Hong Kong Certificate of Education Examination
Mathematics Paper I

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Makers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

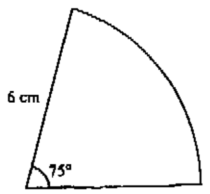
5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

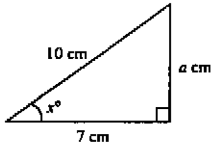
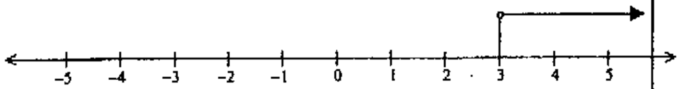
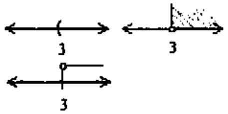
6. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).
   a. The symbol (*u-1*) should be used to denote 1 mark deducted for *u*. At most deduct ***1 mark*** for *u* for the whole paper.
   b. The symbol (*pp-1*) should be used to denote 1 mark deducted for *pp*. At most deduct ***2 marks*** for *pp* for the whole paper. For similar *pp*, deduct 1 mark for the first time that it occurs. Do not penalize candidates twice in the paper for the same *pp*.
   c. At most deduct 1 mark in each question. Deduct the mark for *u* first if both marks for *u* and *pp* may be deducted in the same question.
   d. In any case, do not deduct any marks for *pp* or *u* in those steps where candidates could not score any marks.

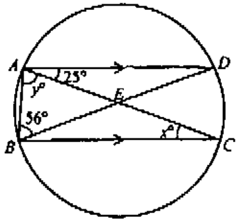
7. Marks entered in the Page Total Box should be the NET total scored on that page.

8. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to', 'f.t.' stands for 'follow through' and 'or equivalent' means 'accepting equivalent forms of the equation which may have not been simplified but without uncollected like terms'. Steps which can be skipped are shaded whereas alternative answers are enclosed with $\boxed{\text{rectangles}}$ or (brackets). All fractional answers must be simplified.

Solution	Marks	Remarks
1. When $C=30$,		
$30=\frac{5}{9}(F-32)$	1M	substituting $C=30$
$\frac{30\times9}{5}=F-32$ (or $270=5F-160$)	1A	removing brackets
$F=86$	1A	
$\because C=\frac{5}{9}(F-32)$		
$\therefore F=\frac{9}{5}C+32$ (or $9C=5F-160$)	$\boxed{1A}$	removing brackets
When $C=30$,		
$F=\frac{9}{5}\times30+32$	$\boxed{1M}$	substituting $C=30$
$F=86$	$\boxed{1A}$	
	(3)	
2. $\frac{x^{-3}y}{x^2}=\frac{y}{x^2x^3}$ (or $x^{-2}x^{-3}y$)	1M	applying $a^{-n}=\frac{1}{a^n}$
$=\frac{y}{x^{2+3}}$ (or $x^{-2-3}y$)	1M	applying $a^m a^n=a^{m+n}$
$=\frac{y}{x^5}$	1A	
	(3)	
3. Area of the sector $=\frac{75}{360}(6^2\pi)$ cm²	1M+1A	1M for ratio or area of circle
≈ 23.6 cm² (or $7\frac{1}{2}\pi$ cm²)	1A	r.t. 23.6 or $\frac{15}{2}\pi$, 7.5π
Area of the sector $=\frac{1}{2}\times6^2\times\frac{75}{180}\pi$ cm²	$\boxed{1M}$+$\boxed{1A}$	1M for $\frac{1}{2}r^2\theta$ or correct value of θ
≈ 23.6 cm² (or $7\frac{1}{2}\pi$ cm²)	$\boxed{1A}$	r.t. 23.6 or $\frac{15}{2}\pi$, 7.5π
	(3)	



