spaces provided on this cover.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
7. The diagrams in this paper are not necessarily drawn to scale.

Candidate Number	
Centre Number	
Seat Number	

	Marker's Use Only	Examiner's Use Only
	Marker No.	Examiner No.
Section A Question No.	Marks	Marks
1–2		
3–4		
5–6		
7–8		
9		
10		
11		
12		
13		
Section A Total		

Checker's Use Only	Section A Total		

Section B Question No.*	Marks	Marks
Section B Total		

To be filled in by the candidate.

Checker's Use Only	Section B Total		

Checker No.	

FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area × height
PYRAMID	Volume	=	$\frac{1}{3}$ × base area × height

Page total

SECTION A(1) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

1. Simplify $\dfrac{(ab^2)^2}{a^5}$ and express your answer with positive indices. (3 marks)

2. In Figure 1, the radius of the sector is 6 cm. Find the area of the sector in terms of π . (3 marks)

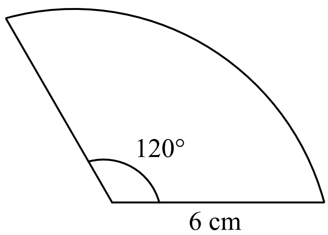

Figure 1

Page total

3. In Figure 2 , OP and OQ are two perpendicular straight roads where $OP = 100$ m and $OQ = 80$ m . (3 marks)

(a) Find the value of θ .

(b) Find the bearing of P from Q .

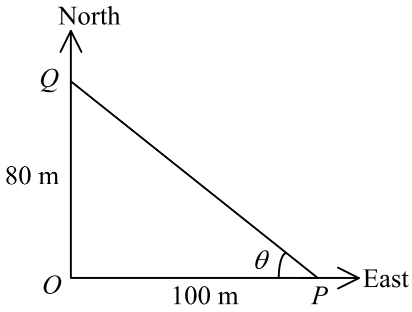

Figure 2

4. Let $f(x) = x^3 - 2x^2 - 9x + 18$. (3 marks)

(a) Find $f(2)$.

(b) Factorize $f(x)$.

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5. For the set of data 4, 4, 5, 6, 8, 12, 13, 13, 13, 18, find (4 marks)
(a) the mean,
(b) the mode,
(c) the median,
(d) the standard deviation.

6. The radius of a circle is 8 cm . A new circle is formed by increasing the radius by 10% . (4 marks)

(a) Find the area of the new circle in terms of π .

(b) Find the percentage increase in the area of the circle.

Page total

7. (a) Solve the inequality $3x+6 \geq 4+x$.

(b) Find all integers which satisfy both the inequalities $3x+6 \geq 4+x$ and $2x-5<0$.

(4 marks)

8. In Figure 3, the straight line $L: x-2y+8=0$ cuts the coordinate axes at A and B . (4 marks)

(a) Find the coordinates of A and B .

(b) Find the coordinates of the mid-point of AB .

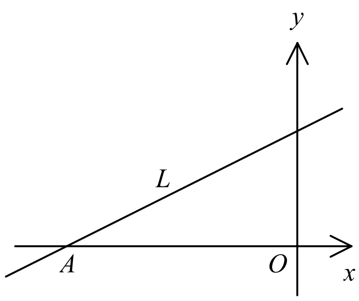

Figure 3

Page total

9. In Figure 4, BD is a diameter of the circle $ABCD$. $AB = AC$ and $\angle BDC = 40°$. Find $\angle ABD$.
(5 marks)

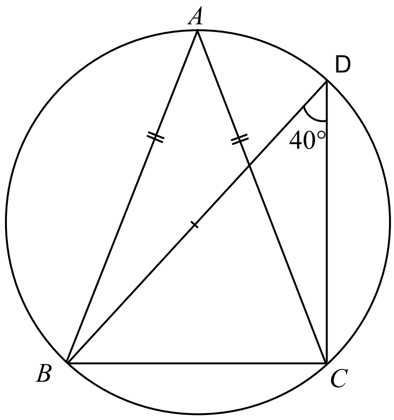

Figure 4

Section A(2) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

10. In Figure 5, ABC is a triangle in which $\angle BAC = 20^\circ$ and $AB = AC$. D , E are points on AB and F is a point on AC such that $BC = CE = EF = FD$.

(a) Find $\angle CEF$. (4 marks)

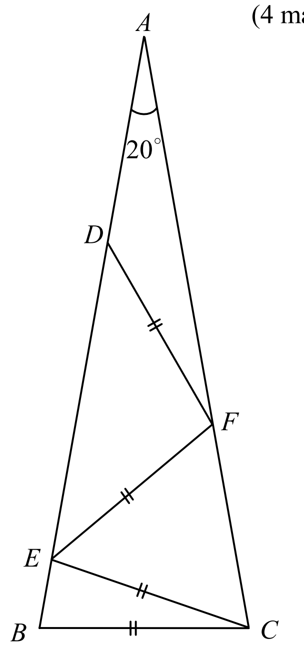

Figure 5

(b) Prove that $AD = DF$. (3 marks)

Page total

11. The area of a paper bookmark is A cm^2 and its perimeter is P cm . A is a function of P . It is known that A is the sum of two parts, one part varies as P and the other part varies as the square of P . When $P = 24$, $A = 36$ and when $P = 18$, $A = 9$.

(a) Express A in terms of P . (3 marks)

(b) (i) The best-selling paper bookmark has an area of 54 cm^2 . Find the perimeter of this bookmark.

