香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九七年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1997

數學 試卷一
MATHEMATICS PAPER I

本評卷參考乃考試局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。

After the examinations, marking schemes will be available for reference at the Teachers' Centres.

97-CE-MATHS I-1

Hong Kong Certificate of Education Examination
Mathematics Paper I

NOTES FOR MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, provided that the method used is sound.

2. In a question consisting of several parts each depending on the previous parts, marks may be awarded to steps or methods correctly deduced from previous erroneous answers. However, marks for the corresponding answers should NOT be awarded. In the marking scheme, marks are classified as:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Others	awarded for correctly completing a proof or arriving at an answer given in a question.

3. Use of notation different from those in the marking scheme should not be penalised.

4. Each mark deducted for *poor presentation* (p.p.) should be denoted by [pp–1] :
   a. At most deduct 1 mark for (p.p.) in each question, up to a maximum of 3 marks for the whole paper.
   b. For similar (p.p.), deduct 1 mark for the first time that it occurs.
      i.e. do not penalise candidates twice in the paper for the same p.p.

5. Each Mark deducted for *wrong/no unit* (u.) should be denoted by [u–1] :
   a. No mark can be deducted for (u.) in Section A.
   b. At most deduct 1 mark for (u.) for the whole paper.

6. Marks entered in the Page Total Box should be the NET total scored on that page.

Solution	Marks	Remarks
1. (a) $x^2 - 9 = (x-3)(x+3)$	2A	
(b) $ac + bc - ad - bd = (a+b)c - (a+b)d$	1A	
$= (a+b)(c-d)$	1A	
	(4)	
2. (a) $\sqrt{27} - \sqrt{12} = 3\sqrt{3} - 2\sqrt{3}$	1A	For simplifying either term
$= \sqrt{3}$	1A	
(b) $\frac{1}{2\sqrt{3}+\sqrt{2}} = \frac{2\sqrt{3}-\sqrt{2}}{(2\sqrt{3}+\sqrt{2})(2\sqrt{3}-\sqrt{2})}$	1A	
$= \frac{2\sqrt{3}-\sqrt{2}}{(2\sqrt{3})^2-(\sqrt{2})^2}$	1A	can be omitted
$= \frac{2\sqrt{3}-\sqrt{2}}{10}$ (or $\frac{\sqrt{3}}{5} - \frac{\sqrt{2}}{10}$, $\frac{\sqrt{2}(\sqrt{6}-1)}{10}$)	1A	
	(5)	
3. (a) $\frac{x^3y^2}{x^{-3}y} = x^{3-(-3)}y^{2-1}$	1M	For applying $a^m a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$ or $\frac{1}{a^n} = a^{-n}$
$= x^6 y$	1A	
(b) $\frac{\log 8 + \log 4}{\log 16} = \frac{\log 2^3 + \log 2^2}{\log 2^4}$	1M	For expressing the numbers as powers of a common number
$= \frac{3\log 2 + 2\log 2}{4\log 2}$	1M	For applying $\log a^n = n\log a$
<u>OR</u> $\frac{\log 8 + \log 4}{\log 16} = \frac{\log 32}{\log 16}$		
$= \frac{\log 2^5}{\log 2^4}$	1M	
$= \frac{5\log 2}{4\log 2}$	1M	
$= \frac{5}{4}$ (or 1.25)	1A	
	(5)	

