香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九五年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1995

數學　試卷一
MATHEMATICS PAPER I

本評卷參考乃考試局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

在今年考試結束後，各科評卷參考將存放於北角教師中心，供教師參閱。

Each year after the examinations, marking schemes will be available for reference at the North Point Teachers' Centre.

只限教師參閱 FOR TEACHERS' USE ONLY

Hong Kong Certificate of Education Examination
Mathematics Paper I

NOTES FOR MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits all the marks allocated to that part, provided that the method used is sound.

2. In a question consisting of several parts each depending on the previous parts, marks may be awarded to steps or methods correctly deduced from previous erroneous answers. However, marks for the corresponding answers should NOT be awarded. In the marking scheme, marks are classified as:

'M' marks -	awarded for correct methods being used;
'A' marks -	awarded for the accuracy of the answers;
Others -	awarded for correctly completing a proof or arriving at an answer given in a question.

3. Use of notation different from those in the marking scheme should not be penalised.

4. Each mark deducted for *poor presentation* (p.p.) should be denoted by [pp-1] :

   a. At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   b. For similar p.p., deduct 1 mark for the first time that it occurs.
   i.e. do not penalise candidates twice in the paper for the same p.p.

5. Each Mark deducted for *wrong/no unit* (u.) should be denoted by [u-1] :

   a. No mark can be deducted for u. in Section A.

   b. At most deduct 1 mark for u. for the whole paper.

6. Marks entered in the Page Total Box should be the NET total scored on that page.

Solution	Marks	Remarks
1. (a) $x \geq 2$	1A	
(b) $(x-1)^2(x+5)$	1A	
(c) $135°$ (or $\frac{3\pi}{4}$)	1A	
(d) 13	1A	
(e) 6	1A	
	(5)	
2. (a) $(a+b)^2-(a-b)^2 = (a^2+2ab+b^2)-(a^2-2ab+b^2)$	1A	
$= 4ab$	1A	
OR $(a+b)^2-(a-b)^2 = [(a+b)+(a-b)][(a+b)-(a-b)]$	1A	
$= 4ab$	1A	
(b) Remainder $= (-2)^3+1$	1A	
$= -7$	1A	
OR By long division		Or by synthetic division
$x^2 - 2x + 4$		
$x+2)\ x^3 + 0 + 0 + 1$		
$x^3 + 2x^2$		
$-2x^2$		
$-2x^2 - 4x$		
$4x + 1$		
$4x + 8$		
-7	2A	
	(4)	
3. (a) $S_{20} = \frac{20}{2}[2(1)+4(20-1)]$	1A	
$= 780$	1A	
(b) $S_\infty = \frac{9}{1-\frac{1}{3}}$	1A	
$= 13\frac{1}{2}$ (or 13.5)	1A	
	(4)	

Solution	Marks	Remarks
4. (a) Mr. Lee paid		
$2400000×(1+30%)	1A	
= $3120000	1A	
(b) $\frac{3000000-3120000}{3120000}\times 100\%$	1M	
$\approx -3.85\%$	1A	r.t. -3.85%
i.e. Mr. Lee lost 3.85%.		
	(4)	
5. (a) $x : y+1 = 4 : 5$		
$\frac{x}{y+1} = \frac{4}{5}$	1A	Can be omitted
$x = \frac{4(y+1)}{5}$	1A	Or other equivalent forms
(b) Sub. $x = \frac{4(y+1)}{5}$ into $2x+9y = 97$,		
$\frac{8(y+1)}{5} + 9y = 97$	1M	
$y=9$ and $x=8$	1A + 1A	
OR Let $x=4k$, $y+1=5k$ for some $k\neq 0$.		
Then $2(4k)+9(5k-1)=97$	1M	
$k=2$		
$x=8$ and $y=9$	1A + 1A	
	(5)	
6. $2\sin^2\theta+5\sin\theta-3 = 0$		
$(2\sin\theta-1)(\sin\theta+3) = 0$	1A	Can be omitted
$\sin\theta=\frac{1}{2}$ or $\sin\theta=-3$ (rej.)	1A + 1A	
$\theta = 30°$ or $150°$ ($\frac{\pi}{6}$ or $\frac{5\pi}{6}$)	1A + 1A	Deduct 1A for each excess answer
	(5)	

