

### FORMULAS FOR REFERENCE

SPHERE	Surface area	$= 4\pi r^2$
	Volume	$= \frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	$= 2\pi rh$
	Volume	$= \pi r^2 h$
CONE	Area of curved surface	$= \pi rl$
	Volume	$= \frac{1}{3}\pi r^2 h$
PRISM	Volume	$= \text{base area} \times \text{height}$
PYRAMID	Volume	$= \frac{1}{3} \times \text{base area} \times \text{height}$

### SECTION A (39 MARKS)

Answer ALL questions in this section.

There is no need to start each question on a fresh page.

1. In Figure 1, the cumulative frequency polygon shows the distribution of the marks of 80 students in a Mathematics test.

- (a) From the figure, write down the median of the distribution.
- (b) Copy the table below onto your answer book and complete it.

Marks	No. of Students
20 - 29	
30 - 39	
40 - 49	
50 - 59	
60 - 69	

Hence find the mean mark of the students in the test.

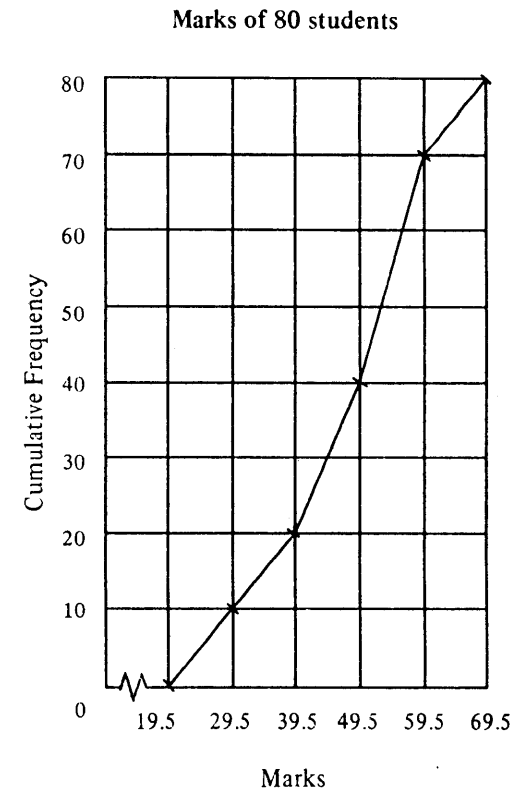


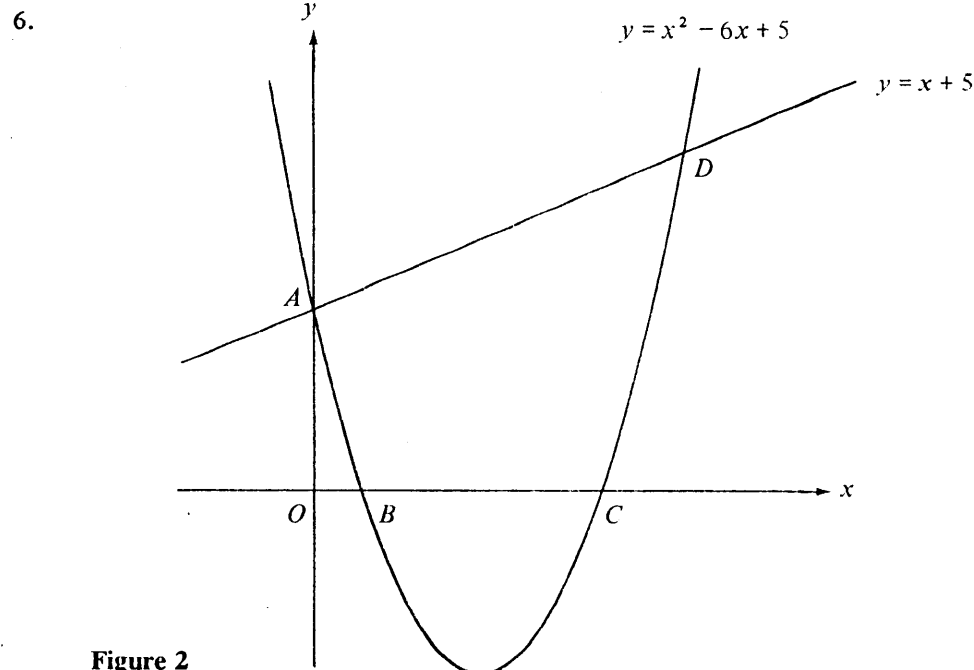
Figure 1

(5 marks)