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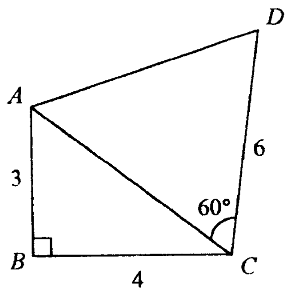
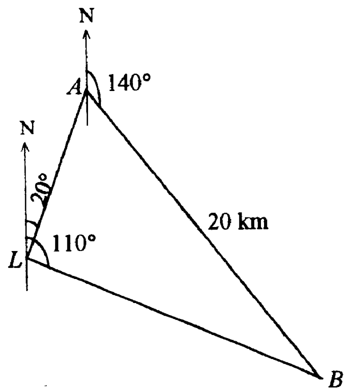
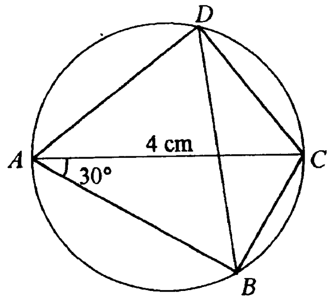
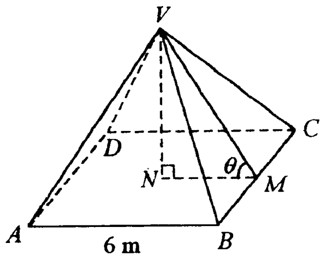
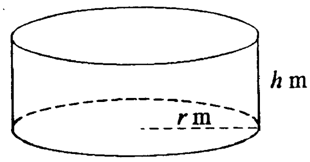
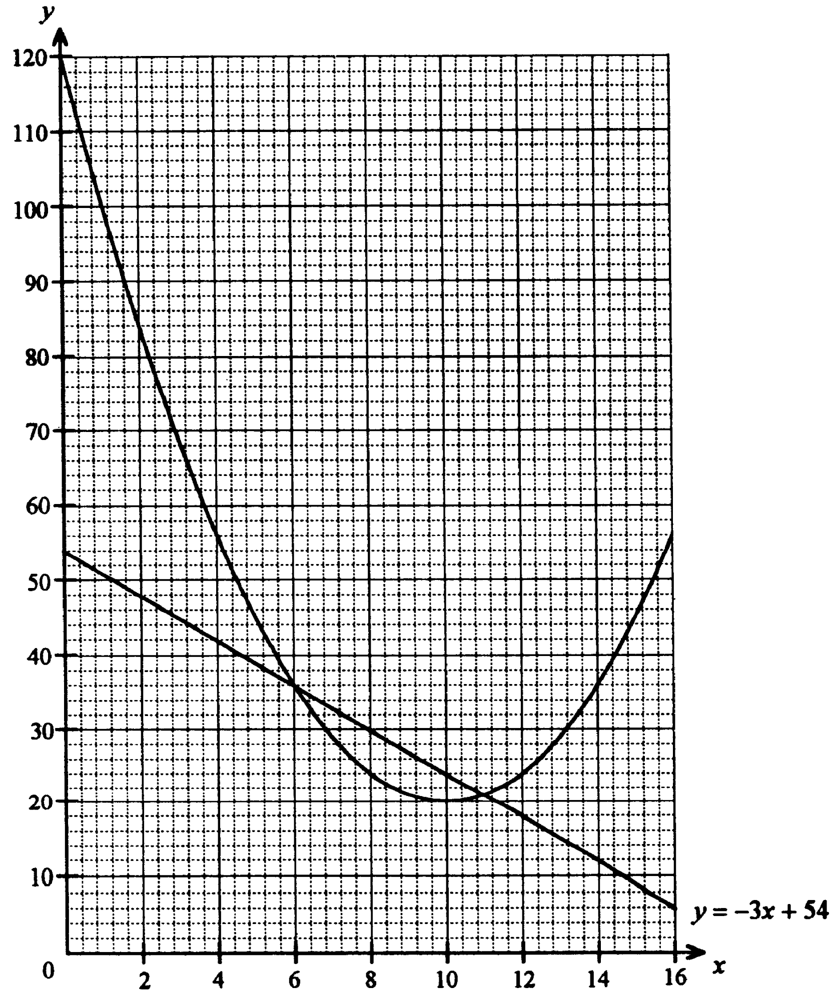
Solution	Marks	Remarks
Note: In question 4, accept graphical solutions if no algebraic expressions as answers are provided. Withhold 1 mark for having equal signs in inequalities.		
4. (i) $2x - 17 > 0$ $x > \frac{17}{2}$	1A	
(ii) $x^2 - 16x + 63 > 0$ $(x-7)(x-9) > 0$	1A	For factorization, can be omitted
$x < 7$ or $x > 9$	2A	
The range of values of x which satisfy both the inequalities in (i) and (ii): $x > 9$	1A	
	(5)	
5. 		
(a) $AC = 5$	1A	
(b) $AD = \sqrt{5^2 + 6^2 - 2(5)(6)(\cos 60^\circ)}$	1M	For the cosine rule
$= \sqrt{31}$ (or 5.57)	1A	r.t. 5.57
(c) Area of $\Delta ACD = \frac{1}{2}(5)(6)\sin 60^\circ$	1M	
$= \frac{15}{2}\sqrt{3}$ (or 13.0)	1A	r.t. 13.0
	(5)	

