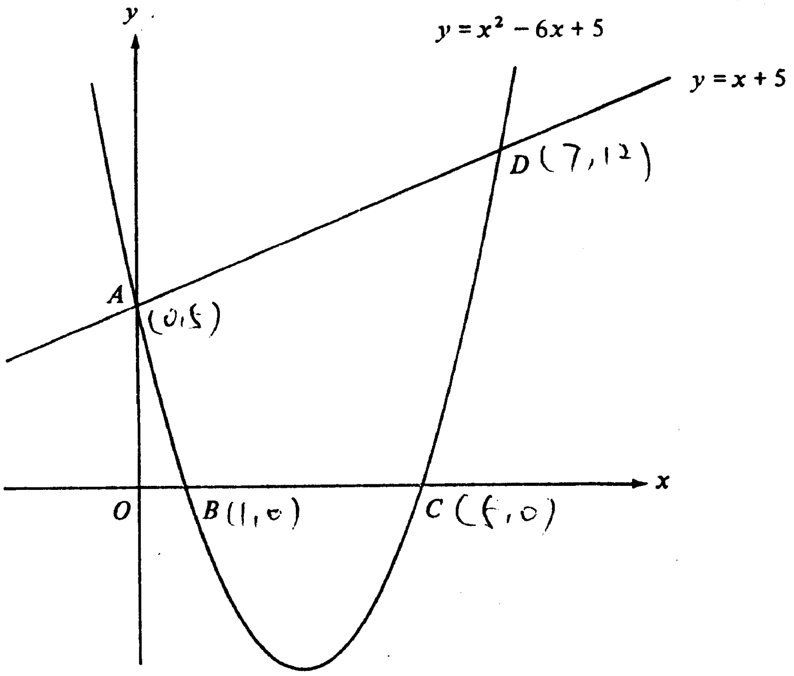
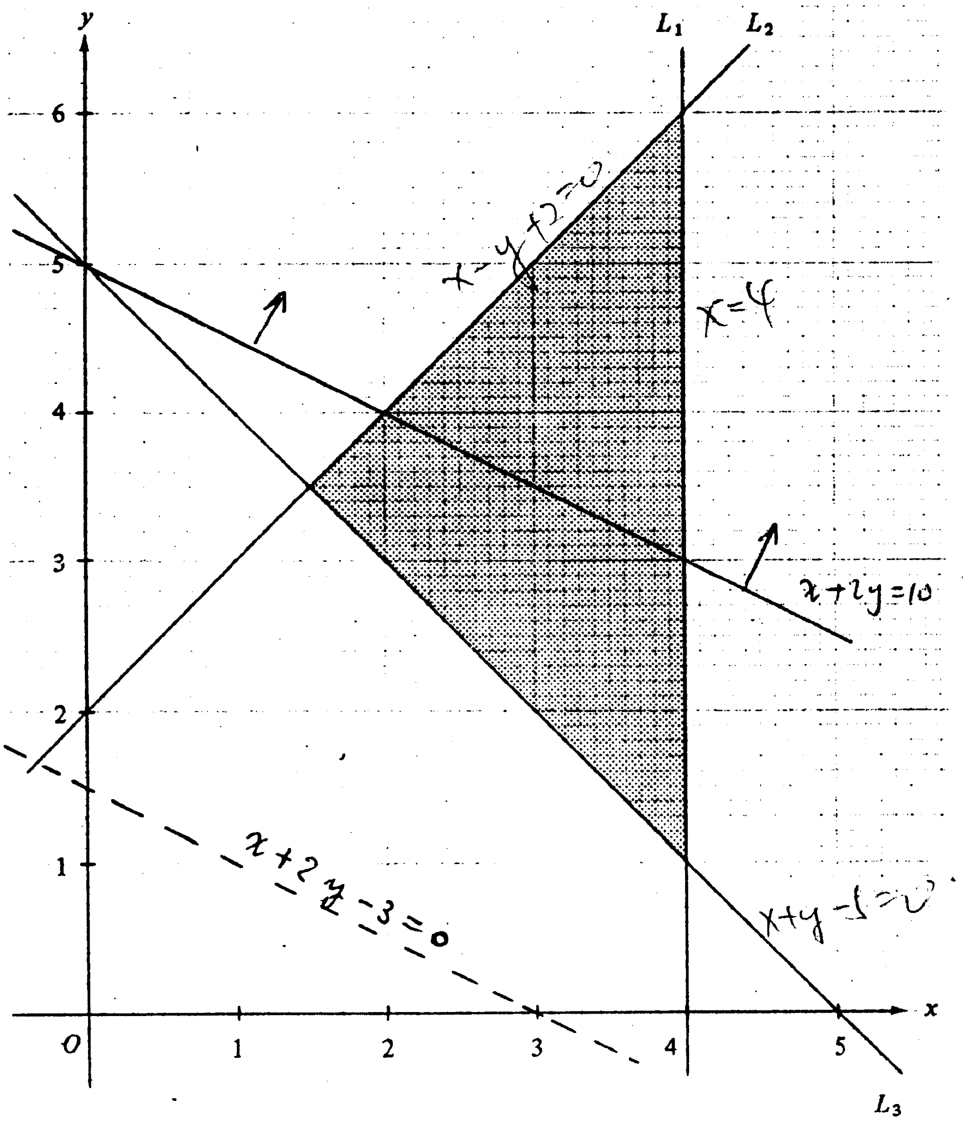
<table>
<tr><th>Solutions</th><th>Marks</th><th></th></tr>
<tr><td>1. (a) Median mark = 49.5</td><td>1A</td><td></td></tr>
<tr><td>(b)
<table>
<tr><th>Marks</th><th>No. of Students</th></tr>
<tr><td>20 - 29</td><td>10</td></tr>
<tr><td>30 - 39</td><td>10</td></tr>
<tr><td>40 - 49</td><td>20</td></tr>
<tr><td>50 - 59</td><td>30</td></tr>
<tr><td>60 - 69</td><td>10</td></tr>
</table></td><td>1A+1A</td><td>1A for any two correct, 1A for the rest</td></tr>
<tr><td>Mean mark</td><td></td><td></td></tr>
<tr><td>$= \frac{24.5 \times 10 + 34.5 \times 10 + 44.5 \times 20 + 54.5 \times 30 + 64.5 \times 10}{80}$</td><td>1M</td><td>Accept 25,35,etc,<br>denominator = $\Sigma f$</td></tr>
<tr><td>$= \frac{3760}{80}$</td><td></td><td></td></tr>
<tr><td>= 47 ........................................</td><td>1A</td><td>Must show working</td></tr>
<tr><td></td><td>5</td><td></td></tr>
<tr><td>2. (a) $x \propto \frac{y^2}{z}$<br>$x = \frac{ky^2}{z}$ for some constant $k$</td><td>1A</td><td>For either</td></tr>
<tr><td>$18 = \frac{k(3)^2}{2}$ ....................................</td><td>1M</td><td>Substituting for $k$</td></tr>
<tr><td>$k = 4$</td><td>1A</td><td></td></tr>
<tr><td>i.e. $x = \frac{4y^2}{z}$</td><td></td><td></td></tr>
<tr><td>(b) Putting $y = 1$ , $z = 4$ , $x = \frac{4(1)^2}{4}$ ............</td><td>1M</td><td>Substituting for k</td></tr>
<tr><td>= 1</td><td>1A</td><td></td></tr>
<tr><td></td><td>5</td><td></td></tr>
<tr><td>3. (a) $\frac{150000}{15}$ = £10000 ..........................</td><td>1A</td><td></td></tr>
<tr><td>(b) $10000(0.146)\left(\frac{30}{365}\right)$</td><td>1A</td><td></td></tr>
<tr><td>= £120</td><td></td><td></td></tr>
<tr><td>Amount = £(10000 + 120)</td><td>1M</td><td></td></tr>
<tr><td>= £ 10120 ..........................</td><td>1A</td><td>Accept 10100 ~ 10120</td></tr>
<tr><td>(c) 14.50 × 10120</td><td></td><td></td></tr>
<tr><td>= HK$146740 ................................</td><td>1A</td><td>Accept 146000 ~ 147000</td></tr>
<tr><td></td><td>5</td><td></td></tr>
</table>