(ii) The manufacturer of the bookmarks wants to produce a gold miniature similar in shape to the best-selling paper bookmark. If the gold miniature has an area of 8 cm^2 , find its perimeter.

(5 marks)

Page total

12. Two hundred students participated in a summer reading programme. Figure 6 shows the cumulative frequency polygon of the distribution of the numbers of books read by the participants.

The cumulative frequency polygon of the distribution of the numbers of books read by the participants

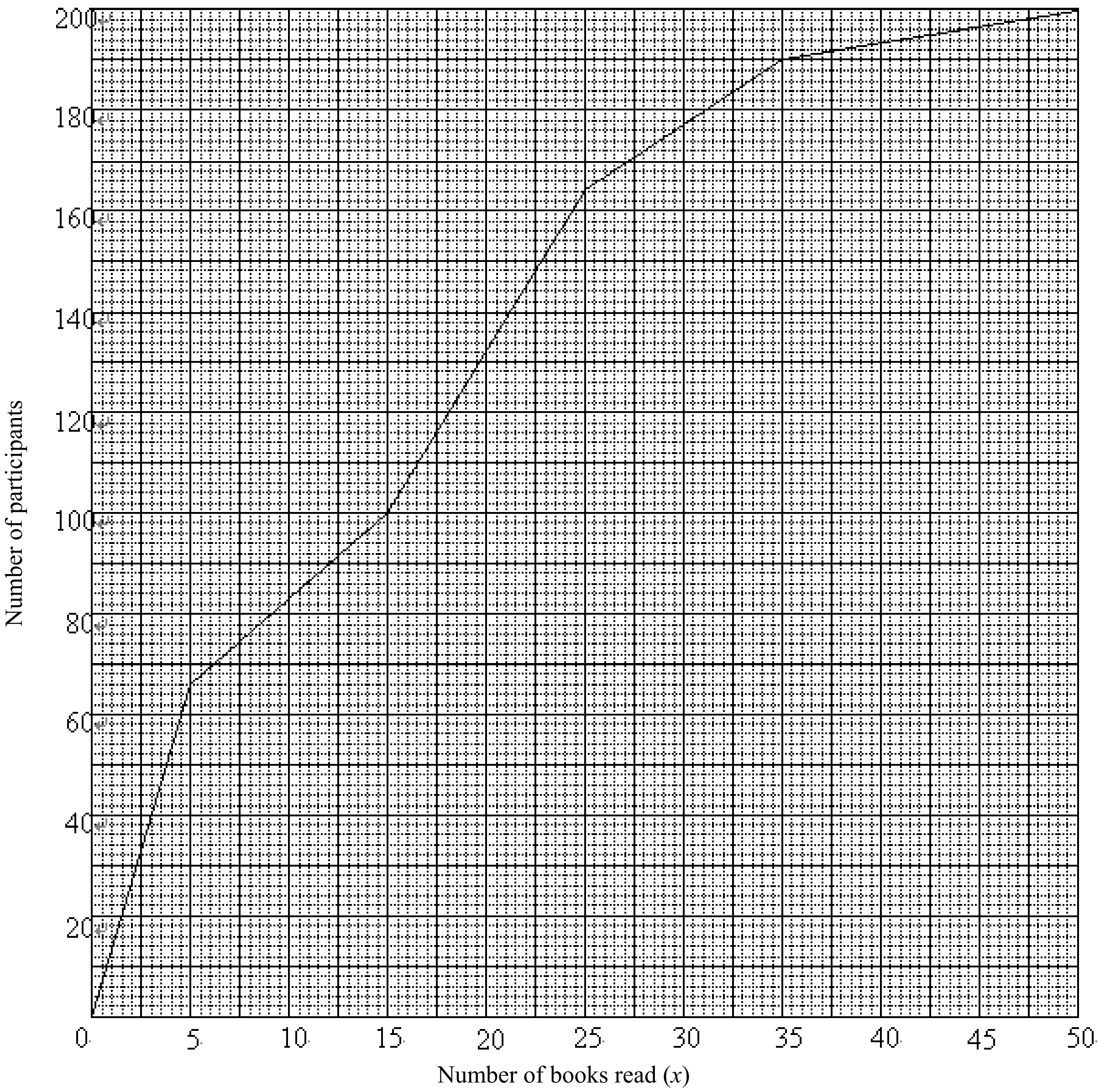

Figure 6

Page total

(a) The table below shows the frequency distribution of the numbers of books read by the participants. Using the graph in Figure 6, complete the table. (1 mark)

Number of books read (x)	**Number of participants**	**Award**
$0 < x \le 5$	66	Certificate
$5 < x \le 15$		Book coupon
$15 < x \le 25$	64	Bronze medal
$25 < x \le 35$		Silver medal
$35 < x \le 50$	10	Gold medal

(b) Using the graph in Figure 6 , find the inter-quartile range of the distribution. (2 marks)

(c) Two participants were chosen randomly from those awarded with medals.
Find the probability that (6 marks)

(i) they both won gold medals;

(ii) they won different medals.

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13. A line segment AB of length 3 m is cut into three equal parts AC_1, C_1C_2 and C_2B as shown in Figure 7(a).

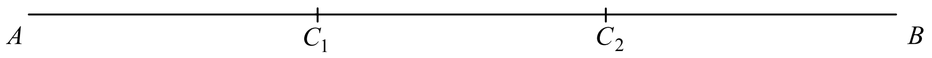

Figure 7(a)

On the middle part C_1C_2, an equilateral triangle $C_1C_2C_3$ is drawn as shown in Figure 7(b) .