Solution	Marks	Remarks
6. 		
(a) $\angle LAB = 180° - 140° + 20° = 60°$ (or $\angle LBA = 30°$)	1A	
$\because$ $\angle ALB = 110° - 20° = 90°$		
$\therefore$ ΔALB is right-angled at L		
$LB = 20 \sin 60°$ km	1M	
$= 10\sqrt{3}$ km (or 17.3 km)	1A	r.t. 17.3
(b) $\angle ABL = 30°$		
Let ϕ be the bearing of L from B.		
Then $\phi = 360° - 30° - 40° = 290°$	1M	
$\therefore$ The bearing of L from B is $290°$. (or N70°W)	1A	
	(5)	
7. (a) The height of the smaller cone : the height of the larger cone		
$= 2 : 3$	1A	
(b) Total surface area of the smaller cone : total surface area of the larger cone		
$= 4 : 9$	1M	
The cost of painting the larger cone $= \$32 \times \frac{9}{4}$	1M	
$= \$72$	1A	
	(4)	
8. (a) $\alpha + \beta = \frac{7}{2}$	1A	
$\alpha\beta = 2$	1A	
(b) $(\alpha + 2) + (\beta + 2) = (\alpha + \beta) + 4$		
$= \frac{7}{2} + 4$		
$= \frac{15}{2}$	1A	
$(\alpha + 2)(\beta + 2) = \alpha\beta + 2(\alpha + \beta) + 4$		
$= (2) + 2(\frac{7}{2}) + 4$	1M	
$= 13$	1A	
$\therefore$ The required equation is $2x^2 - 15x + 26 = 0$.	1A	Or equivalent
	(6)	

Solution	Marks	Remarks
9. 		
(a) $\angle BDC = 30°$	1A	
$\angle ADC = 90°$	1A	can be omitted
$\angle ADB = 90° - 30°$		
$= 60°$	1A	
(b) $\overset{\frown}{AB} : \overset{\frown}{BC} = 60 : 30$		
$= 2 : 1$	1A	Accept 2
(c) $AB : BC = 4\cos 30° : 4\sin 30°$ (or $\tan 60°$)	1M	For finding AB and BC
$= \sqrt{3} : 1$ (or $1.73 : 1$, $1 : 0.577$)	1A	Accept $\sqrt{3}$ etc. Numerical ans. r.t. 1.73, 0.577
	(6)	
10. (a) Population at the end of 1998 $= 300\,000(1+2\%)^2$	1A	
$= 312\,120$	1A	r.t. 312000
OR Population at the end of 1997 $= 300\,000(1+2\%) = 306\,000$ Population at the end of 1998 $= 306\,000(1+2\%) = 312\,120$	1M+1A	r.t. 312000
(b) If $300\,000(1+2\%)^n = 330\,000$,	1A	
then $1.02^n = 1.1$		
$n \log 1.02 = \log 1.1$	1M	
$n \approx 4.81$	1A	Accept $n = 5$
$\therefore$ The population will exceed 330 000 at the end of 2001.	1A	
OR Population at the end of 1999 $= 300\,000(1+2\%)^3 \approx 318\,362$	1M	1M for calculating the populations of any two years
Population at the end of 2000 $= 300\,000(1+2\%)^4 \approx 324\,730$	1A	r.t. 325000
Population at the end of 2001 $= 300\,000(1+2\%)^5 \approx 331\,224$	1A	r.t. 331000
$\therefore$ The population will exceed 330 000 at the end of 2001.	1A	
	(6)	