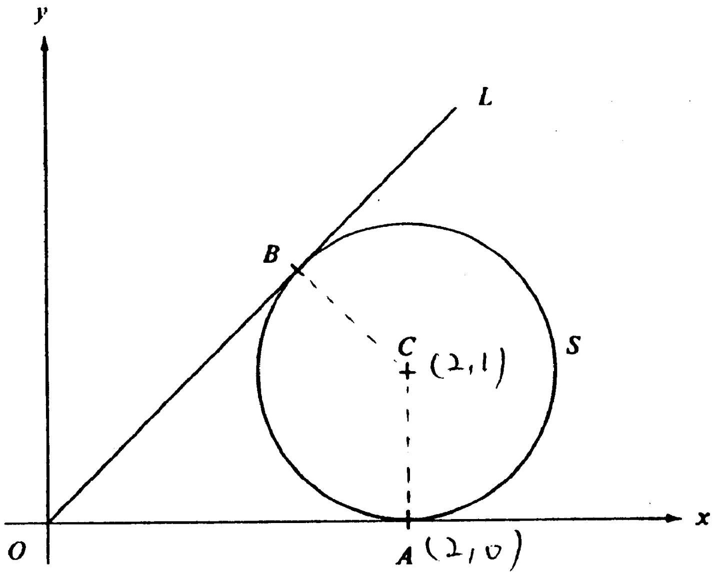
| Solutions | Marks | |
|---|---|---|
| 4. (a) $2a = 3b = 5c$ | | |
| $\frac{2a}{30} = \frac{3b}{30} = \frac{5c}{30}$ .................................. | 1M | |
| $\therefore \ a : b : c = 15 : 10 : 6$ | 2A | Correct ratio not in this form, 1A only |
| **Alternatively** | | |
| $\frac{a}{b} = \frac{3}{2}, \quad \frac{b}{c} = \frac{5}{3}$ | | |
| Writing $\frac{a}{b} = \frac{15}{10}, \quad \frac{b}{c} = \frac{10}{6}$ | 1M | |
| $\therefore \ a : b : c = 15 : 10 : 6$ | 2A | See above |
| (b) $\left.\begin{array}{l} a = 15k \\ b = 10k \\ c = 6k \end{array}\right\}$ | 1M | for either |
| $a - b + c = (15 - 10 + 6)k$ | | |
| $= 55$ | 1M | |
| $k = 5$ | | |
| $c = 30$ | 1A | |
| | 6 | |
| 5. $\sin^2\theta - 3\cos\theta - 1 = 0$ | | |
| $1 - \cos^2\theta - 3\cos\theta - 1 = 0$ .................................. | 1M | $\sin^2\theta = 1 - \cos^2\theta$ |
| $\cos^2\theta + 3\cos\theta = 0$ | 1A | |
| $\cos\theta(\cos\theta + 3) = 0$ | | |
| $\cos\theta = 0$ or $\cos\theta = -3$ (rejected) | 1A+1A | Accept $\cos\theta = 0$ |
| $\therefore \ \theta = 90^\circ$ or $270^\circ$ $\left(\frac{\pi}{2} \text{ or } \frac{3\pi}{2}\right)$ ......................... | 1A+1A | Withhold 1 mark for each extraneous answer |
| | 6 | |

