FORMULAS FOR REFERENCE

SPHERE	Surface area	$= 4\pi r^2$
	Volume	$= \frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	$= 2\pi rh$
	Volume	$= \pi r^2 h$
CONE	Area of curved surface	$= \pi rl$
	Volume	$= \frac{1}{3}\pi r^2 h$
PRISM	Volume	= base area × height
PYRAMID	Volume	$= \frac{1}{3}$ × base area × height

SECTION A (39 marks)
Answer ALL questions in this section.
There is no need to start each question on a fresh page.
In questions 1–2, working steps are not required and you need to give the answers only.

1. (a) What is the simple interest on $100 for 6 months at 3% p.a. ?

(b) In Figure 1, find h .

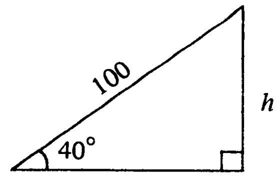


Figure 1

(c) In Figure 2, find x .

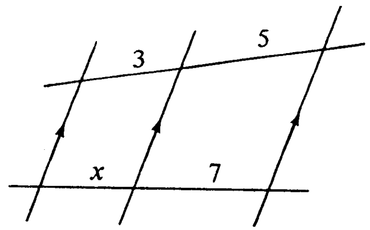


Figure 2

(d) In Figure 3, find a point (x, y) in the shaded region (including the boundary) at which the value of $x + 2y$ is

(i) greatest,
(ii) least.

What are these greatest and least values?

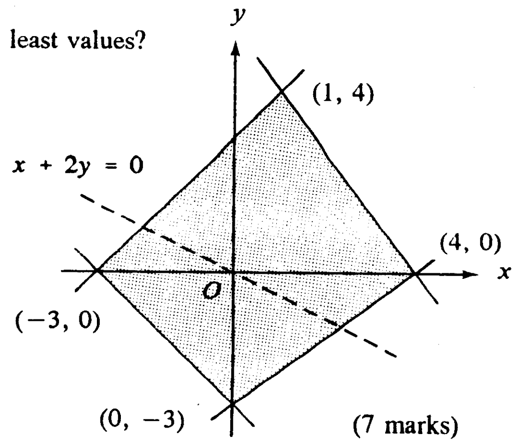


Figure 3

(7 marks)

2. (a) Let $f(x) = \dfrac{x^2 + 1}{x - 1}$. Find $f(3)$.

(b) If $2xy + 3 = 6x$, express y in terms of x.

(c) Simplify $\dfrac{1}{x - 1} - \dfrac{1}{x + 1}$.

(d) Find the remainder when $x^3 + x^2$ is divided by $x - 1$.

(e) Find the H.C.F. and L.C.M. of $6x^2y^3$ and $4xy^2z$.

(f) If $(x - 1)(x + 2) \equiv x^2 + rx + s$, find r and s.

(g) Rationalize $\dfrac{1}{\sqrt{3} - 1}$.

(9 marks)

3. Solve $\dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \dfrac{3}{2}$ for $0^\circ \le \theta < 360^\circ$.

(4 marks)

4. Solve the inequality $x^2 - x - 2 < 0$.

Hence solve the inequality $(y - 100)^2 - (y - 100) - 2 < 0$.

(6 marks)

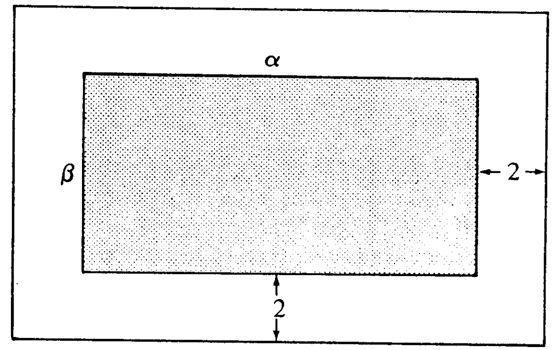
5. (a) If $9^x = \sqrt{3}$, find x.

(b) Simplify and express with positive indices $x\left(\dfrac{x^{-1}}{y^2}\right)^{-3}$.

(6 marks)

6.

Figure 4



The length α and the breadth β of a rectangular photograph are the roots of the equation $2x^2 - mx + 500 = 0$. The photograph is mounted on a piece of rectangular cardboard, leaving a uniform border of width 2 as shown in Figure 4.