Solution	Marks	Remarks
11. (a) (i) Mean = 64.4	1A	r.t. 64.4
(ii) Mode = 95	1A	
(iii) Median = 78	1A	
(iv) Standard deviation = 30.6	1A	r.t. 30.6
(b) This is because the distribution of marks in the Mathematics test is biased (to the high end).	1	
(c) (i) Let the student scored x marks in the English test.		
$\frac{x-63}{15}=0.4$	1A	
$x=69$	1A	
(ii) (I) Percentage of classmates scored fewer marks than Lai Wah in the Mathmatics test		
$=\frac{17}{35}\times 100\%$		
$\approx 48.6\%$ (or $48\frac{4}{7}\%$)	1A	r.t. 48.6
(II) The standard score of Lai Wah in the English test		
$=\frac{78-63}{15}$		
$=1$	1A	Or 84%
$\because$ The marks of the English test is normally distributed		
$\therefore$ More than half (or about 84%) of her classmates scored less than her.		
Hence Lai Wah performed better in the English test than in the Mathematics test relative to her classmates.	1	
OR (II) The standard score of Lai Wah in the English test		
$=\frac{78-63}{15}$		
$=1$	1A	
The standard score of Lai Wah in the Mathematics test		
$=\frac{78-64.4}{30.6}$		
≈ 0.44		
$\therefore$ Lai Wah performed better in the English test than in the Mathematics test relative to her classmates.	1	
(iii) The mean of the marks in the English test after the wrong mark has been corrected		
$=63+\frac{10}{35}$ (or $\frac{63\times 35+10}{35}$)	1A	
≈ 63.3	1A	r.t. 63.3

Solution	Marks	Remarks
12.		
 		
(a) (i) $VN = 3\tan\theta$ m	1A	
$VM = \frac{3}{\cos\theta}$ m (or $3\sqrt{1+\tan^2\theta}$ m)	1A	
(ii) Capacity $= \frac{1}{3}\cdot 6^2\cdot 3\tan\theta$ m^3 .....(1)	1M	For either (1) or (2)
$= 36\tan\theta$ m^3	1A	
Total surface area $= 4\cdot\frac{6}{2}\cdot\frac{3}{\cos\theta}$ m^2 .....(2)		
$= \frac{36}{\cos\theta}$ m^2 (or $\frac{36}{\sqrt{1+\tan^2\theta}}$ m^2)	1A	
(b) (i) $\because$ The base areas of the greenhouses are the same		
$\therefore$ $\pi r^2 = 36$		
$r = \frac{6\sqrt{\pi}}{\pi}$ (or $\frac{6}{\sqrt{\pi}}$)	1A	
(ii) $\because$ The capacities of the greenhouses are the same		
$\therefore$ $36h = 36\tan\theta$ (or $\pi\left(\frac{6}{\sqrt{\pi}}\right)^2 h = 36\tan\theta$)	1M	
$h = \tan\theta$	1A	
(iii) If the total surface areas of the greenhouses are equal, then		
$\pi r^2 + 2\pi rh = \frac{36}{\cos\theta}$		
$36 + 2\pi\cdot\frac{6}{\sqrt{\pi}}\cdot\tan\theta = \frac{36}{\cos\theta}$	1M	
$36 + 12\sqrt{\pi}\tan\theta = \frac{36}{\cos\theta}$		
$3 + \sqrt{\pi}\tan\theta = \frac{3}{\cos\theta}$	1	
(iv) $\because$ $3 + \sqrt{\pi}\tan 61^\circ - \frac{3}{\cos 61^\circ}$ (≈ 0.00960) > 0	1M+1A	r.t. 0.01
$3 + \sqrt{\pi}\tan 62^\circ - \frac{3}{\cos 62^\circ}$ (≈ -0.0567) < 0		r.t. −0.06
$\therefore$ (*) has a root between 61° and 62° .		