| Solutions | Marks | |
|---|---|---|
| 6. (a) Putting $x = 0$ , $y = 5$ . | | |
| $\therefore$ $A = (0,5)$ .................................. | 1A | OR The coordinates of $A$ are $x$=0, $y$=5 |
| Putting $y = 0$ , $x^2 - 6x + 5 = 0$ | | |
| $(x - 1)(x - 5) = 0$ | | |
| $x = 1$ or $5$ | | |
| $\therefore$ $B = (1,0)$ .................................. | 1A | |
| $C = (5,0)$ .................................. | 1A | |
| (b) Putting $y = x + 5$ (or $x = y - 5$) | | |
| $x + 5 = x^2 - 6x + 5$ $(y = (y - 5)^2 - 6(y - 5) + 5)$ | 1A | |
| $x^2 - 7x = 0$ | | |
| $x(x - 7) = 0$ | | |
| $x = 0$ or $7$ | | |
| At $D$ , $x = 7$ .................................. | 1A | |
| $\therefore$ $y = 12$ | 1A | |
| i.e. $D = (7, 12)$ | | |
|  | | |
| | 6 | |
| (a) $\alpha + \beta = -\frac{20}{10}$ $(= -2)$ | 1A | |
| $4^{\alpha} \times 4^{\beta} = 4^{\alpha + \beta}$ .................................. | 1A | |
| $= 4^{-2}$ $\left(= \frac{1}{16} = 0.0625\right)$ | 1A | |
| (b) $\alpha\beta = \frac{1}{10}$ | 1A | |
| $\log_{10}\alpha + \log_{10}\beta = \log_{10}\alpha\beta$ .................................. | 1A | |
| $= \log_{10}\frac{1}{10}$ | | |
| $= -1$ .................................. | 1A | |
| | 6 | |