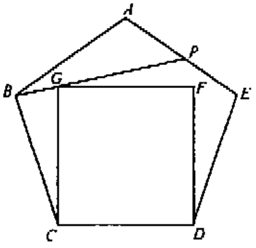
Solution	Marks	Remarks
4. $a^2+7^2=10^2$ (or $a=\sqrt{10^2-7^2}$)	1A	
$a=\sqrt{51}$ (or 7.14)	1A	r.t. 7.14
$\cos x^\circ=\frac{7}{10}$ (or $\sin x^\circ=\frac{\sqrt{51}}{10}$, $\tan x^\circ=\frac{\sqrt{51}}{7}$)	1M	
$x\approx 45.6$	1A	r.t. 45.6, u–1 for $x\approx 45.6^\circ$, $x\approx 45^\circ 34'$, $x^\circ\approx 45.6^\circ$, $x^\circ\approx 45.6$
$\cos x^\circ=\frac{7}{10}$	1A	
$x\approx 45.6$	1A	
$a\approx 10\sin 45.6^\circ$ (or $a\approx 7\tan 45.6^\circ$)	1M	
$a\approx 7.14$	1A	
	(4)	
5. $\frac{11-2x}{5}<1$		
$11-2x<5$ (or $\frac{11}{5}-\frac{2}{5}x<1$)	1A+1A	For any 2 of these 3 steps, 1A for each. 2 of these 3 steps can be omitted.
$-2x<-6$		
$2x>6$ (or $6<2x$, $\frac{2}{5}x>\frac{6}{5}$)		
$x>3$	1A	
	1M	or 
	(4)	
6. f(–3) (or $2(-3)^3+6(-3)^2-2(-3)-7$)	2A	
$=-1$	1A	
∴ The remainder is –1 .		
$2+0-2$ $1+3\,\overline{)\,2+6-2-7}$ $2+6$ $-2-7$ $-2-6$ -1	$\boxed{2A}$	
∴ Remainder = –1	$\boxed{1A}$	
	(3)	

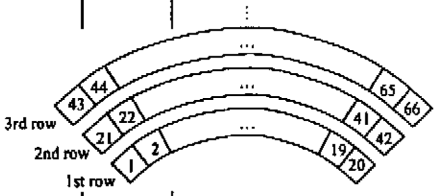
Solution	Marks	Remarks
7. $x=25$	1A	u–1 for $x=25^\circ$, $x^\circ=25^\circ$
∵ $\angle ADB=x^\circ$	1M	applying ∠s in same segment
∴ $y=180-56-25-x$	1M	
$=74$	1A	u–1 for $y=74^\circ$, $y^\circ=74^\circ$
	(4)	
8. Actual area = 220×5000^2 cm²	2M	for $\times 5000^2$, ignore unit
$=\frac{220\times 5000^2}{100^2}$ m²	1M	for $\div 100^2$, pp–1 for not handling units properly
$=550\,000$ m² (or area in m² = 550 000)	1A	
	(4)	
9. (a) Slope of $L=\frac{4-0}{-4-6}$		
$=-\frac{2}{5}$ (or –0.4)	1A	
(b) Equation of L :		
$y=-\frac{2}{5}(x-6)$ (or $\frac{y-4}{x+4}=-\frac{2}{5}$)	1M	
$y=-\frac{2}{5}x+\frac{12}{5}$ (or $2x+5y-12=0$)	1A	or equivalent
(c) When $x=0$,	1M	
$y=\frac{12}{5}$. (or $y=2.4$)	1A	
∴ $C=(0,\frac{12}{5})$.		
	(5)	