2. In a joint variation,  $x$  varies directly as  $y^2$  and inversely as  $z$ . Given that  $x = 18$  when  $y = 3$ ,  $z = 2$ ,
- express  $x$  in terms of  $y$  and  $z$ ,
  - find  $x$  when  $y = 1$ ,  $z = 4$ .
- (5 marks)

3. A man buys some British pounds (£) with 150 000 Hong Kong dollars (HK\$) at the rate £1 = HK\$15.00 and puts it on fixed deposit for 30 days. The rate of interest is 14.60% per annum.
- How much does he buy in British pounds?
  - Find the amount in British pounds at the end of 30 days. (Suppose 1 year = 365 days and the interest is calculated at simple interest.)
  - If he sells the amount in (b) at the rate of £1 = HK\$14.50, how much does he get in Hong Kong dollars?
- (5 marks)

4. Let  $2a = 3b = 5c$ .
- Find the ratio  $a : b : c$ .
  - If  $a - b + c = 55$ , find  $c$ .
- (6 marks)
5. Solve  $\sin^2 \theta - 3\cos \theta - 1 = 0$  for  $0^\circ \leq \theta < 360^\circ$ .
- (6 marks)



The curve  $y = x^2 - 6x + 5$  meets the  $y$ -axis at  $A$  and the  $x$ -axis at  $B$  and  $C$  as shown in Figure 2.

- Find the coordinates of  $A$ ,  $B$  and  $C$ .
  - The line  $y = x + 5$  passes through  $A$  and meets the curve again at  $D$ . Find the coordinates of  $D$ .
- (6 marks)
7. Let  $\alpha$  and  $\beta$  be the roots of the equation  $10x^2 + 20x + 1 = 0$ . Without solving the equation, find the values of
- $4^\alpha \times 4^\beta$ ,
  - $\log_{10} \alpha + \log_{10} \beta$ .
- (6 marks)

**SECTION B (60 marks)**

Answer any FIVE questions from this section.

Each question carries 12 marks.

8. In Figure 3,  $L_1$  is the line  $x = 4$ ,  $L_2$  is the line passing through the point  $(0, 2)$  with slope 1, and  $L_3$  is the line passing through the points  $(5, 0)$  and  $(0, 5)$ .

(a) Find the equations of  $L_2$  and  $L_3$ . (3 marks)

(b) Write down the three inequalities which determine the shaded region, including the boundary. (3 marks)

(c) Suppose  $P = x + 2y - 3$  and  $(x, y)$  is any point satisfying all the inequalities in (b).  
 (i) Find the point  $(x, y)$  at which  $P$  is a minimum. What is this minimum value of  $P$ ?  
 (ii) If  $P \geq 7$ , by adding a suitable straight line to Figure 3, find the range of possible values of  $x$ . (6 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page
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8.(Cont'd) If you attempt Question 8, fill in the details in the first three boxes above and tie this sheet inside your answer book.