| Solutions | Marks | |
|---|---|---|
| 8. (a) $L_2 : y - 2 = 1(x - 0)$ | | |
| $x - y = -2$ (or $x - y + 2 = 0$, etc.) | 2A | |
| $L_3 : \frac{x}{5} + \frac{y}{5} = 1$ | 1A | 2+1 |
| i.e. $x + y = 5$ (or $x + y - 5 = 0$, etc.) | | |
| | 3 | |
| (b) The region is determined by the inequalities | | Withhold 1 mark if '=' omitted or for each extraneous constraint Note other equivalent forms |
| $x \le 4$ | 1A | |
| $x - y \ge -2$ .................................. | 1A | |
| $x + y \ge 5$ | 1A | |
| | 3 | |
| (c) (i) Drawing the line $x + 2y - 3 = c$ . ......... | 1M<br>+<br>1A | OR Finding the values of $P$ at any vertex |
| $P$ is minimum at the point (4, 1) and the minimum value of $P = 4 + 2(1) - 3 = 3$ . | 1A | At (4,6), $P$=13<br>(4,1), $P$=3<br>(1.5,3.5), $P$=5.5 |
| (ii) $x + 2y - 3 \ge 7$<br>$x + 2y \ge 10$ | 1A | |
| Drawing $x + 2y = 10$ in the figure. | 1A | |
| The possible range of values of $x$ is $2 \le x \le 4$ . | 1A | |
| | 6 | |



| Solutions | Marks | |
|---|---|---|
| 9. (a) $C = (2,1)$ | 1A | |
| $A = (2,0)$ | 1A | |
| | 2 | |



| | | |
|---|---|---|
| (b) Putting $y = mx$ in $S$ | 1M | Let $\angle COA = \theta$ |
| $x^2 + (mx)^2 - 4x - 2mx + 4 = 0$ | | $\tan\theta = \frac{1}{2}$ 1M |
| | | $\angle BOA = 2\theta$ 1M |
| $(1 + m^2)x^2 - (4 + 2m)x + 4 = 0$ ................. | 1A | $\therefore m = \tan 2\theta$ 1A |
| | | $= \frac{2\tan\theta}{1 - \tan^2\theta}$ 1A |
| | | $= \frac{2 \times \frac{1}{2}}{1 - \frac{1}{4}} = \frac{4}{3}$ 1A |
| For tangency, $(4 + 2m)^2 - 4(1 + m^2)(4) = 0$ | 1M | |
| $3m^2 - 4m = 0$ | 1A | |
| $m = \frac{4}{3}$ as $m \neq 0$ | 1A | |
| | 5 | |
| (c) (i) As $OA$, $OB$ are tangents, $\angle OAC = 90°$ and $\angle OBC = 90°$ | 1 | For either |
| $\therefore \angle OAC + \angle OBC = 180°$ | | |
| So $O$, $A$, $C$, $B$ are concyclic. | 1 | |
| (ii) As $\angle OAC = 90°$, $OC$ is a diameter of the required circle, | | |
| whose centre $= (1, \frac{1}{2})$ and radius $= \frac{\sqrt{5}}{2}$ . | 1A+1A | |
| Equation of the circle is $(x - 1)^2 + (y - \frac{1}{2})^2 = \frac{5}{4}$ | 1A | |
| i.e. $x^2 + y^2 - 2x - y = 0$ | | |
| | 5 | |