Solution	Marks	Remarks
10. (a) $10x^2+9x-22=0$		
$(x+2)(10x-11)=0$ (or $x=\frac{-9\pm\sqrt{9^2+4\times10\times22}}{2\times10}$)	1A	
$x=-2$ or $\frac{11}{10}$ (or $x=-2$ or 1.1)	1A	
	(2)	
(b) $10000(1+r\%)^2+9000(1+r\%)=22000$	1M+1A	1M for $10000(1+r\%)^2$
$[10000(1+r\%)+9000](1+r\%)=22000$	$\boxed{\text{1M}}+\boxed{\text{1A}}$	1M for $10000(1+r\%)+9000$
		pp−1 for confusing r with $r\%$
$10(1+r\%)^2+9(1+r\%)-22=0$ (or $r^2+290r-3000=0$, $10(r\%)^2+29(r\%)-3=0$)		
From (a), $1+r\%=1.1$	1M	for choosing '+ve' value from 1 '+ve' and 1 '−ve' roots, provided that the original equation must be correct
$r=10$	1A	
	(4)	
11. (a) Missing value in 1st table = 66	1A	
Missing value in 2nd table = 20	1A	
	(2)	
(b) An estimate of the mean		
$=\frac{210\times3+230\times13+250\times30+270\times20+290\times9}{75}$ (seconds)	1M	
≈ 255 seconds	1A	r.t. 255
	(2)	
(c) Median ≈ 254 seconds (or 255 seconds)	1A	r.t. 254 or 255
	(1)	
(d) Number of songs have lengths greater than 220 seconds but not greater than 260 seconds		
$=13+30$ (or $46-3$)		
$=43$	1A	
Percentage required $=\frac{43}{75}\times100\%$		
$\approx 57.3\%$ (or $57\frac{1}{3}\%$)	1A	r.t. 57.3
	(2)	

Solution	Marks	Remarks
12. (a) Numbers having two zero digits are 100, 200, …, 900.		
Probability required $=\frac{9}{900}$	1A	for numerator
$=\frac{1}{100}$ (or 0.01)	1A	
Probability required $=\frac{1}{10}\times\frac{1}{10}$	$\boxed{\text{1A}}$	
$=\frac{1}{100}$ (or 0.01)	$\boxed{\text{1A}}$	
	(2)	
(b) Numbers having no zero digits are 111, 112, …, 119 … 911, 912, …, 919 121, 122, …, 129 … 921, 922, …, 929 ⋮ ⋮ 191, 192, …, 199 … 991, 992, …, 999		
Probability required $=\frac{9\times9\times9}{900}$	1A	for numerator
$=\frac{81}{100}$ (or 0.81)	1A	
Probability required $=\frac{9}{10}\times\frac{9}{10}$	$\boxed{\text{1A}}$	
$=\frac{81}{100}$ (or 0.81)	$\boxed{\text{1A}}$	
	(2)	
(c) Numbers having exactly one zero digit are 101, 102, …, 109, 110, 120, …, 190 201, 202, …, 209, 210, 220, …, 290 ⋮ ⋮ 901, 902, …, 909, 910, 920, …, 990		
Probability required $=\frac{9\times9+9\times9}{900}$	1A	for numerator
$=\frac{9}{50}$ (or 0.18)	1A	
Probability required $=1-\frac{1}{100}-\frac{81}{100}$	$\boxed{\text{1M}}$	
$=\frac{9}{50}$ (or 0.18)	$\boxed{\text{1A}}$	
Probability required $=\frac{1}{10}\times\frac{9}{10}\times2$	$\boxed{\text{1A}}$	
$=\frac{9}{50}$ (or 0.18)	$\boxed{\text{1A}}$	
	(2)	

