

### FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	$\pi rl$
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area $\times$ height
PYRAMID	Volume	=	$\frac{1}{3} \times$ base area $\times$ height

### SECTION A (39 marks)

Answer ALL questions in this section.

There is no need to start each question on a fresh page.

In questions 1–3, working steps are not required and you need to give the answers only.

- Express  $30^\circ$  in radians, leaving your answer in terms of  $\pi$ .
  - Find  $x$  if  $\sin x = \frac{1}{2}$  and  $90^\circ < x < 180^\circ$ .
  - Simplify  $\frac{1 - \sin^2 A}{\cos A}$ . (3 marks)
- If  $\log x = p$  and  $\log y = q$ , express  $\log xy$  in terms of  $p$  and  $q$ .
  - Find the remainder when  $x^3 - 2x^2 + 3x - 4$  is divided by  $x - 1$ .
  - Rationalize  $\frac{1}{\sqrt{3} + \sqrt{2}}$ . (3 marks)

3. In Figure 1, the shaded region, including the boundary, is determined by three inequalities.

- (a) Write down the three inequalities.
- (b) How many points  $(x, y)$ , where  $x$  and  $y$  are both integers, satisfy the three inequalities in (a)?

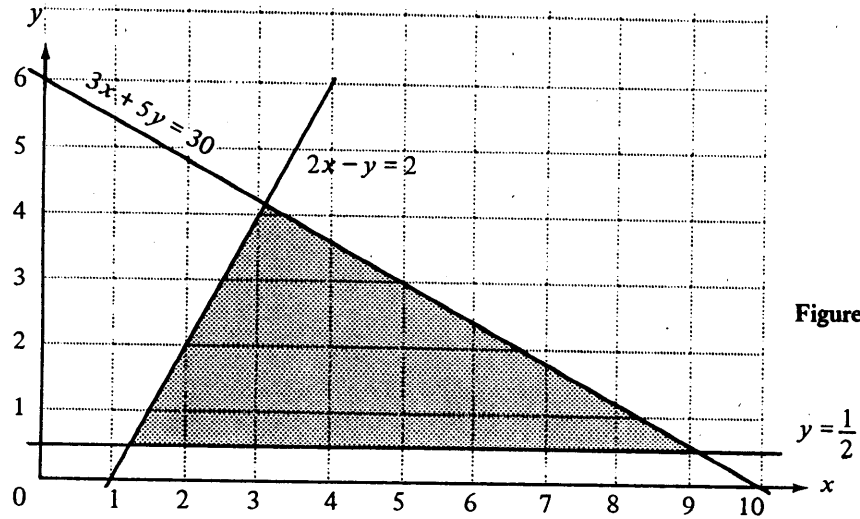


Figure 1

4. (a) Factorize (4 marks)

(i)  $x^2 - 2x$ ,

(ii)  $x^2 - 6x + 8$ .

- (b) Simplify  $\frac{1}{x^2 - 2x} + \frac{1}{x^2 - 6x + 8}$ . (5 marks)

5.  $L_1$  is the line passing through the point  $A(10, 5)$  and perpendicular to the line  $L_2: x - 2y + 5 = 0$ .

- (a) Find the equation of  $L_1$ .
- (b) Find the intersection point of  $L_1$  and  $L_2$ . (6 marks)

6. Find the range of values of  $k$  so that the quadratic equation  $x^2 + 2kx + (k + 6) = 0$  has two distinct real roots.

(6 marks)

7.

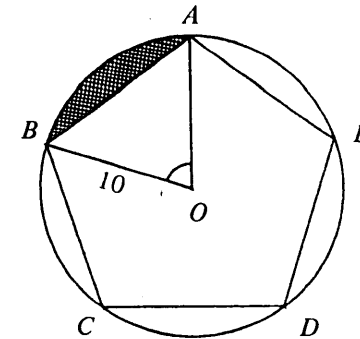


Figure 2

In Figure 2,  $ABCDE$  is a regular pentagon inscribed in a circle with centre  $O$  and radius 10.

- (a) Find  $\angle AOB$  and the area of triangle  $OAB$ .
- (b) Find the area of the shaded part in the figure.

(6 marks)

8. In a sports competition, the mean score of a team of  $m$  men and  $n$  women is 70.

- (a) Find the total score of the team in terms of  $m$  and  $n$ .
- (b) If the mean score of the men is 75 and the mean score of the women is 62, find the ratio  $m : n$ .
- (c) If there are altogether 39 persons in the team, find the number of men.

(6 marks)

**SECTION B (60 marks)**

Answer any FIVE questions from this section.

Each question carries 12 marks.

9.