**Alternatively**

| | | |
|---|---|---|
| (1) Let the circle be $x^2 + y^2 + ax + by + c = 0$ | | |
| Values of $a$, $b$, $c$ obtained by substitution | 1A+1A+1A | |
| (2) As $OC$ is a diameter, the circle is | | |
| $\frac{y - 0}{x - 0} \cdot \frac{y - 1}{x - 2} = -1$ | 2A | |
| i.e. $x^2 + y^2 - 2x - y = 0$ | 1A | |

| Solutions | Marks | |
|---|---|---|
| 10. (a) (i) The probability that the candidate fails on the first attempt but passes on the second is $(1 - 0.7) \times 0.7$ | 1A<br>+<br>1M | $1 - 0.7$<br>$p \times 0.7$ |
| $= 0.21$ .................................. | 1A | |
| (ii) The probability of passing Part *A* in no more than 2 attempts is $0.7 + 0.21$ | 1M+1A | |
| $= 0.91$ .................................. | 1A | Alternatively<br>1A for any two:<br>0.6×0.7<br>0.3×0.6×0.7<br>0.4×0.6×0.7<br>0.3×0.4×0.6×0.7<br>$p_1+P_2+p_3+p_4 =$ 1M<br>Ans. 2A |
| (iii) The probability of passing Part *B* in no more than 2 attempts is $0.6 + 0.4 \times 0.6$ | 1A | |
| $= 0.84$ .................................. | 1A | |
| ∴ the required probability $= 0.91 \times 0.84$ | 1M | |
| $= 0.764$ (0.7644) | 1A | |
| | 10 | |
| (b) No. expected $= 0.764 \times 10000$ | 1M | |
| $= 7640$ (7644)....................... | 1A | |
| | 2 | |
| 11. | | |

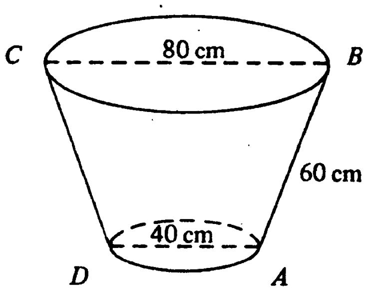


Figure 5a

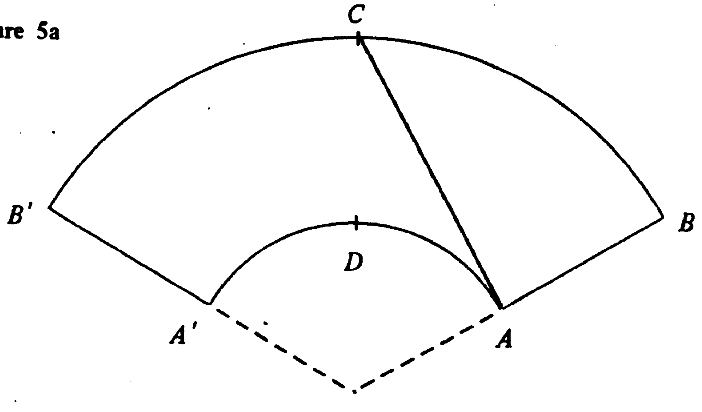
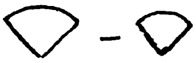
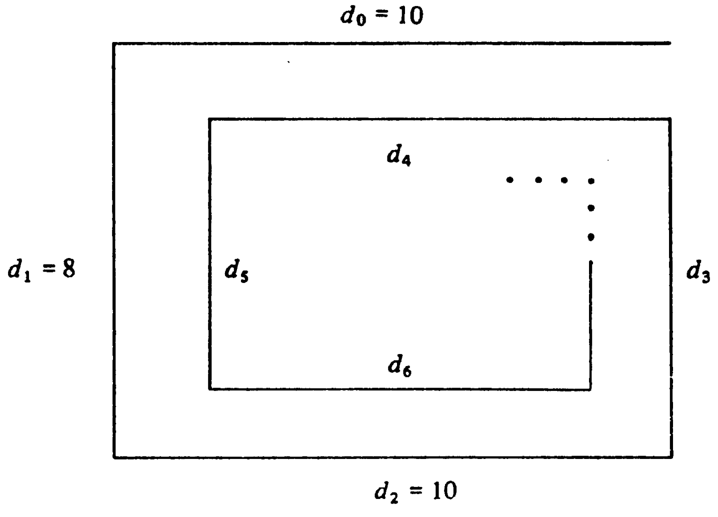