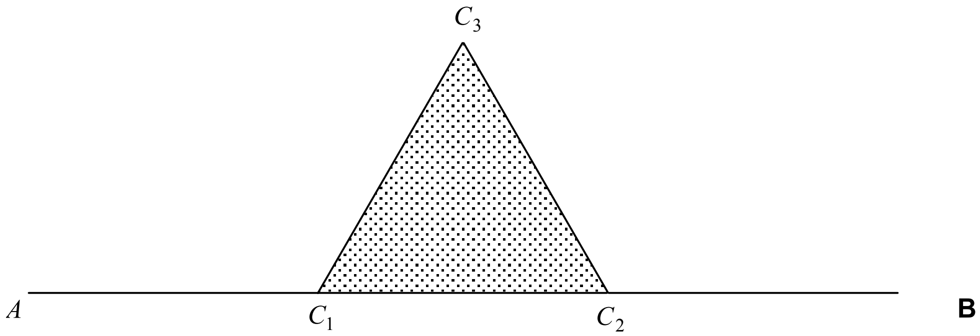

Figure 7(b)

(a) Find, in surd form, the area of triangle $C_1C_2C_3$. (2 marks)

Page total

(b) Each of the line segments AC_1, C_1C_3, C_3C_2 and C_2B in Figure 7(b) is further divided into three equal parts. Similar to the previous process, four smaller equilateral triangles are drawn as shown in Figure 7(c). Find, in surd form, the total area of all the equilateral triangles.

(3 marks)

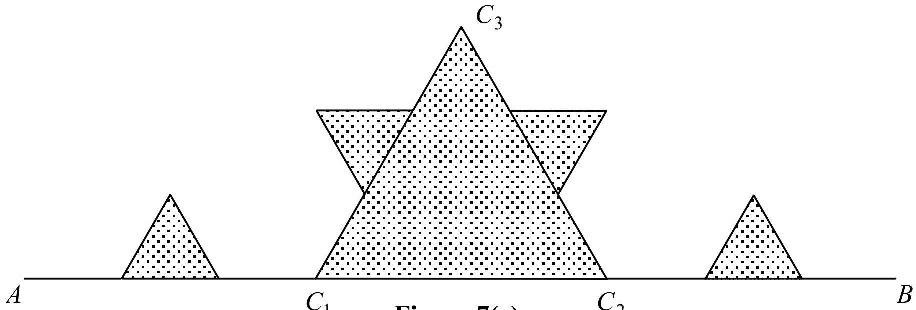

Figure 7(c)

(c) Figure 7(d) shows all the equilateral triangles so generated when the previous process is repeated again. What would the total area of all the equilateral triangles become if this process is repeated indefinitely? Give your answer in surd form.

(4 marks)

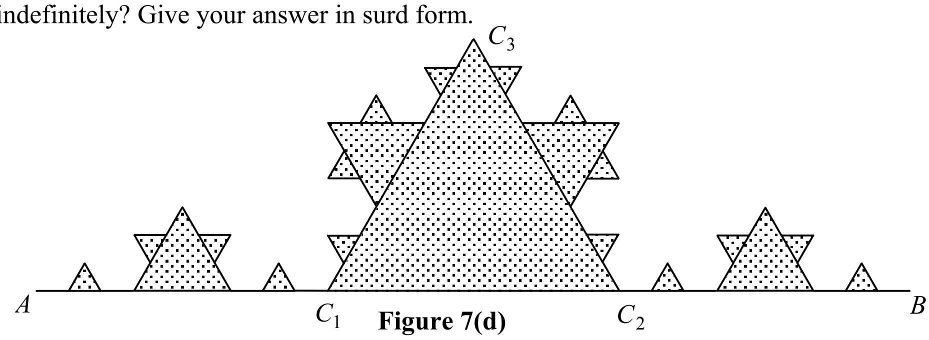

Figure 7(d)

Page total

SECTION B (33 marks)
Answer any THREE questions in this section and write your answers in the spaces provided.
Each question carries 11 marks.

14. In Figure 8, AB is a straight track 900 m long on the horizontal ground. E is a small object moving along AB. ST is a vertical tower of height h m standing on the horizontal ground. The angles of elevation of S from A and B are 20° and 15° respectively. $\angle TAB = 30°$.

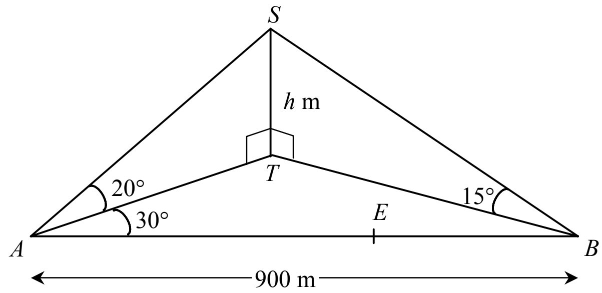

Figure 8

(a) Express AT and BT in terms of h. Hence find h. (5 marks)

(b) (i) Find the shortest distance between E and S.

(ii) Let θ be the angle of elevation of S from E. Find the range of values of θ as E moves along AB.

(6 marks)

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15. (a) Figure 9(a) shows two vessels of the same height 24 cm, one in the form of a right circular cylinder of radius 6 cm and the other a right circular cone of radius 9 cm. The vessels are held vertically on two horizontal platforms, one of which is 5 cm higher than the other. To begin with, the cylinder is empty and the cone is full of water. Water is then transferred into the cylinder from the cone until the water in both vessels reaches the same horizontal level. Let h cm be the depth of water in the cylinder.