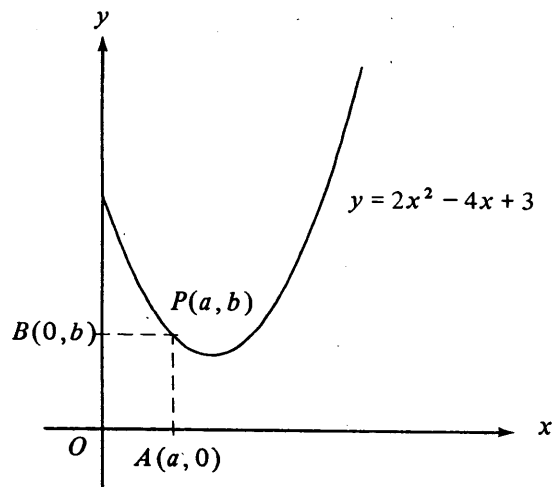


Figure 3

Figure 3 shows the graph of  $y = 2x^2 - 4x + 3$ , where  $x \geq 0$ .

$P(a, b)$  is a variable point on the graph. A rectangle  $OAPB$  is drawn with  $A$  and  $B$  lying on the  $x$ - and  $y$ - axes respectively.

- (a) (i) Find the area of rectangle  $OAPB$  in terms of  $a$ .
- (ii) Find the two values of  $a$  for which  $OAPB$  is a square. (6 marks)
- (b) Suppose the area of  $OAPB = \frac{3}{2}$ .
- (i) Show that  $4a^3 - 8a^2 + 6a - 3 = 0$  .....(\*) .
- (ii) Show that there is a root of (\*) lying between 1.2 and 1.3 .

Hence use the method of bisection to find this root, correct to 2 decimal places.

(6 marks)

10.

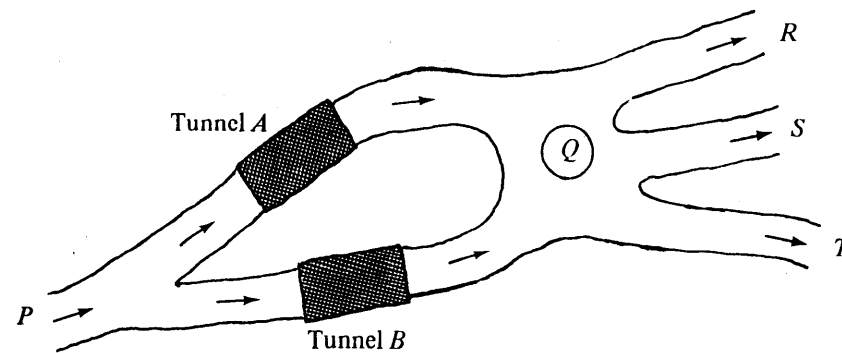


Figure 4

Figure 4 shows a one-way road network system from Town  $P$  to Towns  $R$ ,  $S$  and  $T$ . Any car leaving Town  $P$  will pass through either Tunnel  $A$  or Tunnel  $B$  and arrive at Towns  $R$ ,  $S$  or  $T$  via the roundabout  $Q$ . A survey shows that  $\frac{2}{5}$  of the cars leaving  $P$  will pass through Tunnel  $A$ . The survey also shows that  $\frac{1}{7}$  of all the cars passing through the roundabout  $Q$  will arrive at  $R$ ,  $\frac{2}{7}$  at  $S$ , and  $\frac{4}{7}$  at  $T$ .

- (a) Find the probabilities that a car leaving  $P$  will
- (i) pass through Tunnel  $B$ ,
- (ii) not arrive at  $T$ ,
- (iii) arrive at  $R$  through Tunnel  $B$ ,
- (iv) pass through Tunnel  $A$  but not arrive at  $R$ . (6 marks)
- (b) Two cars leave  $P$ . Find the probabilities that
- (i) one of them arrives at  $R$  and the other one at  $S$ ,
- (ii) both of them arrive at  $S$ , one through Tunnel  $A$  and the other one through Tunnel  $B$ . (6 marks)

11.

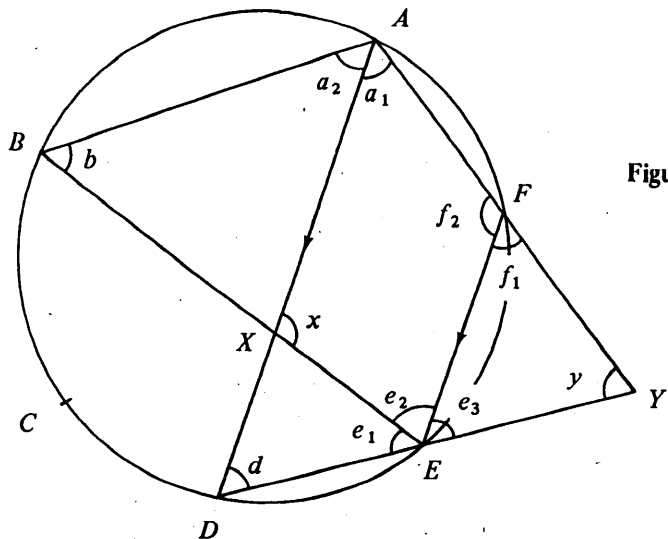


Figure 5

Answers to this question should be written in the blanks provided on p.8 - p.9 .