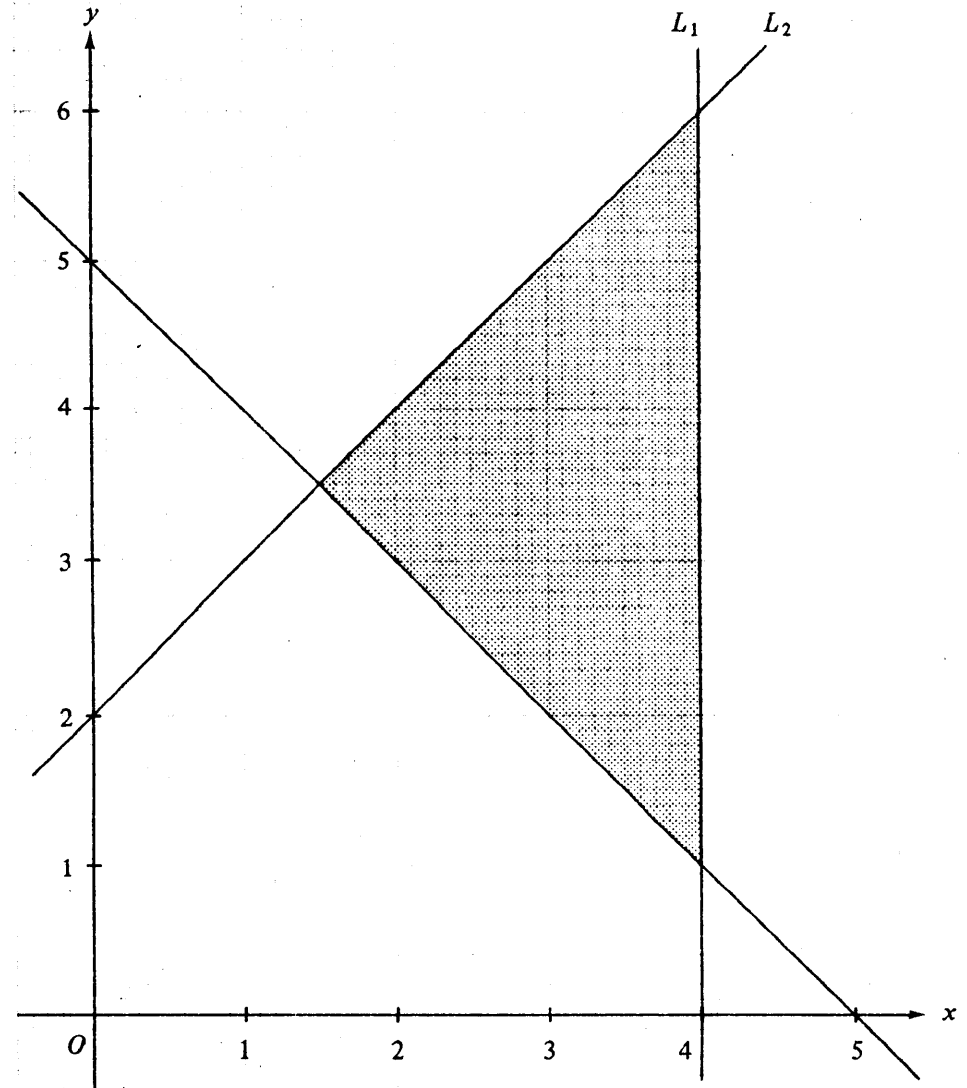


Figure 3

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9.