Solution	Marks	Remarks
13. (a) (i) From the graph, y is minimum when $x = 10$		
$\therefore$ Number of belts in a batch = 10	1A	
(ii) From the graph, $y < 90$ when $x \geq 2$	1M	Accept $x > 1.6$, $x \geq 1.6$ or $x = 2, 3, 4, \ldots$
i.e. $x = 2, 3, \ldots, 11$	1A	Accept $2 \leq x \leq 11$
$\therefore$ Number of belts in a batch = 2, 3, 4, ..., 11		
(b) (i) $144 = 3^2 - 17(3) + c$, $c = 186$	1A	
(ii) If $H = 120$, then $x^2 - 17x + 186 = 120$	1M	
$x^2 - 17x + 66 = 0$		
$x^2 - 20x + 120 = -3x + 54$		
By adding the line $y = -3x+54$ on the graph,	1A	
	1A	$x = 6 \pm 0.2$, 11 ± 0.2
$x = 6$ or 11 (rej.)	1A+1A	
$\therefore$ The required number of handbags is 6.		
(iii) Total cost of 10 belts and 6 handbags		
$= \$[10 \times (10^2 - 20 \times 10 + 120) + 6(6^2 - 17 \times 6 + 186)]$	1A	
$= \$[10 \times 20 + 6 \times 120]$		
$= \$920$		
Total income for selling the belts and handbags		
$= \$[6 \times 100 + 4 \times 300 + 4 \times 10 + 2 \times 60]$	1A	
$= \$1960$		
$\therefore$ She gained \$1040.	1A	

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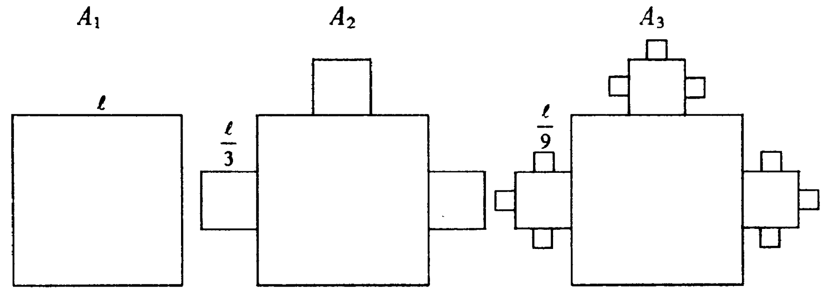
Solution	Marks	Remarks
14. (a) (i) $P(0 < T \le 200) = \frac{40}{50} \cdot \frac{39}{49}$	1A	
$= \frac{156}{245}$ (or 0.637)	1A	r.t. 0.637 (Ref. A_1)
(ii) $P(500 \le T \le 700) = \frac{10}{50} \cdot \frac{40}{49} + \frac{40}{50} \cdot \frac{10}{49}$	1M+1A	1M for $p_1p_2 + p_2p_1$ 1A for either term
$= 2 \cdot \frac{10 \times 40}{50 \times 49}$		
$= \frac{16}{49}$ (or 0.327)	1A	r.t. 0.327 (Ref. A_2)
(iii) $P(1000 \le T \le 1200) = \frac{10}{50} \cdot \frac{9}{49}$	1A	
$= \frac{9}{245}$ (or 0.0367)	1A	r.t. 0.0367 (Ref. A_3)
(iv) $P(T > 1200) = 0$	1A	Accept 'impossible', 'no chance'
(b) Let the total weight obtained in the afternoon be T'.		
(i) $P(T' < 450 \text{ or } T' > 850)$		
$= \frac{156}{245} + \frac{9}{245}$	1M	For $A_1 + A_3$
$= \frac{33}{49}$ (or 0.673)	1A	r.t. 0.673
(ii) $P(\lvert T - T' \rvert > 200)$		
$= 1 - \left(\frac{156}{245}\right)^2 - \left(\frac{16}{49}\right)^2 - \left(\frac{9}{245}\right)^2$	1M	For $1 - A_1^2 - A_2^2 - A_3^2$
$= \frac{29208}{60025}$ (or 0.487)	1A	r.t. 0.487
<u>OR</u>		
$= \frac{156}{245}\left(\frac{16}{49} + \frac{9}{245}\right) + \frac{16}{49}\left(\frac{156}{245} + \frac{9}{245}\right) + \frac{9}{245}\left(\frac{156}{245} + \frac{16}{49}\right)$	1M	
$= \frac{29208}{60025}$ (or 0.487)	1A	r.t. 0.487