(a) Find the area of the photograph.

(b) Find, in terms of m,

(i) the perimeter of the photograph,

(ii) the area of the border.

(7 marks)

SECTION B (60 marks)
Answer any FIVE questions from this section.
Each question carries 12 marks.

7. The following frequency table shows the distribution of the scores of 200 students in a Mathematics examination.

Frequency Table

Score	Frequency
0 – 9	20
10 – 19	40
20 – 29	60
30 – 39	50
40 – 49	20
50 – 59	10

Cumulative Frequency Table

Score (less than)	Cumulative Frequency
9.5	
19.5	
29.5	
39.5	
49.5	
59.5	

(a) Copy the cumulative frequency table onto your answer book and fill in the blanks. (2 marks)

(b) (i) Draw the cumulative frequency polygon on the graph paper (p.6) and determine the interquartile range.

(ii) If the pass percentage is set at 60%, determine the pass score from the cumulative frequency polygon. (6 marks)

(c) Find the mean and standard deviation of the distribution of scores. (Working steps need not be shown.) (2 marks)

(d) The teacher found that the scores were too low. He added 20 to each score. Write down the mean and the standard deviation of the new set of scores. (2 marks)

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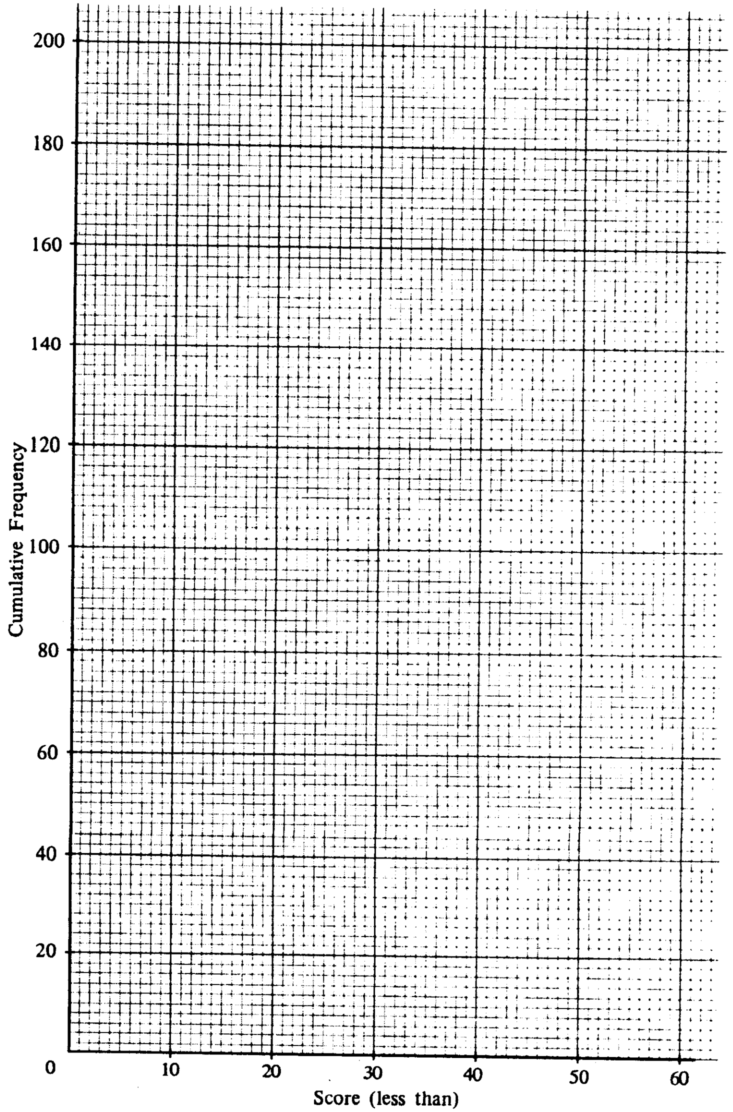
7. (Cont'd) **If you attempt Question 7, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**