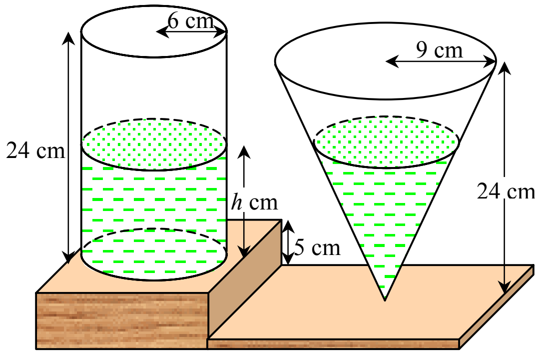

Figure 9(a)

(i) Show that $h^3 + 15h^2 + 843h - 13699 = 0$.

(ii) It is known that the equation in (a)(i) has only one real root. Show that the value of h lies between 11 and 12 . Using the method of bisection, find h correct to 1 decimal place.

(9 marks)

(b) Figure 9(b) shows a set up which is modified from the one in Figure 9(a). The lower part of the cone is cut off and sealed to form a frustum of height 19 cm . The two vessels are then held vertically on the same horizontal platform. To begin with, the cylinder is empty and the frustum is full of water. Water is then transferred into the cylinder from the frustum until the water in both vessels reaches the same horizontal level. Find the depth of water in the cylinder.

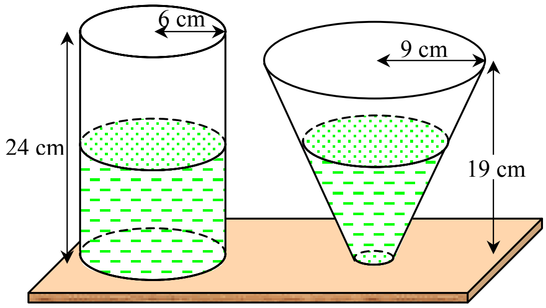

Figure 9(b)

(2 marks)

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16.

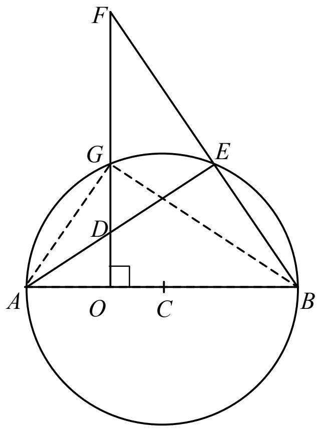

Figure 10

In Figure 10, AB is a diameter of the circle $ABEG$ with centre C. The perpendicular from G to AB cuts AB at O. AE cuts OG at D. BE and OG are produced to meet at F.
Mary and John try to prove $OD \cdot OF = OG^2$ by using two different approaches.

(a) Mary tackles the problem by first proving that $\Delta AOD \sim \Delta FOB$ and $\Delta AOG \sim \Delta GOB$.
Complete the following tasks for Mary.

(i) Prove that $\Delta AOD \sim \Delta FOB$.

(ii) Prove that $\Delta AOG \sim \Delta GOB$.

(iii) Using (a)(i) and (a)(ii), prove that $OD \cdot OF = OG^2$.

(7 marks)

(b) John tackles the same problem by introducing a rectangular coordinate system in Figure 10 so that the coordinates of C, D and F are $(c, 0)$, $(0, p)$ and $(0, q)$ respectively, where c, p and q are positive numbers. He denotes the radius of the circle by r.
Complete the following tasks for John.

(i) Express the slopes of AD and BF in terms of c, p, q and r.

(ii) Using (b)(i), prove that $OD \cdot OF = OG^2$.

(4 marks)

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17. (a) Figure 11 shows two straight lines L_1 and L_2. L_1 cuts the coordinate axes at the points $(5k, 0)$ and $(0, 9k)$ while L_2 cuts the coordinate axes at the points $(12k, 0)$ and $(0, 5k)$, where k is a positive integer. Find the equations of L_1 and L_2.

(2 marks)

(b) A factory has two production lines A and B. Line A requires 45 man-hours to produce an article and the production of each article discharges 50 units of pollutants. To produce the same article, line B requires 25 man-hours and discharges 120 units of pollutants. The profit yielded by each article produced by the production line A is \$3 000 and the profit yielded by each article produced by the production line B is \$2 000.

(i) The factory has 225 man-hours available and the total amount of pollutants discharged must not exceed 600 units. Let the number of articles produced by the production lines A and B be x and y respectively. Write down the appropriate inequalities and by putting $k = 1$ in Figure 11, find the greatest possible profit of the factory.

(ii) Suppose now the factory has 450 man-hours available and the total amount of pollutants discharged must not exceed 1200 units. Using Figure 11, find the greatest possible profit.

(9 marks)