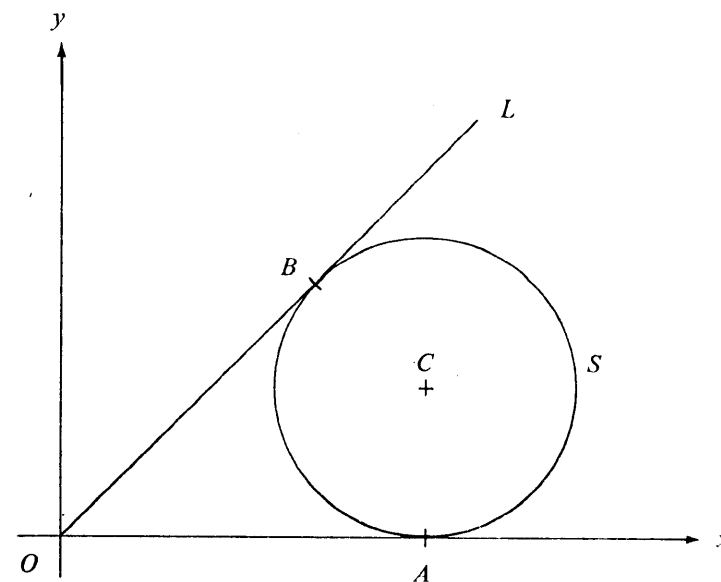


Figure 4

In Figure 4, the circle  $S : x^2 + y^2 - 4x - 2y + 4 = 0$  with centre  $C$  touches the  $x$ -axis at  $A$ . The line  $L : y = mx$ , where  $m$  is a non-zero constant, passes through the origin  $O$  and touches  $S$  at  $B$ .

- (a) Find the coordinates of  $C$  and  $A$ .  
(2 marks)
- (b) Show that  $m = \frac{4}{3}$ .  
(5 marks)
- (c) (i) Explain why the four points  $O, A, C, B$  are concyclic.  
(ii) Find the equation of the circle passing through these four points.  
(5 marks)

10. The practical test for a driving licence consists of two independent parts  $A$  and  $B$ . To pass the practical test, a candidate must pass in both parts. If a candidate fails in any one of these parts, the candidate may take that part again. Statistics show that the passing percentages for Part  $A$  and Part  $B$  are 70% and 60% respectively.

- (a) A candidate takes the practical test. Find the probabilities that the candidate
- fails Part  $A$  on the first attempt and passes it on the second attempt,
  - passes Part  $A$  in no more than two attempts,
  - passes the practical test in no more than two attempts in each part.
- (10 marks)
- (b) In a sample of 10 000 candidates taking the practical test, how many of them would you expect to pass the practical test in no more than two attempts in each part?

(2 marks)

11.

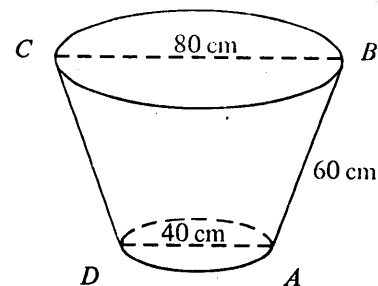


Figure 5a

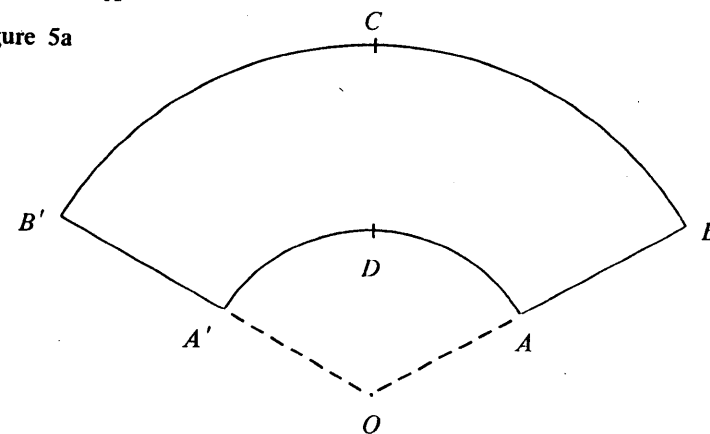


Figure 5b

Figure 5a shows a metal bucket. Its slant height  $AB$  is 60 cm. The diameter  $AD$  of the base is 40 cm and the diameter  $BC$  of the open top is 80 cm. The curved surface of the bucket is formed by the thin metal sheet  $ABB'A'$  shown in Figure 5b, where  $\widehat{ADA'}$  and  $\widehat{BCB'}$  are arcs of concentric circles with centre  $O$ .

- Find  $OA$  and  $\angle AOA'$ . (5 marks)
- Find the area of the metal sheet  $ABB'A'$ , leaving your answer in terms of  $\pi$ . (3 marks)
- There is an ant at the point  $A$  on the outer curved surface of the bucket. Find the shortest distance for it to crawl along the outer curved surface of the bucket to reach the point  $C$ . (4 marks)

(4 marks)

12.