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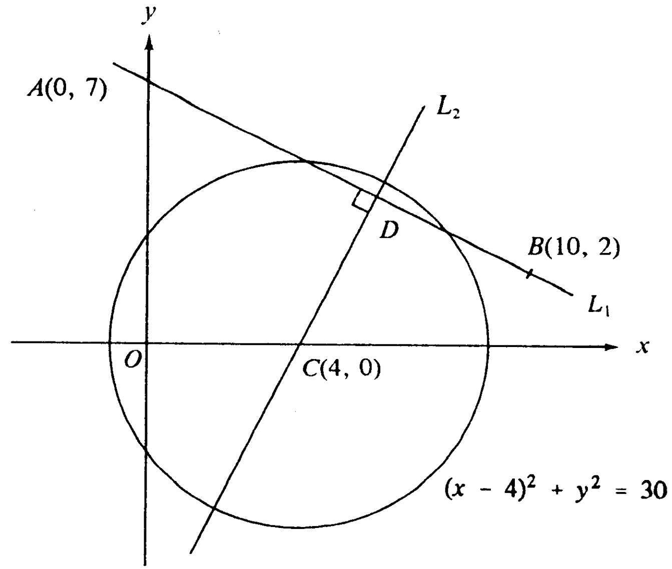
8. In Figure 5, L_1 is the line passing through $A(0, 7)$ and $B(10, 2)$; L_2 is the line passing through $C(4, 0)$ and perpendicular to L_1 ; L_1 and L_2 meet at D .

Figure 5



(a) Find the equation of L_1 . (2 marks)

(b) Find the equation of L_2 and the coordinates of D . (4 marks)

(c) P is a point on the line segment AB such that $AP : PB = k : 1$. Find the coordinates of P in terms of k .

If P lies on the circle $(x - 4)^2 + y^2 = 30$, show that

$$2k^2 - 16k + 7 = 0 \quad \text{.........................(*)}$$

Find the roots of equation (*).

Furthermore, if P lies between A and D , find the value of $\dfrac{AP}{PB}$.

(6 marks)

9.

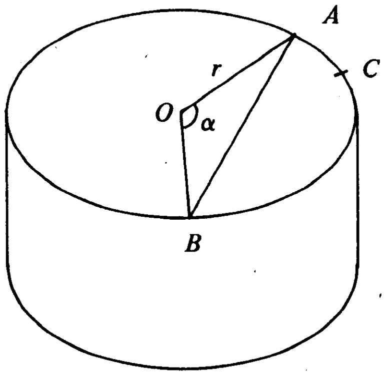


Figure 6

Figure 6 shows a right circular cylinder. O is the centre and r is the radius of its top face. A chord AB divides the area of the top face in the ratio 4:1 and subtends an angle α radians at O. C is a point on the minor arc AB.

(a) (i) Find the area of the sector $OACB$ in terms of r and α.

(ii) Find the area of the segment ACB in terms of r and α.

(iii) Show that $\sin \alpha = \alpha - \frac{2\pi}{5}$.

(iv) Show that the value of α lies between 2.1 and 2.2.

(v) Use the method of bisection to find the value of α, correct to 2 decimal places.

(10 marks)

(b) The cylinder is cut along AB into 2 parts by a plane perpendicular to its top face. Find the ratio of the curved surface areas of the two parts in the form $k : 1$, where $k > 1$.

(2 marks)

10. Consider the food production and population problems of a certain country. In the 1st year, the country's annual food production was 8 million tonnes. At the end of the 1st year its population was 2 million. It is assumed that the annual food production increases by 1 million tonnes each year and the population increases by 6% each year.

(a) Find, in million tonnes, the annual food production of the country in

(i) the 3rd year,
(ii) the nth year.

(2 marks)

(b) Find, in million tonnes, the total food production in the first 25 years.

(2 marks)

(c) Find the population of the country at the end of

(i) the 3rd year,
(ii) the nth year.

(2 marks)

(d) Starting from the end of the first year, find the minimum number of years it will take for the population to be doubled.

(3 marks)

(e) If the 'annual food production per capita' (i.e. $\frac{\text{annual food production in a certain year}}{\text{population at the end of that year}}$) is less than 0.2 tonne, the country will face a food shortage problem. Determine whether the country will face a food shortage problem or not at the end of the 100th year.