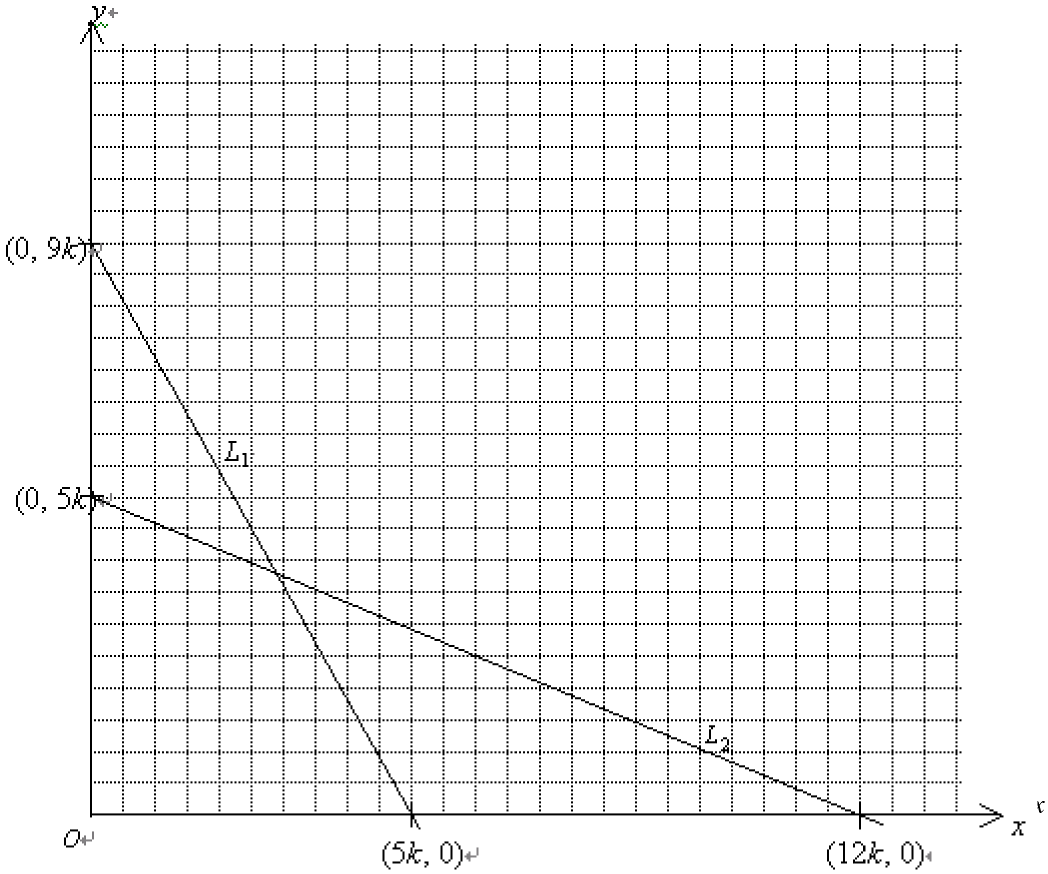

Figure 11

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2002

Mathematics 1
Section A(1)

1. $\dfrac{(ab^2)^2}{a^5} = \dfrac{b^4}{a^3}$

2. Area = 12π cm^2

3. (a) θ is $38.7°$.

   (b) The bearing of P from Q is $129°$.

4. (a) $f(2) = 0$

   (b) $f(x) = (x-2)(x-3)(x+3)$

5. (a) Mean = 9.6
   (b) Mode = 13
   (c) Median = 10
   (d) Standard deviation = 4.59

6. (a) The area of the new circle is 77.44π cm^2 .

   (b) The percentage increase in area is 21% .

7. (a) $x \geq -1$

   (b) The required integers are -1 , 0 , 1 , 2 .

8. (a) The coordinates of A are (−8, 0) .
   The coordinates of B are (0, 4) .

   (b) The mid-point is (−4, 2) .

9. $\angle ABD = 20°$

10. (a) $\because \quad AB = AC$

$\therefore \quad \angle B = \dfrac{180° - 20°}{2} = 80°$

$\because \quad BC = CE$

$\therefore \quad \angle CEB = \angle B = 80°$

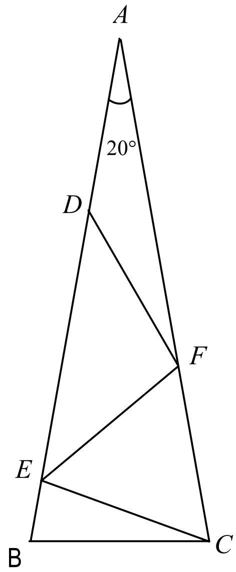
$\therefore \quad \angle BCE = 180° - 80° - 80° = 20°$

$\therefore \quad \angle ECF = \angle ACB - \angle BCE$

$\quad\quad = 60°$

$\because \quad CE = EF$

$\therefore \quad \angle CEF = 60°$



(b) $\angle DEF = 180° - 60° - 80°$ (adj. ∠s on st. line)

$\quad\quad = 40°$

$\because \quad EF = FD$

$\therefore \quad \angle FDE = \angle DEF$

$\quad\quad = 40°$ (base ∠s of isos. Δ)

In $\triangle ADF$,

$\angle DFA = 40° - 20°$ (ext ∠of Δ)

$\quad\quad = 20°$

$\quad\quad = \angle DAF$

$\therefore \quad AD = DF$ (base ∠s of Δ =)

Section A(2)

11. (a) Let $A = aP + bP^2$, where a and b are constants.

Sub. $P = 24$, $A = 36$,

$24a + 576b = 36$

$2a + 48b = 3$ ………… (1)

Sub. $P = 18$, $A = 9$,

$18a + 324b = 9$

$2a + 36b = 1$ ………… (2)

Solving (1) and (2)

$a = -\frac{5}{2}$

$b = \frac{1}{6}$

$\therefore \quad A = -\frac{5}{2}P + \frac{1}{6}P^2$

(b) (i) When $A = 54$,

$-\frac{5}{2}P + \frac{1}{6}P^2 = 54$

$P^2 - 15P - 324 = 0$

$P = 27$ or $P = -12$ (rejected)

$\therefore$ the required perimeter is 27 cm.

(ii) Let P' cm be the perimeter of the gold bookmark.