Solution	Marks	Remarks
7. (a) $3^x = \frac{1}{3^{\frac{3}{2}}}$	1A	For $\sqrt{27}=3^{\frac{3}{2}}$
$= 3^{-\frac{3}{2}}$	1M	For applying $\frac{1}{a^n}=a^{-n}$
$x = -\frac{3}{2}$	1A	
OR $3^x \cdot 3^{\frac{3}{2}} = 1$	1A	
$3^{x+\frac{3}{2}} = 1$	1M	
$x = -\frac{3}{2}$	1A	
(b) $\log x = \log 48 - 2\log 4$		
$= \log 48 - \log 4^2$	1M	For applying $n\log a=\log a^n$
$= \log \frac{48}{16}$	1M	For applying $\log a-\log b=\log \frac{a}{b}$
$= \log 3$		
$x = 3$	1A	
OR $\log x+\log 4^2 = \log 48$	1M	
$\log (4^2 x) = \log 48$	1M	
$x = 3$	1A	
	(6)	
8. (a) $x^2-3x-4=k$	1A	
$x^2-3x-k-4=0$		
$\therefore$ (i) $\alpha+\beta=3$	1A	Answer only
(ii) $\alpha\beta=-k-4$	1A	
(b) By section formula, $\frac{2\alpha+\beta}{3}=0$	1A	
$2\alpha+\beta=0.$		
Solving $\begin{cases}\alpha+\beta=3\\2\alpha+\beta=0\end{cases}$, we have		
$\alpha=-3$ (or $\beta=6$)	1A	
$k=(-3)^2-3(-3)-4$		
$= 14$	1A	
OR If $BP=2PA$, then $\beta=-2\alpha$	1A	
By (a), $\begin{cases}\alpha-2\alpha=3\\\alpha(-2\alpha)=-k-4\end{cases}$		
$\Rightarrow$ $\alpha=-3$	1A	
$k=14$	1A	
	(6)	

Solution	Marks	Remarks
9. (a) (i) Total number of students in S2 = 180	1A	
(ii) The median of the yearly average score = 59 (or 60)	1A	
(b) (i) The minimum yearly average score in Group One = 75	1A	
(ii) The minimum yearly average score in Group Two = 45 (or 44)	1A	

(c)

Yearly average score (x)	Class mark	Frequency
$20 < x < 30$	25	12
$30 < x \leq 40$	35	20
$40 < x \leq 50$	45	28
$50 < x \leq 60$	55	32
$60 < x \leq 70$	65	28
$70 < x \leq 80$	75	30
$80 < x \leq 90$	85	22
$90 < x \leq 100$	95	8