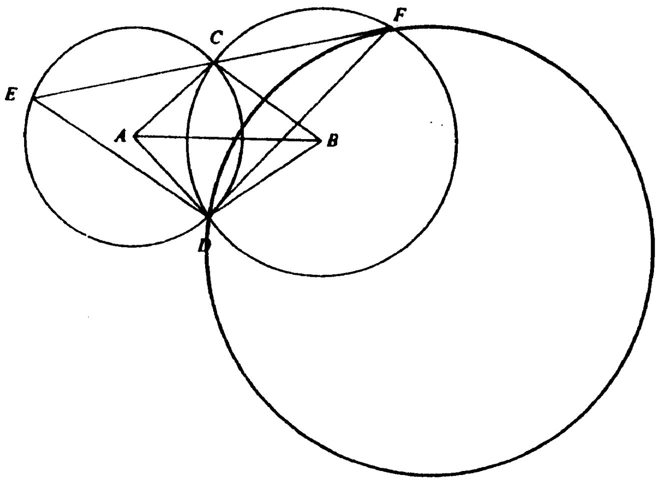
Fig 5b

| Solutions | Marks | |
|---|---|---|
| 11. (a) Let $\angle AOA' = \theta$ | | |
| $OA \times \theta = 40\pi$<br>$OB \times \theta = 80\pi$ | 1A | For either |
| $\frac{OA}{OB} = \frac{40\pi}{80\pi} \left(= \frac{1}{2}\right)$ | 1A | |
| $\frac{OA}{OA + 60} = \frac{1}{2}$ | | |
| $OA = 60$ cm | 1A | |
| $60\theta = 40\pi$ (or $120\theta = 80\pi$) | 1M | |
| $\theta = \frac{2}{3}\pi$ (= 120°) | 1A | |
| | 5 | |
| **Alternatively** | | |
| From Fig.5a, by similiar triangles,<br>$\frac{OA}{OB} = \frac{40}{80}$ $(= \frac{1}{2})$ | 2A | |
| $\frac{OA}{OA + 60} = \frac{1}{2}$<br>$\therefore$ $OA = 60$ cm | 1A | |
| Let $\angle AOA' = \theta$<br>$60\theta = 40\pi$ (or $120\theta = 80\pi$) | 1M | |
| $\theta = \frac{2}{3}\pi$ | 1A | |
| (b) Area of $ABB'A'$ = $\frac{1}{3}\pi 120^2 - \frac{1}{3}\pi 60^2$ | 1M<br>+ 1M | Area of sector  |
| $= 3600\pi \text{cm}^2$ | 1A | |
| | 3 | |
| (c) The shortest distance = distance between $A$ and $C$ in Figure 5b. | 1M | Attempt to find $AC$ |
| $\angle AOC = \frac{120}{2} = 60°$ | 1A | |
| $AC^2 = OA^2 + OC^2 - 2(OA)(OC)\cos 60°$ | 1M | $\angle CAO = 90°$ 1 |
| $= 60^2 + 120^2 - 2(60)(120)(\frac{1}{2})$ | | $\sin 60° = \frac{AC}{OC}$ |
| $= 10800$ | | $\therefore AC = 60\sqrt{3}$cm 1A |
| $\therefore$ $AC$ = 104 cm (103.923) .................... | 1A | (= 104) |
| | 4 | |

| Solutions | Marks | |
|---|---|---|
| 12. (a) $d_3 = 0.9 d_1$ | | |
| $= 7.2$ | 1A | |
| $d_5 = d_3 \times 0.9 = 6.48$ | 1A | |
| $d_{2n-1} = 8(0.9)^{n-1}$ | 2A | |
| | 4 | |
| (b) $d_6 = 10 \times 0.9^2 = 8.1$ ........................ | 1A | |
| $d_{2n} = 10 \times 0.9^{n-1}$ | 1A | |
| | 2 | |
| (c) (i) $d_1 + d_3 + d_5 + \cdots + d_{2n-1}$ | | |
| $= 8 + 8(0.9) + 8(0.9)^2 + \ldots + 8(0.9)^{n-1}$ | | |
| $= \dfrac{8[1-(0.9)^n]}{1-0.9}$ ......................... | 1M | Attempting to sum as G.P. |
| $= 80(1-0.9^n)$ | 1A | |
| (ii) $d_2 + d_4 + d_6 + \cdots + d_{2n}$ | | |
| $= 10 + 10(0.9) + 10(0.9)^2 + \cdots + 10(0.9)^{n-1}$ | | |
| $= \dfrac{10[1-(0.9)^n]}{1-0.9}$ | | |
| $= 100(1-0.9^n)$ ...................... | 1A | |
| | 3 | |
| (d) $d_0 + d_1 + d_2 + d_3 + \cdots$ | | |
| $= 10 + (d_1 + d_3 + d_5 + \cdots) + (d_2 + d_4 + d_6 + \cdots)$ | 1M | Grouping even and odd terms |
| $= 10 + \dfrac{8}{1-0.9} + \dfrac{10}{1-0.9}$ .......................... | 1M | Either infinite sum |
| $= 190$ ............................... | 1A | |
| | 3 | |