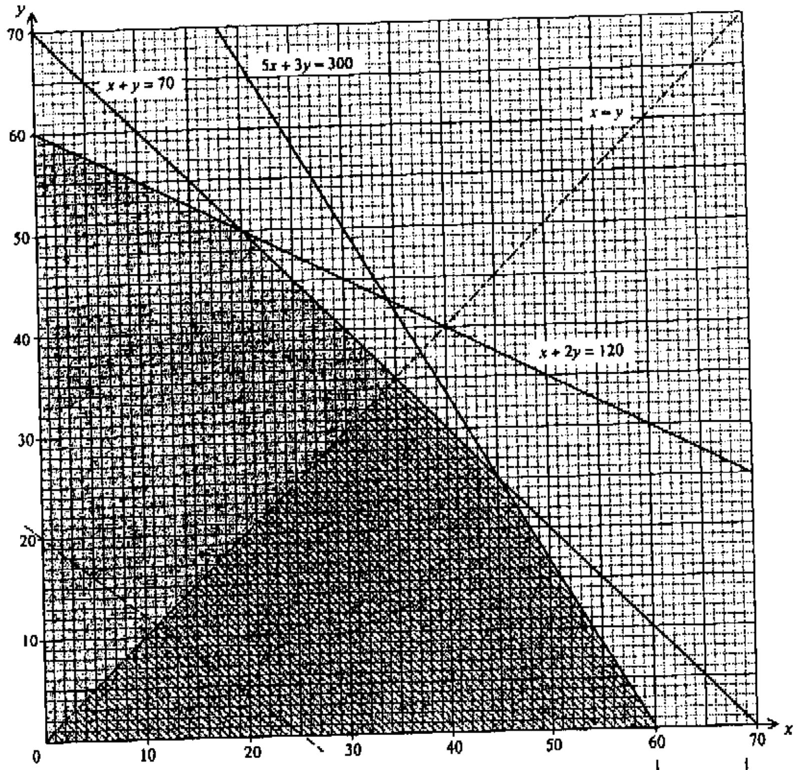
Solution	Marks	Remarks
13. (a) Size of each interior angle of the pentagon = $\frac{(5-2)\times180^\circ}{5}=108^\circ$	1A	
$\angle BCG=108^\circ-90^\circ=18^\circ$	1A	
$\angle CBG=\frac{180^\circ-18^\circ}{2}=81^\circ$	1M	
$\angle ABP=108^\circ-81^\circ=27^\circ$	1A	
$\angle APB=180^\circ-27^\circ-108^\circ=45^\circ$	1A (5)	
(b) $\because\ \frac{AP}{\sin 27^\circ}=\frac{AB}{\sin 45^\circ}$		
$\therefore\ AP=\frac{\sin 27^\circ}{\sin 45^\circ}AB$		
$=\frac{\sin 27^\circ}{\sin 45^\circ}AE$ (or $\frac{AP}{\sin 27^\circ}=\frac{AE}{\sin 45^\circ}$ etc.)	1M	
$\approx 0.642AE$ (or $AE\approx 1.56AP$)		1 d.p. is sufficient
$\therefore$ AP is longer than PE.	1M+1 (3)	





Solution	Marks	Remarks
14. (a) Number of seats in the last row $=20+2(50-1)$	1A	
$=118$	1A (2)	
(b) Total number of seats in the first n rows $=\frac{n}{2}[2\times20+2(n-1)]$	1A	
$=n^2+19n$		
If $n^2+19n=2000$, then (or $n^2+19n\ge2000$)	1M	
$n^2+19n-2000=0$		
$n=\frac{-19\pm\sqrt{19^2-4(-2000)}}{2}$		
$n=36.2$ or -55.2 (or $n\approx36.2$ only)	1A	r.t. 36.2, −55.2
$\therefore$ The seat numbered 2000 can be found in the 37th row.	1A	
Let $f(n)=n^2+19n$. $\because$ $f(36)=1980$ $f(37)=2072$ $\therefore$ The seat numbered 2000 can be found in the 37th row.	$\}\boxed{1M}+\boxed{1A}$ $\boxed{1A}$	
	(4)	

Solution	Marks	Remarks
15. (a) x and y satisfy the following conditions:		
$1000(40x)+800(30y)\le2400000$ or $5x+3y\le300$	1A	*Withhold 1 mark for any "<".*
$1000(10x)+800(25y)\le1200000$ or $x+2y\le120$	1A	
$x+y\le70$	1A	
x, y are non-negative integers		



Solution	Marks	Remarks
Draw the straight lines $5x+3y=300$, $x+2y=120$ and $x+y=70$.	1A+1A	1A for any correct line. 1A for all. Accept dotted lines or lines without labeling. Position of lines should not lie outside 1 small grid at the edges.
Let \$P(x, y) be the profit generated by x boxes of brand A mixed nuts and y boxes of brand B mixed nuts. Then $P(x,y)=800x+1000y$ $=200(4x+5y)$	1A	
By drawing parallel lines of $4x+5y=0$,	1M	check the line on graph
$\because$ $P(0,0)=0$, $P(0,60)=60000$, $P(20,50)=66000$, $P(45,25)=61000$ and $P(60,0)=48000$	$\boxed{1M}$	
$P(x,y)$ attains its maximum at $(20,50)$.	1A	f.t.
$\therefore$ The profit is the greatest when $x=20$ and $y=50$.	(8)	

Solution	Marks	Remarks
(b) In addition to the conditions in (a), x , y should also satisfy $y < x$. The feasible solution becomes the shaded region.	1A	or drawing $y = x$ in the figure
By considering lines parallel to $4x + 5y = 0$ (or testing points) , $P(x, y)$ attains its maximum at $(36, 34)$.	1A	
$\therefore$ The greatest profit is $62800.	1A (3)	

Solution	Marks	Remarks

16.