1A For the class marks

1A + 1A For the frequencies, 1A for any 3 being correct

(d) $\bar{x} = 59.6$ — 1A r.t. 59.6

$s = 19.0$ — 1A r.t. 19.0

(e) $\bar{x}-s = 40.6$

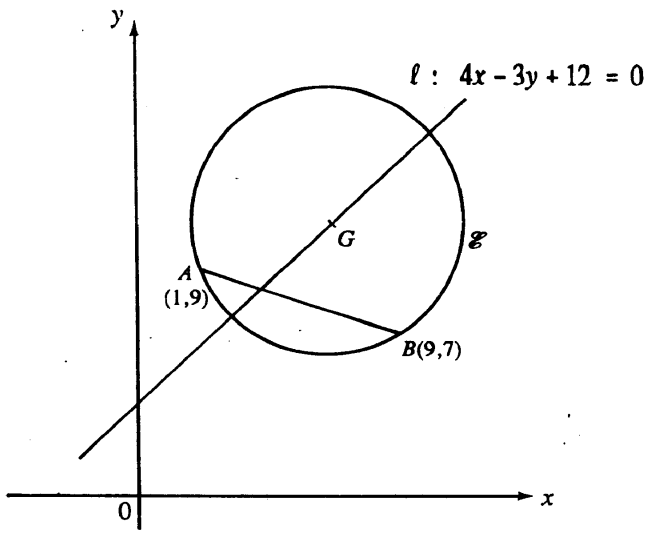
$\bar{x}+s = 78.6$

Number of students whose scores lie within $\bar{x}-s$ and $\bar{x}+s \approx 146-34$ — 1A + 1A 1A for 146, accept 144-146; 1A for 34, accept 32-34.

$= 112$ — Accept 110-114

The percentage is $\frac{112}{180} \times 100\% \approx 62.2\%$ — 1A Accept answers r.t. 61.1-63.3

Solution	Marks	Remarks
10. (a) Equation of AB : $\frac{y-9}{x-1} = \frac{7-9}{9-1}$	1A	
$y-9=-\frac{1}{4}(x-1)$		
$y=-\frac{x}{4}+\frac{37}{4}$ (or $x+4y=37$)	1A	or other equivalent forms
(b) Mid-point of AB = (5,8)	1A	
Slope of the perpendicular bisector of AB = 4	1M	Or letting eqtn. be $y=4x+k$
Equation of the perpendicular bisector of AB is		
$y-8=4(x-5)$		
$y=4x-12$	1A	or other equivalent forms
OR $(x-1)^2+(y-9)^2=(x-9)^2+(y-7)^2$	2A	
$16x-4y-48=0$	1A	
$4x-y-12=0$		
Solving $y=4x-12$ and $4x-3y+12=0$,		
we have $4x-3(4x-12)+12=0$	1M	For reducing into 1 unknown
$x=6$, $y=12$	1A	
$\therefore G=(6,12)$		
(c) Equation of the circle is		
$(x-6)^2+(y-12)^2=(6-1)^2+(12-9)^2$	1M	
$(x-6)^2+(y-12)^2=34$	1A	
or $x^2+y^2-12x-24y+146=0$		
(d) (i) Let the mid-point of DE be (x,y), then		
$\frac{x+5}{2}=6$, $\frac{y+8}{2}=12$	1M	For both eqtns.
$x=7$, $y=16$	1A	
(ii) Equation of DE is		
$y-16=-\frac{1}{4}(x-7)$		
$x+4y-71=0$	1A	or other equivalent forms



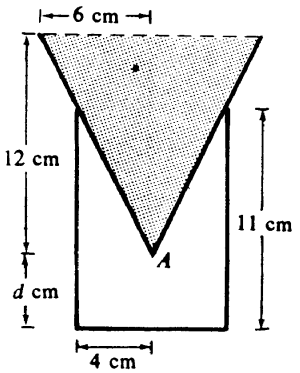
Solution	Marks	Remarks
11. (a) (i) $p = 1-\frac{4}{5}$		
$= \frac{1}{5}$ (or 0.2)	1A	
(ii) $q = 0$	1A	Accept answers given in reasonable order
$r = 1$	1A	
(b) (i) The probability that China will win the Championship		
$= \frac{1}{2}\times\frac{1}{2}$	1M	1M for $\frac{1}{2}\times p_1$, can be omitted
$= \frac{1}{4}$ (or 0.25)	1A	
(ii) (I) The probability that Wai Ming will study for the test		
$= \frac{1}{2}\times\frac{1}{3}+\frac{1}{2}$	1M+1M+1A	1M for $\frac{1}{2}\times p_2$, 1M for p_3+p_4
$= \frac{2}{3}$ (or 0.667)	1A	r.t. 0.667
(II) The probability that Wai Ming passes the test		
$= \frac{2}{3}\times\frac{4}{5}$	1M+1A	1M for answer of (b)(ii)(I)$\times p_5$
$= \frac{8}{15}$ (or 0.533)	1A	r.t. 0.533