(3 marks)

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11. **Answers to this question should be written in the blanks provided on p.12 – p.13 .**

Figure 7

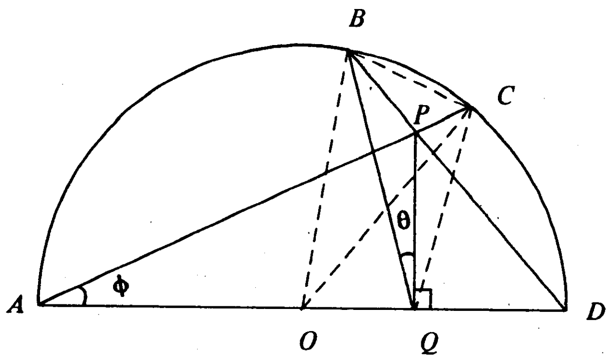

Figure 7 shows a semicircle with diameter AD and centre O . The chords AC and BD meet at P . Q is the foot of the perpendicular from P to AD .

(a) Show that A , Q , P , B are concyclic.

(3 marks)

(b) Let $\angle BQP = \theta$. Find, in terms of θ ,

(i) $\angle BQC$,

(ii) $\angle BOC$.

(6 marks)

(c) Let $\angle CAD = \phi$. Find $\angle CBQ$ in terms of ϕ .

(3 marks)

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If you attempt Question 11, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.

Answers to Question 11

(a) Join AB .

$\angle ABD = 90°$ (______)

$\angle AQP =$ ______ (Given)

$\therefore$ $\angle ABD + \angle AQP =$ ______

$\therefore$ $AQPB$ is a cyclic quadrilateral, i.e. A , Q , P , B are concyclic. (Opp. $\angle$s supp.)

(b) (i) Join CD .

Using the same argument as in (a), it can be shown that $PQDC$ is also a cyclic quadrilateral.

$\therefore$ $\angle PQC =$ ______ ($\angle$s in the same segment)

Consider the cyclic quadrilateral $ADCB$.

$\angle BDC =$ ______ ($\angle$s in the same segment)

As $AQPB$ is a cyclic quadrilateral,

$\angle BAP =$ ______ ($\angle$s in the same segment)

Go on to the next page

Answers to Question 11 (Cont'd)

Page Total

$\therefore \quad \angle BQC = \angle BQP + \angle PQC$

In terms of θ ,

$\angle BQC =$ [ ]

(ii) Consider the given semicircle.

$\angle BOC = 2 \times \angle BAC$ ([ ])

In terms of θ ,

$\angle BOC =$ [ ]

(c) Solution :

12.

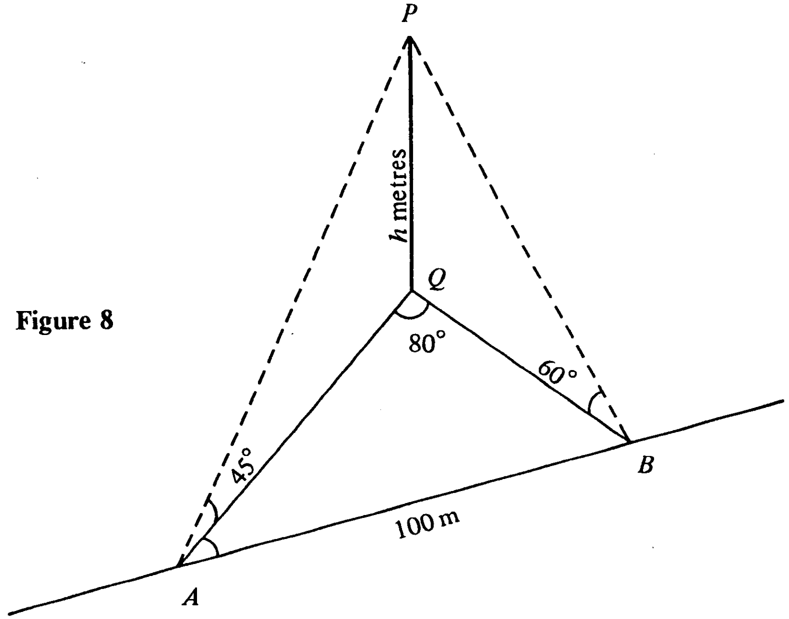


Figure 8

In Figure 8, PQ is a vertical television tower h metres high. A and B are two points 100m apart on a straight road in front of the tower with A , B and Q on the same horizontal ground and $\angle AQB = 80°$. The angles of elevation of P from A and B are 45° and 60° respectively.