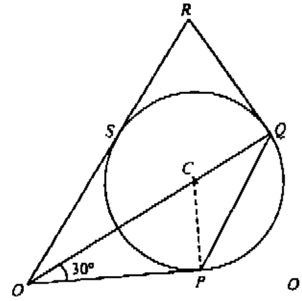


Figure 9A

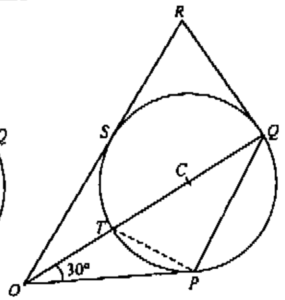


Figure 9B

Solution	Marks	Remarks
(a) Refer to Figure 9A,		
(L1)... $\angle OPC = 90°$ (tangent ⊥ radius)		(tangent properties) [切線⊥半徑]、[切線性質／定理]
(L2)... $\angle PCO = 180° - 90° - 30° = 60°$ (∠ sum of Δ)		[Δ內角和]
(L3)... $\angle PQO = \frac{1}{2}\angle PCO = 30°$ (∠ at centre twice ∠ at circumference)		(∠ at centre = 2 × ∠ at circumference) [圓心角兩倍於圓周角]、[圓心角是圓周角的兩倍]、[圓心角=2×圓周角]
Refer to Figure 9A, and let $\angle CQP = x$.		
(L4)... $\angle OPC = 90°$ (tangent ⊥ radius)		(tangent properties) [切線⊥半徑]、[切線性質／定理]
(L5)... $\angle PCO = 180° - 90° - 30° = 60°$ (∠ sum of Δ)		[Δ內角和]
(L6)... $\therefore$ $CP = CQ$ (radius)		
(L7)... $\therefore$ $\angle CPQ = \angle CQP = x$ (base ∠s of isos. Δ)		[等腰Δ底角]
(L8)... $2x = \angle PCO = 60°$ (ext. ∠ of Δ)		[Δ的外角]
(L9)... $x = 30°$		
Refer to Figure 9B, and let $\angle CQP = x$.		
(L10)... $\angle TPO = \angle CQP = x$ (∠ in alt. segment)		[交錯弓形的圓周角]、[弦切角定理]
(L11)... $\angle TPQ = 90°$ (∠ in semicircle)		[半圓上的圓周角]
(L12)... $\therefore$ $30° + 90° + 2x = 180°$ (∠ sum of Δ)		[Δ內角和]
(L13)... $x = 30°$		

Marking Scheme :			
Case 1	Any correct proof with correct reasons.	3	
Case 2	Any correct proof without reasons.	1	At most 2 marks
	In addition, any relevant correct argument with correct reason (at most 1 mark).	1	
Case 3	Any relevant correct argument with correct reason.	1	At most 1 mark

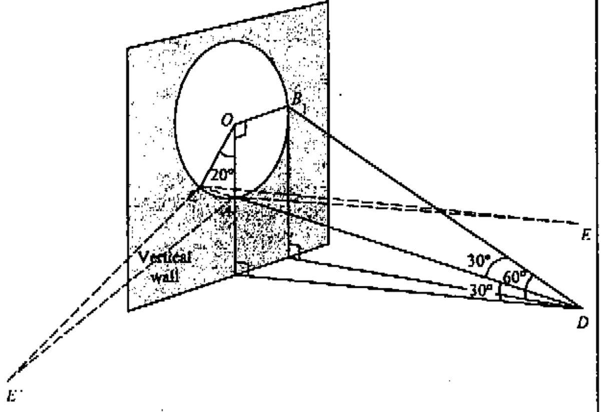
(3)