Solution	Marks	Remarks

15. (a)



(i)

Table 1	$A_1 \to A_2$	$A_2 \to A_3$	$A_3 \to A_4$
Number of squares added	3	9	27
Length of sides of the squares added	$\frac{\ell}{3}$	$\frac{\ell}{9}$	$\frac{\ell}{27}$

Marks: 1A (number of squares added), 1A (length of sides)

(ii) Total area of all the squares in A_4

$= \ell^2 + 3(\frac{\ell}{3})^2 + 9(\frac{\ell}{9})^2 + 27(\frac{\ell}{27})^2$ — 1A+1M (1A for the first three terms; 1M for the 4th term)

$= (1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27})\ell^2$

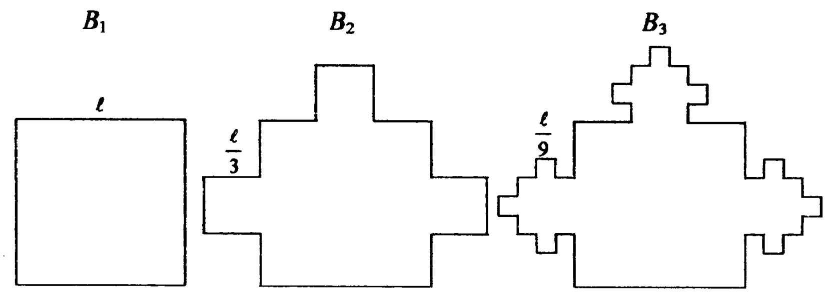
$= \frac{40}{27}\ell^2$ (or $1.48\ell^2$) — 1A (r.t. 1.48)

(iii) $k = (1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots)\ell^2$

$= \frac{1}{1 - \frac{1}{3}}\ell^2$ — 1A

$= \frac{3}{2}\ell^2$ (or $1.5\ell^2$) — 1A

(b)



(i)

Table 2	B_1	B_2	B_3	B_4
Perimeter	4ℓ	6ℓ	8ℓ	10ℓ

Marks: 1A+1A+1A

(ii) Perimeter of B_n

$= 4\ell + (n-1)(2\ell)$

$= 2(n+1)\ell$ (or $(2n+2)\ell$, $2n\ell + 2\ell$) — 1A

The perimeter of B_n would tend to infinity if n increases indefinitely. — 1A

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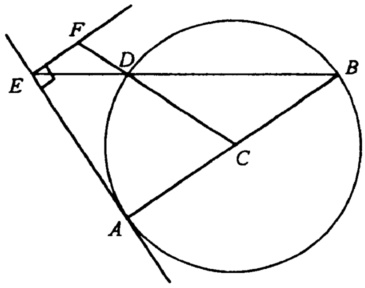
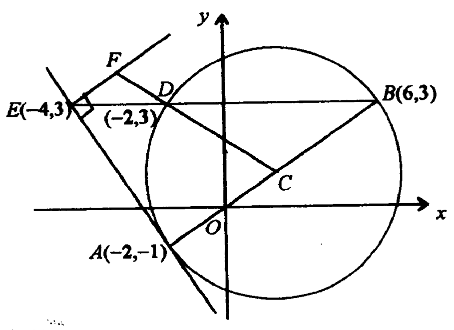
Solution	Marks	Remarks
16. (a)		