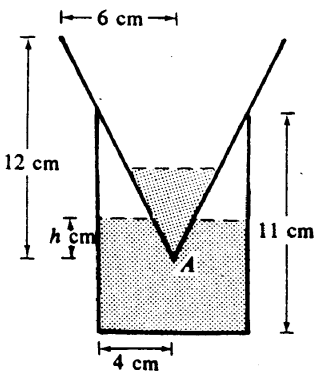
Solution	Marks	Remarks
12. (a) (i) $20x+40y\geq 240$ (or $x+2y\geq 12$)	1A	Withhold 1A for missing equal signs
(ii) $25x+37.5y\leq 300$ (or $2x+3y\leq 24$)	1A	
(iii) $x+y\leq 10$	1A	
(b) (3,5), (3,6), (4,4), (4,5), (5,4), (6,3), (6,4), (7,3	1A + 1A	1A for any 4 correct answers
(c) Let \$$C$ be the amount Mrs. Chiu has to paid, then $C=25x+37.5y$	1A	Can be omitted
$C(3,5)=262.5$ $C(6,3)=262.5$ $C(3,6)=300$ $C(6,4)=300$ $C(4,4)=250$ $C(7,3)=287.5$		
Optional: $C(4,5)=287.5$ $C(5,4)=275$	1M+1A	1M for correctly substituting 1 point in (b)
OR By drawing parallel lines of $25x+37.5y=0$ (or $2x+3y=0$)	1M+1A	
The least amount = \$(25×4+37.5×4) = \$250	1	
(d) (i) (3,6), (6,4)	1A	
(ii) Let N be the number of chocolates, then $N=20x+40y$	1A	Can be omitted
$\because N(3,6)=300$ $N(6,4)=280$ $\therefore$ The greatest number of chocolates that Mrs. Chiu can buy is 300.	1	

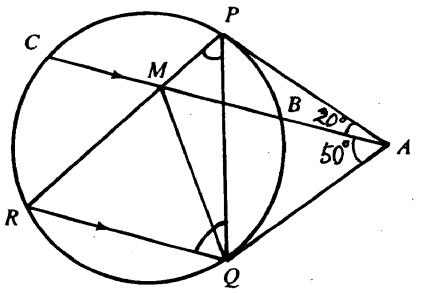
Solution	Marks	Remarks
13. (a) Volume of water $= \frac{1}{3}\pi(6)^2(12)$ cm^3		
$= 144\pi$ cm^3	1A	
(b) $\frac{11-d}{4} = \frac{12}{6}$ (or $\frac{d}{6} = \frac{6}{12}$)	1M + 1A	
$11-d = 8$		
$d = 3$	1A	
(c) (i) Volume of water that has leaked out when the water level in the cylindrical reaches A		
$= \pi(4)^2(3)$ cm^3		
$= 48\pi$ cm^3	1A	
(ii) $\pi(4)^2(h+3) - \frac{1}{3}\pi(\frac{h}{2})^2 h = 104\pi$	1A	
$16h + 48 - \frac{h^3}{12} = 104$		
$h^3 - 192h + 672 = 0$ .....(*)	1	
(iii) Let $f(h) = h^3-192h+672$, then $f(0)=672>0$ and $f(6)=-264<0$	1M	Testing the signs are different
$\therefore$ equation (*) has a root between 0 and 6.		

Interval	mid-value (h_i)	$f(h_i)$
$0 < h < 6$	3	+ve (123)
$3 < h < 6$	4.5	-ve (-100.875)
$3 < h < 4.5$	3.75	+ve (4.734)
$3.75 < h < 4.5$	4.125	-ve (-49.811)
$3.75 < h < 4.125$	3.9375	-ve (-22.953)
$3.75 < h < 3.9375$	3.84375	-ve (-9.211)

	1M + 1A 1M	1 M for testing sign at mid-value 1A for the correct sign of the function at mid-value 1M for the correct choice of the next interval
$\therefore 3.75 < h < 3.84375$ $h = 3.8$ (correct to 1 decimal place)	1	Check whether it is bounded by the last interval