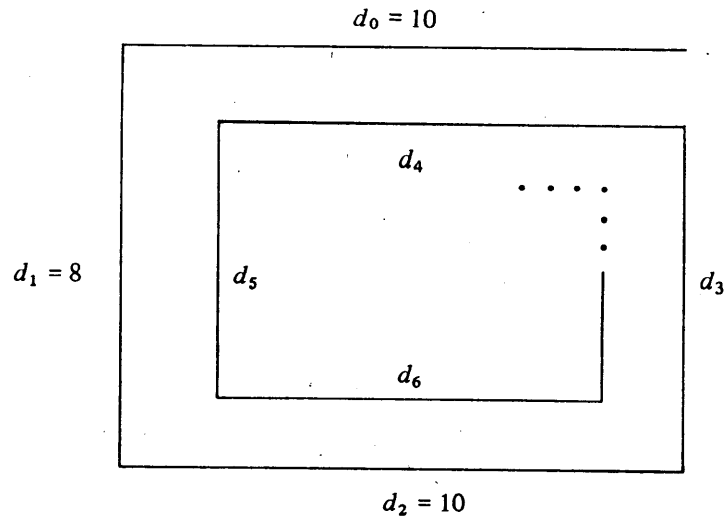


Figure 6

A maze is formed by line segments of lengths  $d_0, d_1, d_2, \dots, d_n, \dots$ , with adjacent line segments perpendicular to each other as shown in Figure 6.

Let  $d_0 = 10, d_1 = 8, d_2 = 10$  and  $\frac{d_{n+2}}{d_n} = 0.9$  when  $n \geq 1$ ,

i.e.  $\frac{d_3}{d_1} = \frac{d_5}{d_3} = \dots = 0.9$  and  $\frac{d_4}{d_2} = \frac{d_6}{d_4} = \dots = 0.9$ .

- Find  $d_3$  and  $d_5$ , and express  $d_{2n-1}$  in terms of  $n$ . (4 marks)
- Find  $d_6$  and express  $d_{2n}$  in terms of  $n$ . (2 marks)
- Find, in terms of  $n$ , the sums
  - $d_1 + d_3 + d_5 + \dots + d_{2n-1}$ ,
  - $d_2 + d_4 + d_6 + \dots + d_{2n}$ . (3 marks)
- Find the value of the sum  $d_0 + d_1 + d_2 + d_3 + \dots$  to infinity. (3 marks)

13.

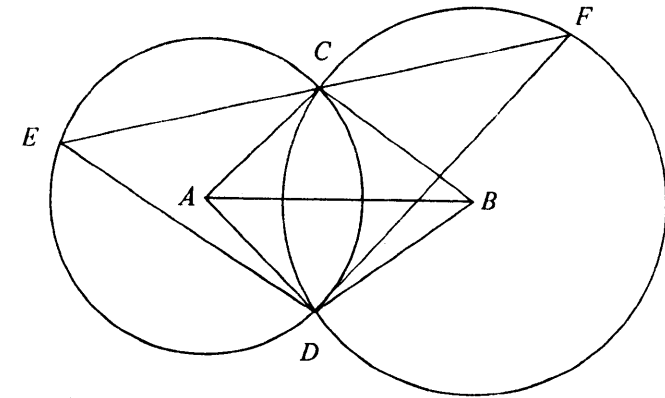


Figure 7

In Figure 7,  $A, B$  are the centres of the circles  $DEC$  and  $DFC$  respectively.  $ECF$  is a straight line.

- Prove that triangles  $ABC$  and  $ABD$  are congruent. (3 marks)
- Let  $\angle FED = 55^\circ, \angle ACB = 95^\circ$ .
  - Find  $\angle CAB$  and  $\angle EFD$ .
  - A circle  $S$  is drawn through  $D$  to touch the line  $CF$  at  $F$ .
    - Draw a labelled rough diagram to represent the above information.
    - Show that the diameter of the circle  $S$  is  $2DF$ . (9 marks)