In Figure 5,  $A, B, C, D, E$  and  $F$  are points on a circle such that  $AD \parallel FE$  and  $\widehat{BCD} = \widehat{AFE}$ .  $AD$  intersects  $BE$  at  $X$ .  $AF$  and  $DE$  are produced to meet at  $Y$ .

- (a) Prove that  $\triangle EFY$  is isosceles. (3 marks)
- (b) Prove that  $BA \parallel DE$ . (1 mark)
- (c) Prove that  $A, X, E, Y$  are concyclic. (3 marks)
- (d) If  $\angle b = 47^\circ$ , find  $\angle f_1$ ,  $\angle y$  and  $\angle x$ . (5 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page
------------------	---------------	-------------	--------------------------

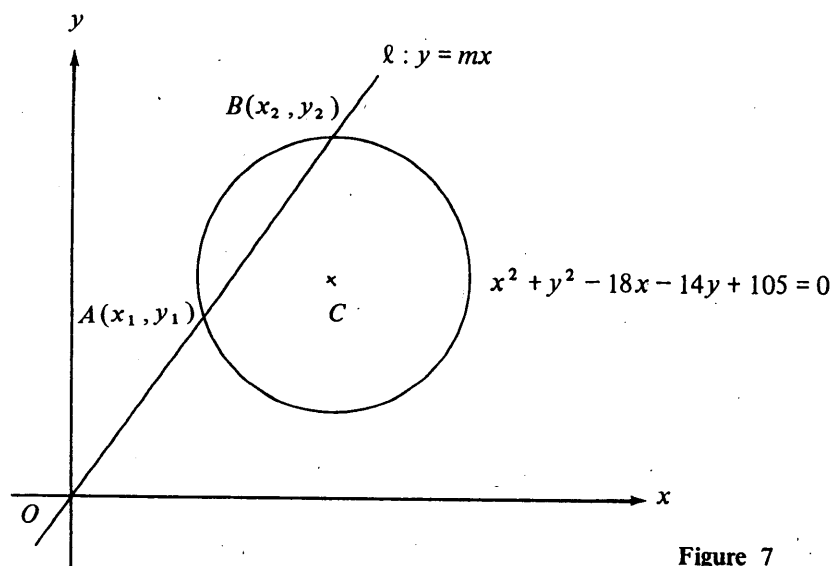
If you attempt Question 11, fill in the details in the first three boxes above and tie this sheet inside your answer book.

Answers to Question 11

- (a) **Proof :**  
 $\angle f_1 =$   (Corr.  $\angle$ s ,  $AD \parallel FE$  )  
 But   $=$   (Ext.  $\angle$  , cyclic quad.)  
 $\therefore \angle f_1 = \angle e_3$   
 $\therefore EY =$   (Sides opp. equal  $\angle$ s)  
 i.e.  $\triangle EFY$  is isosceles
- (b) **Proof :**  
 $\widehat{BCD} = \widehat{AFE}$  (Given)  
 $\therefore \angle a_2 =$   (Equal arcs subtend equal  $\angle$ s at circumference)  
 $\therefore BA \parallel DE$  (Alt.  $\angle$ s equal)



13.



In Figure 7, the line  $\ell : y = mx$  passes through the origin and intersects the circle  $x^2 + y^2 - 18x - 14y + 105 = 0$  at two distinct points  $A(x_1, y_1)$  and  $B(x_2, y_2)$ .

- (a) Find the coordinates of the centre  $C$  and the radius of the circle. (2 marks)
- (b) By substituting  $y = mx$  into  $x^2 + y^2 - 18x - 14y + 105 = 0$ , show that  $x_1 x_2 = \frac{105}{1 + m^2}$ . (2 marks)
- (c) Express the length of  $OA$  in terms of  $m$  and  $x_1$  and the length of  $OB$  in terms of  $m$  and  $x_2$ .  
Hence find the value of the product of  $OA$  and  $OB$ . (4 marks)
- (d) If the perpendicular distance between the line  $\ell$  and the centre  $C$  is 3, find the lengths of  $AB$  and  $OA$ . (4 marks)