(a) (i) Express the lengths of AQ and BQ in terms of h .

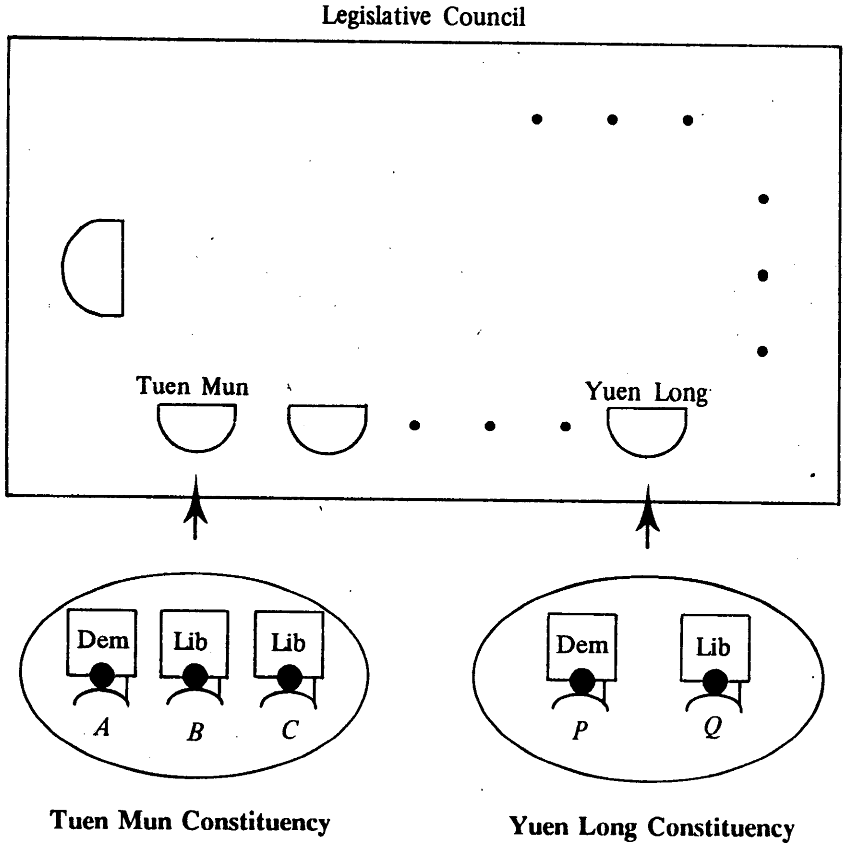
(ii) Find h and $\angle QAB$.

(8 marks)

(b) A person walks from A along the road towards B . At a certain point R between A and B , the person finds that the angle of elevation of P is 50°. How far away is R from A ?

(4 marks)

13.



In a Legislative Council election, each registered voter in a constituency (i.e. district) could select only one candidate in that constituency and cast one vote for that candidate. The candidate who got the greatest number of valid votes won the election in that constituency.

In the Tuen Mun constituency, there were 3 candidates, A, B and C. A belonged to a political party called 'The Democrats'; B and C belonged to a political party called 'The Liberals'.

In the Yuen Long constituency, there were 2 candidates, P and Q. P belonged to 'The Democrats' and Q belonged to 'The Liberals'.

(a) A survey conducted before the election showed that the probabilities of winning the election for A, B and C were respectively 0.65, 0.25 and 0.1 while the probabilities of winning the election for P and Q were respectively 0.45 and 0.55. Calculate from the above data the following probabilities:

(i) The elections in the Tuen Mun and Yuen Long constituencies would both be won by 'The Democrats'.

(ii) The elections in the Tuen Mun and Yuen Long constituencies would both be won by the same party.

(5 marks)

(b) After the election, it was found that in the Tuen Mun constituency there were 40 000 valid votes of which A got 70%, B got 20% and C got 10%; in the Yuen Long constituency, there were 20 000 valid votes of which P got 40% and Q got 60%. Suppose two votes were chosen at random (one after the other with replacement) from the 60 000 valid votes in the two constituencies. What would be the probability that

(i) both votes came from the Tuen Mun constituency and were for 'The Democrats',

(ii) both votes were for 'The Democrats',

(iii) the votes were for different parties?

(7 marks)

END OF PAPER