| Solutions | Marks |
|---|---|
| 13. (a) Consider $\triangle ABC$ and $\triangle ABD$ . | |
| $AB = AB$ (common side) | 1A |
| $BC = BD$ (radii of the same circle) .......... | 1A |
| $CA = DA$ (radii of the same circle) | 1A |
| $\therefore \quad \triangle ABC \cong \triangle ABD$ (SSS) | 3 |
| (b) (i) $\angle CAD = 2\angle FED$ (= 110°) | 1M |
| $\angle CAB = \frac{1}{2}\angle CAD = \angle FED$ | |
| = 55° .................................. | 1A |
| $\angle ABC = 180 - 95 - 55$ | 1M |
| = 30° .................................. | 1A |
| $\therefore \quad \angle EFD = \angle ABC$ | |
| = 30° ............................... | 1A |
| (ii)(1) | |



| Solutions | Marks |
|---|---|
| A labelled diagram showing a cirlce through $D$ touching $CF$ at $F$ . | 1A |
| (2) Through $F$ draw a diameter $FG$ . Join $DG$ . | |
| $\angle DGF = 30°$ ($\angle$ in alt. segment) | 1A |
| $\angle FDG = 90°$ ($\angle$ in a semi-circle) ...... | 1A |
| $\frac{DF}{FG} = \frac{1}{2}$ (= sin30°) | |
| i.e. $FG = 2DF$ ...................... | 1A |
| | 9 |

OR

$\angle DGF = 30°$ 1A

$\angle DFG = 60°$ 1A

$\therefore \angle FDG = 90°$

$FG = 2DF$ 1A

| Alternatively | |
|---|---|
| Through $F$ and $D$, draw the radii $FO$ and $DO$ . | |
| As $OF \perp CF$ , $\angle DFO = 90° - 30° = 60°$ . | 1A |
| As $FO$ and $DO$ are radii of the same circle, | |
| $\angle FDO = 60°$ | 1A |
| $\therefore \triangle DFO$ is equilateral | |
| The diameter $= 2 \times FO = 2 \times DF$ . | 1A |

| Solutions | Marks | |
|---|---|---|
| 14. (a) Consider $\triangle AGH$ . | | |
| $GH = 1000\sin\theta$ m | 1A | |
| $AH = 1000\cos\theta$ m ........................... | 1A | |
| | 2 | |
| (b) $\angle HAB = 30°$ (or $\angle AHB = 60°$) | 1A | |
| $BH = AH\sin 30°$ ............................... | 1A | $BH = GH$<br>$= 1000\sin\theta$<br>1M |
| $= 1000\cos\theta\sin 30°$ | | |
| $= 500\cos\theta$ m .................................. | 1A | |
| Since $\angle GBH = 45°$ , $BH = GH$ | 1M | |
| $500\cos\theta = 1000\sin\theta$ | | |
| $\tan\theta = \frac{1}{2}$ | | |
| $\theta = 26.6°$ (26.565) ......................... | 1A | Accept<br>26°34′ ~ 26°36′ |
| | 5 | |
| (c) $EF = AB = AH\cos 30°$ | | |
| $= 1000\cos 26.565° \times \cos 30°$ | | |
| $= 774.597$m ≈ 775m ......................... | 1A | Accept 774m |
| $BE = CE$ | | |
| $= DF$ | | |
| $= 800$ | | |
| $EH = 800 - 500\cos 26.565°$ | | |
| $= 352.786 = 353$m ......................... | 1A | |
| $\tan\angle FHE = \frac{774.597}{352.786}$ $\left(\text{or } \frac{775}{353}\right)$ | 1M | |
| $\angle FHE \approx 65.5°$ (or $\angle EFH = 24.5°$) | 1A | 65°29′ ~ 65°30′<br>(24°29′ ~ 24°30′) |
| $G$ is S65.5°E of $D$ (or 114°) | 1A | |
| | 5 | |