Solution	Marks	Remarks
(b) (i)		
(L14)... $\angle ROQ = \angle QOP = 30^\circ$ (tangents from ext. pt.)		(tangent properties) [切線性質／定理]
(L15)... $\angle PQO = 30^\circ$ (proved)		
(L16)... ∵ $\angle RQP + \angle POR = 180^\circ$ (opp. ∠s of cyclic quad.)		[圓內接四邊形的對角]
(L17)... ∴ $\angle CQR = 180^\circ - 3\times30^\circ = 90^\circ$		
(L18)... Hence RQ is tangent to circle PQS at Q. (conv. of tangent ⊥ radius)		[切線上半徑的逆定理]

Marking Scheme :		
Case 1 Any correct proof with correct reasons.	3	
Case 2 Any correct proof without reasons.	1	
In addition, any relevant correct argument with correct reason (at most 1 mark).	1	At most 2 marks
Case 3 Any relevant correct argument with correct reason.	1	At most 1 mark

Solution	Marks	Remarks
	(3)	
(b) (ii) ∵ Slope of $OC = \frac{4}{3}$		
∴ Slope of $QR = -\frac{3}{4}$	1M	
$OC = \sqrt{6^2+8^2} = 10$	1A	
$CQ = CP = OC \sin 30^\circ = 5$	1M	
Let the coordinates of Q be (x, y). ∵ $OC : CQ = 10 : 5 = 2 : 1$ ∴ $\frac{2x+1(0)}{3} = 6$ and $\frac{2y+1(0)}{3} = 8$	1M	
Equation of circle: $(x-6)^2+(y-8)^2 = 25$ $x^2+y^2-12x-16y+75=0$ ....(1) Equation of OC: $y = \frac{4}{3}x$ ....(2) Solving (1) and (2), $x^2-12x+27=0$ (or $y^2-16y+48=0$) $x = 3$ (rej.) or 9 (or $y = 4$ (rej.) or 12)	$\boxed{\text{1M}}$	must reject the smaller root
$x = 9$ and $y = 12$		
Hence the equation of QR is $\frac{y-12}{x-9} = -\frac{3}{4}$ $3x+4y-75=0$ (or $y = -\frac{3}{4}x+\frac{75}{4}$)	1A	
	(5)	

Solution	Marks	Remarks
17. (a) (i) $AD = \frac{h}{\sin 30^\circ}$ m $= 2h$ m	1A	u−1 for missing unit
$BD = \frac{h+10}{\sin 60^\circ}$ m $= \frac{2}{\sqrt{3}}(h+10)$ m $= \frac{2\sqrt{3}}{3}(h+10)$ m	1A	
(ii) $AB^2 = 10^2+10^2$ (m²)	1A	or $AB = \sqrt{200}$, $\frac{10}{\sin 45^\circ}$ m etc.
By cosine law, $AB^2 = AD^2+DB^2-2(AD)(DB)\cos\angle ADB$		
$200 = \left(\frac{h}{\sin 30^\circ}\right)^2 + \left(\frac{h+10}{\sin 60^\circ}\right)^2 - 2\left(\frac{h}{\sin 30^\circ}\right)\left(\frac{h+10}{\sin 60^\circ}\right)\cos 30^\circ$	1M+1A	Do not accept setting $AD = BD$
$200 = 4h^2+\frac{4}{3}(h+10)^2-4h(h+10)$		
$h^2-10h-50=0$	1A	or multiples
$h = 13.660$ or -3.660		or $5\pm5\sqrt{3}$
$h \approx 13.7$ or -3.66 (rejected) $\boxed{5+5\sqrt{3} \text{ or } 5-5\sqrt{3} \text{ (rejected)}}$	1A	or $h \approx 13.7$ only
	(7)	