14. (a) Given the G.P.  $a^n, a^{n-1}b, a^{n-2}b^2, \dots, a^2b^{n-2}, ab^{n-1}$ , where  $a$  and  $b$  are unequal and non-zero real numbers, find the common ratio and the sum to  $n$  terms of the G.P. (3 marks)
- (b) A man joins a saving plan by depositing in his bank account a sum of money at the beginning of every year. At the beginning of the first year, he puts an initial deposit of  $\$P$ . Every year afterwards, he deposits 10% more than he does in the previous year. The bank pays interest at a rate of 8% p.a., compounded yearly.
- (i) Find, in terms of  $P$ , an expression for the amount in his account at the end of
- (1) the first year,
  - (2) the second year,
  - (3) the third year.
- (Note: You need not simplify your expressions)
- (ii) Using (a), or otherwise, show that the amount in his account at the end of the  $n$ th year is  $\$54P(1.1^n - 1.08^n)$ . (7 marks)
- (c) A flat is worth  $\$1\,080\,000$  at the beginning of a certain year and at the same time, a man joins the saving plan in (b) with an initial deposit  $\$P = \$20\,000$ . Suppose the value of the flat grows by 15% every year. Show that at the end of the  $n$ th year, the value of the flat is greater than the amount in the man's account. (2 marks)

15.

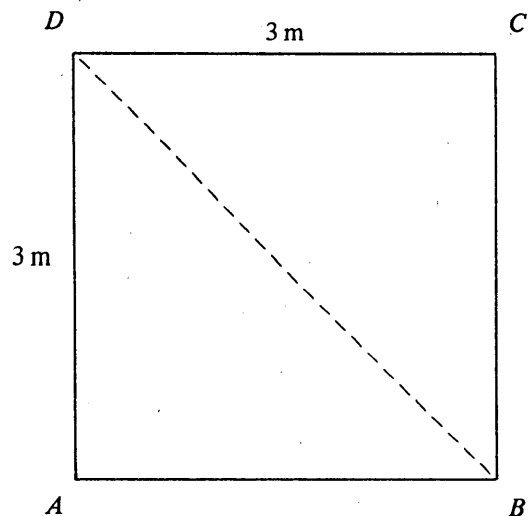


Figure 8a

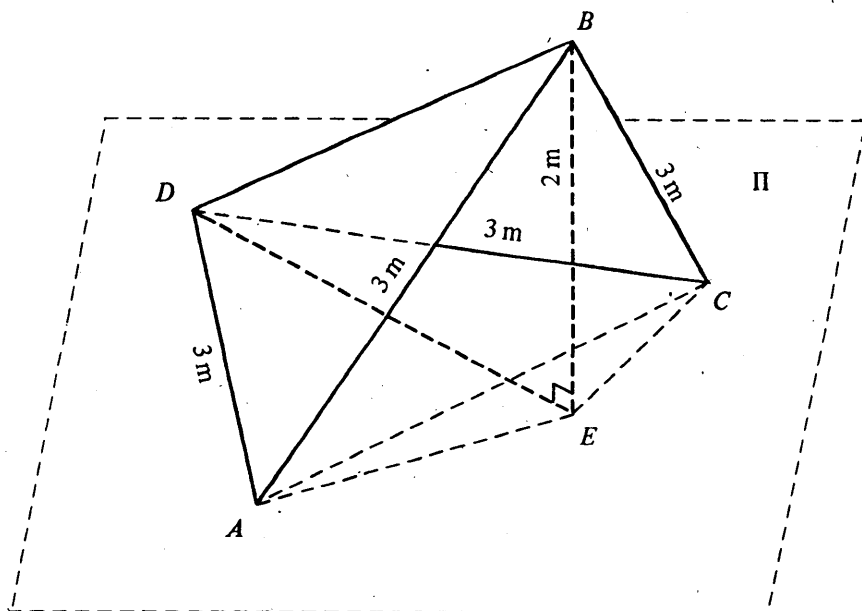


Figure 8b

15.(Cont'd)

In Figure 8a,  $ABCD$  is a thin square metal sheet of side three metres. The metal sheet is folded along  $BD$  and the edges  $AD$  and  $CD$  of the folded metal sheet are placed on a horizontal plane  $\Pi$  with  $B$  two metres vertically above the plane  $\Pi$ .  $E$  is the foot of the perpendicular from  $B$  to the plane  $\Pi$ . (See Figure 8b)

- (a) Find the lengths of  $BD$ ,  $ED$  and  $AE$ , leaving your answers in surd form. (3 marks)
- (b) Find  $\angle ADE$ . (3 marks)
- (c) Find the angle between  $BD$  and the plane  $\Pi$ . (2 marks)
- (d) Find the angle between the planes  $ABD$  and  $CBD$ . (4 marks)

END OF PAPER