$\left(\frac{P'}{27}\right)^2 = \frac{8}{54}$

$P' = 6\sqrt{3}$ (≈ 10.4)

The perimeter of the gold bookmark is $6\sqrt{3}$ (≈ 10.4) cm .

12. (a)

Number of books read (x)	Number of participants	Award
$0 < x \le 5$	66	Certificate
$5 < x \le 15$	**34**	Book coupon
$15 < x \le 25$	64	Bronze medal
$25 < x \le 35$	**26**	Silver medal
$35 < x \le 50$	10	Gold medal

(b) Lower quartile = 3.8
Upper quartile = 22.8
Inter-quartile range = 22.8 − 3.8
= 19

(c) (i) The number of participants who won medals,
$$64 + 26 + 10 = 100$$
The number of participants who won gold medals is 10.
The probability that they both won gold medals
$$= \frac{10}{100} \times \frac{9}{99}$$
$$= \frac{1}{110}$$

(ii) Both won bronze medals
$$P_1 = \frac{64}{100} \times \frac{63}{99} = \frac{112}{275}$$
Both won silver medals
$$P_2 = \frac{26}{100} \times \frac{25}{99} = \frac{13}{198}$$
The probability that they won different medals
$$= 1 - \frac{1}{110} - \frac{112}{275} - \frac{13}{198}$$
$$= \frac{1282}{2475}$$

13. (a) Area of $\Delta C_1C_2C_3 = \frac{1}{2}(1)(1)\sin 60°$

$= \frac{\sqrt{3}}{4}\ \text{m}^2$

(b) Each side of a smaller triangle $= \frac{1}{3}$ m

Area of each smaller triangle $= \frac{1}{2}(\frac{1}{3})(\frac{1}{3})\sin 60° = \frac{\sqrt{3}}{36}\ \text{m}^2$

Total area $= 4\times\frac{\sqrt{3}}{36}+\frac{\sqrt{3}}{4}$

$= \frac{13\sqrt{3}}{36}\ \text{m}^2$

(c) The area

$=\frac{\sqrt{3}}{4}+\frac{4}{9}\times\frac{\sqrt{3}}{4}+\left(\frac{4}{9}\right)^2\times\frac{\sqrt{3}}{4}+\left(\frac{4}{9}\right)^3\frac{\sqrt{3}}{4}+\cdots$

$=\frac{\frac{\sqrt{3}}{4}}{1-\frac{4}{9}}$

$=\frac{9\sqrt{3}}{20}\ \text{m}^2$

Section B

14. (a) $AT = \dfrac{h}{\tan 20°}$ m and $BT = \dfrac{h}{\tan 15°}$ m .

$\because \quad BT^2 = AB^2 + AT^2 - 2AB \cdot AT \cos 30°$

$\therefore \quad \left(\dfrac{h}{\tan 15°}\right)^2 = 900^2 + \left(\dfrac{h}{\tan 20°}\right)^2 - 2(900)\left(\dfrac{h}{\tan 20°}\right)\cos 30°$

$\left(\dfrac{1}{\tan^2 15°} - \dfrac{1}{\tan^2 20°}\right)h^2 + \dfrac{900\sqrt{3}}{\tan 20°}h - 810000 = 0$

$h \approx 153.86 \approx 154$

(b) (i) ES is minimum when $SE \perp AB$ (or $TE \perp AB$) .

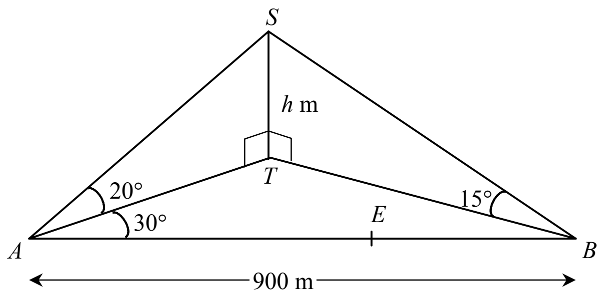
When $TE \perp AB$, $ET = AT \sin 30° = \dfrac{h \sin 30°}{\tan 20°}$ (≈ 211.36)

Shortest distance = $\sqrt{h^2 + (AT \sin 30°)^2}$

$= h\sqrt{1 + \left(\dfrac{\sin 30°}{\tan 20°}\right)^2}$

≈ 261.43

≈ 261 m .



(ii) $\because$ $\tan\theta = \dfrac{h}{ET}$

$\therefore$ θ is maximum when $TE \perp AB$.

$$\tan\theta_{\max} = \frac{h}{AT\sin 30^\circ}$$
$$= \frac{\tan 20^\circ}{\sin 30^\circ}$$

Maximum value of $\theta \approx 36.1^\circ$

Hence $15^\circ \le \theta \le 36.1^\circ$.

15. (a) (i) Total amount of water $= \frac{1}{3}\pi \cdot 9^2 \cdot 24 \ = 648\pi$ cm^3

Volume of water in the cylinder $= \pi \cdot 6^2 h \ = 36\pi h$ cm^3

Volume of water in the cone $= \frac{1}{3}\pi \cdot 9^2 \cdot 24 \cdot \left(\frac{h+5}{24}\right)^3$ cm^3

$$\therefore \quad \frac{3\pi}{64}(h+5)^3 + 36\pi h = 648\pi$$

$$1 - \left(\frac{h+5}{24}\right)^3 = \frac{h}{18}$$

$$h^3 + 15h^2 + 75h + 125 = 768(18-h)$$

$$h^3 + 15h^2 + 75h + 125 + 768h = 13824$$

$$h^3 + 15h^2 + 843h - 13699 = 0$$

(ii) Let f $(h) = h^3 + 15h^2 + 843h - 13699$

$\because$ f(11) $= -1280 < 0$ and f(12) $= 305 > 0$

$\therefore$ The value of h lies between 11 and 12 .