Solution	Marks	Remarks
(i) $\because$ $\angle CAE = 90°$		
$\therefore$ $\angle CAE + \angle FEA = 180°$		
Hence $AB // EF$ (int. $\angle$s supp.)		[同側(旁)內角互補]
Marking Scheme	2	
Case 1 Any correct proof with correct reasons	[2]	
Case 2 Any correct proof without correct reasons	[1]	
Case 3 Any relevant correct argument with correct reason	[1]	
(ii) $\because$ $\angle FDE = \angle CDB$ (vert. opp. $\angle$s)		[對頂角]
$\angle CDB = \angle CBD$ (base $\angle$s, isos. Δ)		[等腰Δ 底角]
$\angle CBD = \angle FED$ (alt. $\angle$s, $AB // EF$)		[(內)錯角，$AB//EF$]
$\therefore$ $\angle FDE = \angle FED$		
Hence $FD = FE$ (sides opp. equal $\angle$s)		Or "base $\angle$s equal", "converse of 'base $\angle$s, isos. Δ' ", "equal $\angle$s, equal sides" [等角對邊相等] 或 [等腰三角形底角等的逆定理] 或 [底角相等] 或 [等邊對等角] 或 [等角對等邊]
<u>OR</u>		
$\because$ $\angle FDE = \angle CDB$ (vert. opp. $\angle$s)		[對頂角]
$\angle FED = \angle CBD$ (alt. $\angle$s, $AB//EF$)		[(內)錯角，$AB//EF$]
$\therefore$ $\Delta CBD \sim \Delta FED$ (equiangular Δs)		Or "AAA" [等角]
$\because$ $CD = CB$		
$\therefore$ $FD = FE$		
Marking Scheme	4	
Case 1 Any correct proof with correct reasons	[4]	
Case 2 Any correct proof without correct reasons	[2]	
In addition, any relevant correct argument with correct reason	[1]	
Case 3 Any relevant correct argument with correct reason	[1]	

Solution	Marks	Remarks
(iii) Let $\mathscr{C}$ be the circle passing through D and touching AE at E.		
$\because$ $\mathscr{C}$ touches AE at E and $EF \perp AE$.		
$\therefore$ the centre of $\mathscr{C}$ lies on the line EF.	1	Pointing out $EF \perp AE$ for AE touching $\mathscr{C}$
$\because$ ED is a chord of $\mathscr{C}$ and $FD = FE$.		
$\therefore$ the centre of $\mathscr{C}$ lies on the perpendicular of DE through F.	1	Pointing out $FD = FE$ for F as centre or FD, FE as radii
F is the intersection of the lines which is the centre of $\mathscr{C}$.		
ACCEPT Consider the circle with F as centre and FD as radius. $\because$ $FD = FE$ $\therefore$ the circle passes through D and E.	1	
$\because$ $EF \perp AE$ and EF is a radius $\therefore$ the circle touches AE at E.	1	
(b) 		
Mid-point of $DE = (-3, 3)$ $\because$ ED is horizontal	1M	For any correct method of finding the x-coordinate of F
$\therefore$ x-coordinate of $F = -3$	1A	
Slope of $AE = -2$ Equation of EF: $\dfrac{y-3}{x+4} = \dfrac{1}{2}$ $x - 2y + 10 = 0$		
Sub. $x = -3$ into EF,	1M	For any correct method of finding the y-coordinate of F
$-3 - 2y + 10 = 0$ $y = \dfrac{7}{2}$	1A	
$\therefore$ $F = (-3, \dfrac{7}{2})$		
Note: Candidate may use equations of other straight lines for finding the coordinates of F: EF : $x - 2y + 10 = 0$ CD : $x + 2y - 4 = 0$ Perpendicular from F to DE : $x = -3$	1M	For correct mehods of finding any two of them
Solving any two of the above equations simultaneously.	1M	
Obtaining $F = (-3, \dfrac{7}{2})$.	1A+1A	

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