Solution	Marks	Remarks
(b) $AC = 2(10\sin 10^\circ)$ (m) $\boxed{\sqrt{10^2+10^2-2(10)(10)\cos 20^\circ} \text{ (m)}}$ $= 20\sin 10^\circ$ (m) ≈ 3.47296 (m)	1A	
$AE = \frac{h}{\sin 25^\circ}$ (m) ≈ 32.3 (m)		
By sine law, $\sin\angle ACE = \frac{AE\sin 5^\circ}{AC}$ $\approx \frac{h\sin 5^\circ}{20\sin 10^\circ \sin 25^\circ}$	1M	
≈ 0.8112 ∴ $\angle ACE = 54.2^\circ$ or 126° $\boxed{54^\circ 13' \text{ or } 126^\circ}$	1A+1A	r.t. 54.2, 126
	(4)	

Solution	Marks	Remarks

18. (a) Let $V = ah^2 + bh^3$ where a, b are non-zero constants. Then 1A

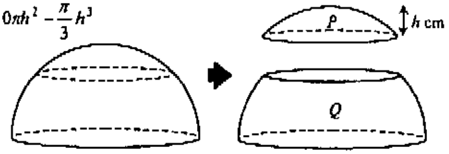
$$\begin{cases} \frac{29}{3}\pi = a+b \\ 81\pi = 9a+27b \end{cases} \qquad \begin{cases} a+b = \frac{29}{3}\pi \text{ ....(1)} \\ a+3b = 9\pi \text{ ....(2)} \end{cases}$$ 1M

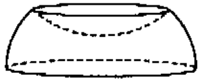
(2) − (1) gives $2b = -\frac{2}{3}\pi$

Hence $b = -\frac{\pi}{3}$ and $a = 10\pi$ 1A

$\therefore \quad V = 10\pi h^2 - \frac{\pi}{3}h^3$

(3)





(b) (i) Surface area $= 2\pi \times 10^2$ (cm²)

≈ 628 cm² (or 200π cm²) 1A r.t. 628

(ii) $\because$ Volume of hemisphere $= \frac{2}{3}\pi \times 10^3$ (cm³) 1A

$\therefore \quad \frac{2}{3}\pi \times 10^3 - 2V = \frac{1400}{3}\pi$

$\frac{2}{3}\pi \times 10^3 - 2(10\pi h^2 - \frac{\pi}{3}h^3) = \frac{1400}{3}\pi$ 1M

$\frac{2}{3}\pi(1000 - 30h^2 + h^3 - 700) = 0$

$h^3 - 30h^2 + 300 = 0$ 1

From the graph in Figure 11.3,

$3.3 < h < 3.4$ (or $3.35 < h < 3.4$ etc.) 1M or claiming to draw $y = -300$, writing $h \approx 3.35$, $h \approx 3.4$ etc.

Let $f(h) = h^3 - 30h^2 + 300$, then $f(3.3) > 0$ and $f(3.4) < 0$.

Using the method of bisection,

Interval	"mid-value"	f(h)
$3.3 < h < 3.4$	3.35	+ve (0.9204)
$3.35 < h < 3.4$	3.375	−ve (−3.2754)
$3.35 < h < 3.375$	3.363	−ve (−1.2583)
$3.35 < h < 3.363$	3.357	−ve (−0.2519)
$3.35 < h < 3.357$	3.354	+ve (0.2507)
$3.354 < h < 3.357$	3.356	−ve (−0.0843)
$3.354 < h < 3.356$	3.355	+ve (0.0832)

1M 1M

use interval ⊆ [0, 5] containing the root as the starting interval testing sign of "mid-value" or any intermediate value

choosing the correct interval

$\therefore \quad 3.355 < h < 3.356$

$h \approx 3.36$ (correct to 2 decimal places) 1A f.t.

Let $f(h) = h^3 - 30h^2 + 300$.

$\because \quad f(3.34) \approx 2.5917$

$f(3.35) \approx 0.9203 \qquad f(3.355) \approx 0.0832$ $\boxed{1M}$+$\boxed{1M}$

$f(3.36) \approx -0.7549$

$f(3.37) \approx -2.4342$

$\therefore \quad h \approx 3.36$ (correct to 2 decimal places) $\boxed{1A}$ f.t.

(8)