a [f(a) < 0]	b [f(b) > 0]	$m = \frac{a+b}{2}$	f(m)
11	12	11.5	−500
11.5	12	11.75	−101
11.75	12	11.875	+101
11.75	11.875	11.8125	+0.224
11.75	11.8125		

$\therefore \quad 11.75 < h < 11.8125$

$h \approx 11.8$ (correct to 1 decimal place)

(b) The situation in Figure 9(b) is the same as the situation in Figure 9(a) if the lower part (5 cm height) of the water of the cone is ignored.
Thus the depth of water in the frustum is

h cm

≈ 11.8 cm

16. (a) (i) In ΔAOD and ΔFOB,

$\angle AOD = \angle FOB = 90°$ (given)

$\because$ $\angle AEB = 90°$ ($\angle$ in semicircle)

$\therefore$ $\angle DAO = 90° - \angle ABE$ ($\angle$ sum of Δ)

On the other hand,

$\angle BFO = 90° - \angle ABE$ ($\angle$ sum of Δ)

$\therefore$ $\angle DAO = \angle BFO$

Hence, $\Delta AOD \sim \Delta FOB$ (AAA)

(ii) In ΔAOG and ΔGOB,

$\angle AOG = \angle GOB = 90°$ (given)

$\because$ $\angle AGB = 90°$ ($\angle$ in semicircle)

$\therefore$ $\angle AGO = 90° - \angle BGO$

$= \angle GBO$ ($\angle$ sum of Δ)

Thus, $\Delta AOG \sim \Delta GOB$ (AAA)

(iii) Hence $\dfrac{OD}{OA} = \dfrac{OB}{OF}$

$OD \cdot OF = OA \cdot OB$

$\because$ $\Delta AOG \sim \Delta GOB$

$\therefore$ $\dfrac{OA}{OG} = \dfrac{OG}{OB}$

i.e. $OA \cdot OB = OG^2$.

Thus $OD \cdot OF = OA \cdot OB = OG^2$

(b) (i) $A = (c - r, 0)$ and $B = (c + r, 0)$.

Slope of $AD = m_{AD} = \dfrac{p}{r - c}$

Slope of $BF = m_{BF} = -\dfrac{q}{r + c}$

(ii) $\because$ $\angle AEB = 90°$ ($\angle$ in semi circle)

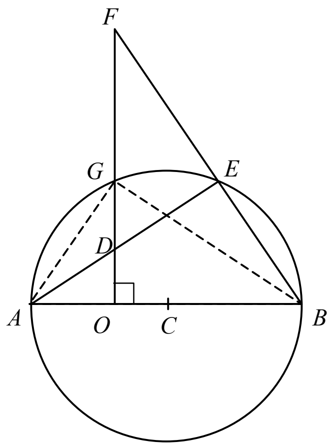
$\therefore$ $m_{AD} \cdot m_{BF} = \dfrac{p}{r - c} \cdot \left(-\dfrac{q}{r + c}\right) = -1$

$pq = r^2 - c^2$

Since $pq = OD \cdot OF$

and $r^2 - c^2 = CG^2 - OC^2 = OG^2$,

we have $OD \cdot OF = OG^2$.