Solution	Marks	Remarks
14. (a)(i) ∵ $\angle PMA = \angle PRQ$ (corr. ∠s, $AC \parallel QR$)	1A	"同位角，$AC \parallel QR$"
and $\angle PRQ = \angle PQA$, (∠ in alt. segment)	1A	"弦切角定理" 或 "交錯弓形的圓周角"
∴ $\angle PMA = \angle PQA$.		
Hence M, P, A and Q are concyclic. (converse of ∠s in same segment)		
(ii) ∵ $\angle RQM = \angle AMQ$, (alt. ∠s, $AC \parallel QR$)	1A	"內錯角，$AC \parallel QR$"
$\angle AMQ = \angle APQ$ (∠s in same segment)	1A	"同弧上的圓周角" 或 "同弓形內的圓周角"
and $\angle APQ = \angle PRQ$, (∠ in alt. segment)	1A	
∴ $\angle RQM = \angle PRQ$.		
Hence $MR = MQ$. (sides opp. equal ∠s)	1A	Or "converse of base ∠s, isos.△", "base ∠s equal", "equal ∠s, equal sides"
		"等角對邊相等" 或 "等腰三角形底角等的逆定理" 或 "底角相等" 或 "等邊對等角" 或 "等角對等邊"
		
(b) $\angle QPR = \angle QAM = 50°$	1A	
$\angle AQP = \frac{180°-20°-50°}{2}$ (or $\angle APQ = 55°$)		
$= 55°$	1A	
$\angle MQR = \angle MRQ = \angle AQP = 55°$	1A	For $\angle MQR=55°$ or $\angle MRQ=55°$
OR $\angle PMQ = 180°-20°-50° = 110°$	1A	
$\angle MQR = \frac{110°}{2} = 55°$ (or $\angle MRQ = 55°$)	1A	
$\angle PQR = \angle MQR + \angle PQM = \angle MQR + \angle PAM = 55°+20° = 75°$	1A	
(c) (i) ∵ ΔRMQ is isosceles (**OR** $RM=QM$ **OR** $\Delta MHR \equiv \Delta MHQ$)		
∴ $RH=QH$	1	
(ii) The perpendicular bisector of a chord of a circle passes through the centre of the circle.	1	
(**OR** MH is the perpendicular bisector of RQ.)		

Solution	Marks	Remarks
15. (a) $OA' = \frac{2}{\tan 30^\circ}$ m		
≈ 3.46 m (or $2\sqrt{3}$ m)	1A	r.t. 3.46
$OB' = \frac{2.6}{\tan 30^\circ}$ m	1A	
≈ 4.50 m (or $(2.6)\sqrt{3}$ m)		
$A'B' = 1.04$ m (or $(0.6)\sqrt{3}$ m)	1A	r.t. 1.04
(b) $\cos\angle BAC = \frac{(0.6)^2+(0.7)^2-(0.8)^2}{2(0.7)(0.6)}$	1A	
$= \frac{1}{4}$		
$\angle BAC \approx 75.5^\circ$ (or $75^\circ 30'$ / $75^\circ 31'$)	1A	r.t. 75.5
$\sin 75.5^\circ = \frac{OD}{0.7\text{ m}}$	1M	
$OD \approx 0.678$ m	1A	r.t. 0.678
(c) Area of the shadow $A'B'C' = \frac{1}{2}(A'B')(OD)$ m^2	1M	With substitution
≈ 0.352 m^2 (or 0.353 m^2)	1A	r.t. 0.352 / 0.353
(d) If the angle of elevation of the sun is less than 30°		
(i) the shadow of AB will be longer than $A'B'$.	1A	
(ii) $\because$ (1) the base of the triangle will be increased;		
(2) the height of the triangle will remain unchanged.		
$\therefore$ the area of the shadow of the road sign will be larger than that of $A'B'C'$.	1+1	1 for correct conclusion with one reason 1 for all being correct