17. (a) Equation of L_1: $\dfrac{y-9k}{x}=-\dfrac{9}{5}$

$9x+5y=45k$

Equation of L_2: $\dfrac{y-5k}{x}=-\dfrac{5}{12}$

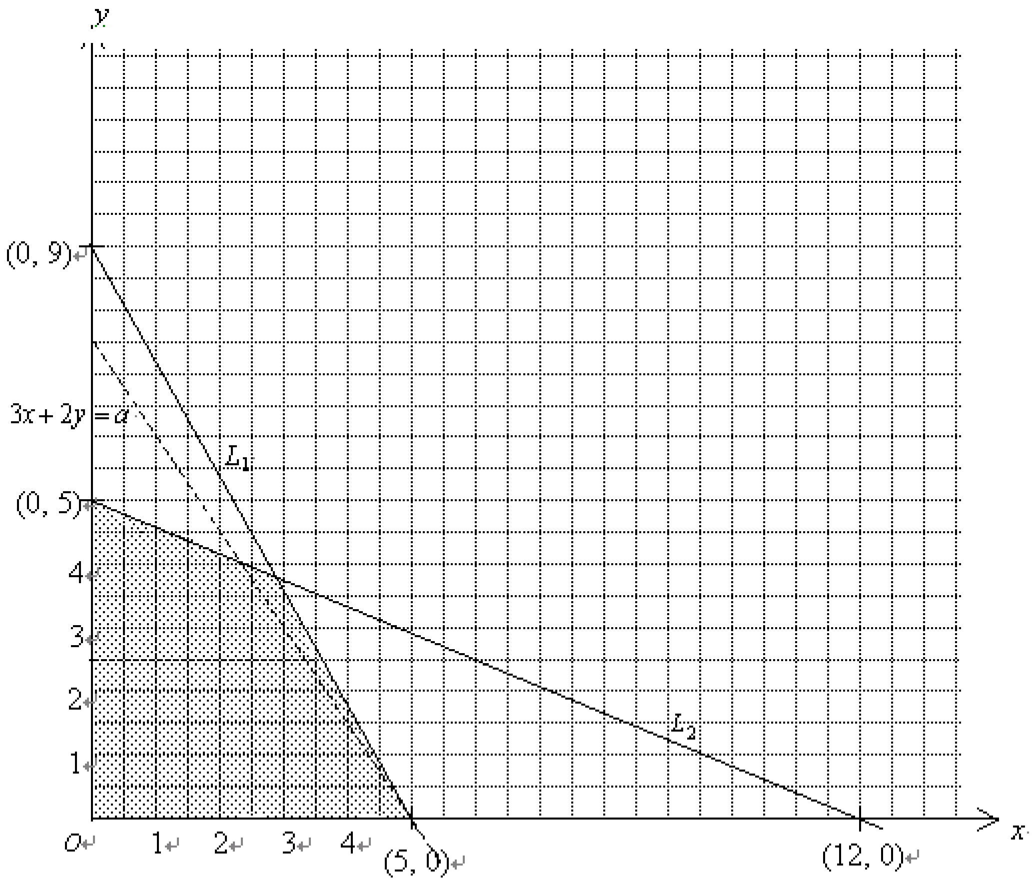
$5x+12y=60k$

(b) (i) Let x and y be respectively the number of articles produced by lines A and B. The constraints are

$$\begin{cases} 45x+25y\le 225 & (\text{or } 9x+5y\le 45)\,, \\ 50x+120y\le 600 & (\text{or } 5x+12y\le 60)\,, \\ x \text{ and } y \text{ are non-negative integers.} \end{cases}$$

The profit is $\$\,1\,000\,(3x+2y)$.

Using the graph in Figure 11 with $k=1$, the feasible solutions are represented by the lattice points in the shaded region below.



From the graph, the most profitable combinations are (3, 3) and (5, 0) .

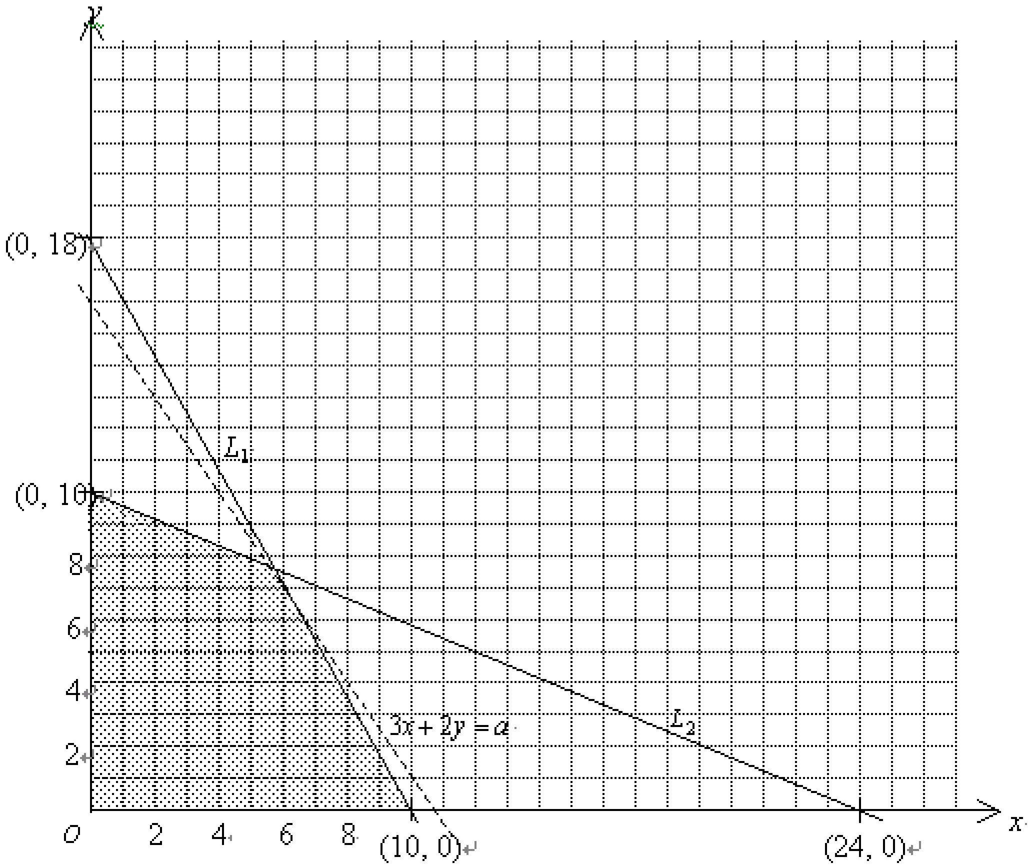
At (3, 3), the profit is \$ 1 000 (9 + 6) = \$ 15 000
At (5, 0), the profit is \$ 1 000 (15 + 0) = \$ 15 000
At (0, 5), the profit is \$ 1 000 (10) = \$ 10 000
At (2, 4), the profit is \$ 1 000 (6 + 8) = \$ 14 000

The greatest possible profit is \$ 15 000 .

(ii) Let x and y be respectively the number of articles produced by production lines A and B. The constraints are

$$\begin{cases} 45x+25y \le 450 & (\text{ or } 9x+5y \le 90), \\ 50x+120y \le 1200 & (\text{ or } 5x+12y \le 120), \end{cases}$$

x and y are non-negative integers.



Using the same graph as in (i) and taking $k = 2$,
the feasible solutions are represented by the lattice points in the shaded region.

From the graph , the most profitable combinations is (6, 7) .

The greatest possible profit is

$ 1 000 (18 + 14) = $ 32 000

END OF PAPER