89-CE
MATHS
PAPER I

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1989

MATHEMATICS PAPER I

8.30 am-10.30 am (2 hours)

This paper must be answered in English

Attempt ALL questions in Section A and any FIVE questions in Section B.

Full marks will not be given unless the method of solution is shown.

FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area × height
PYRAMID	Volume	=	$\frac{1}{3}$ × base area × height

SECTION A **Answer ALL questions in this section.**
There is no need to start each question on a fresh page.
Geometry theorems need not be quoted when used.

1. The monthly income of a man is increased from \$8000 to \$9000.

   (a) Find the percentage increase.

   (b) After the increase, the ratio of his savings to his expenditure is 3 : 7 for each month. How much does he save each month?

   (4 marks)

2. Consider $x + 1 > \frac{1}{5}(3x + 2)$.

   (a) Solve the inequality.

   (b) In addition, if $-4 \leqslant x \leqslant 4$, find the range of x .

   (4 marks)

3. Given that $(x + 1)$ is a factor of $x^4 + x^3 - 8x + k$, where k is a constant,

   (a) find the value of k ,

   (b) factorize $x^4 + x^3 - 8x + k$.

   (6 marks)

4. AB is a diameter of a circle and M is a point on the circumference. C is a point on BM produced such that $BM = MC$.

   (a) Draw a diagram to represent the above information.

   (b) Show that AM bisects $\angle BAC$.

   (6 marks)

5. (a) Solve the simultaneous equations $\begin{cases} x + 2y = 5 \\ 5x - 4y = 4 \end{cases}$

(b) Given that $\begin{cases} \dfrac{a}{c} + \dfrac{2b}{c} = 5 \\ \dfrac{5a}{c} - \dfrac{4b}{c} = 4 \end{cases}$, where a, b and c are non-zero numbers, using the result of (a), find $a : b : c$.

(6 marks)

6.

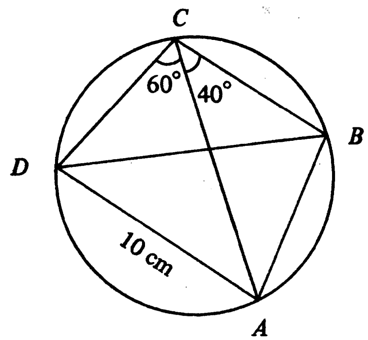

Figure 1

In Figure 1, $ABCD$ is a cyclic quadrilateral with $AD = 10$ cm, $\angle ACD = 60°$ and $\angle ACB = 40°$.

(a) Find $\angle ABD$ and $\angle BAD$.

(b) Find the length of BD in cm, correct to 2 decimal places.

(6 marks)

7. Rewrite the equation $3 \tan \theta = 2 \cos \theta$ in the form $a \sin^2 \theta + b \sin \theta + c = 0$, where a, b and c are integers.

Hence solve the equation for $0° \leqslant \theta < 360°$.

(7 marks)

SECTION B

Answer any FIVE questions from this section.
Each question carries 12 marks.

8.

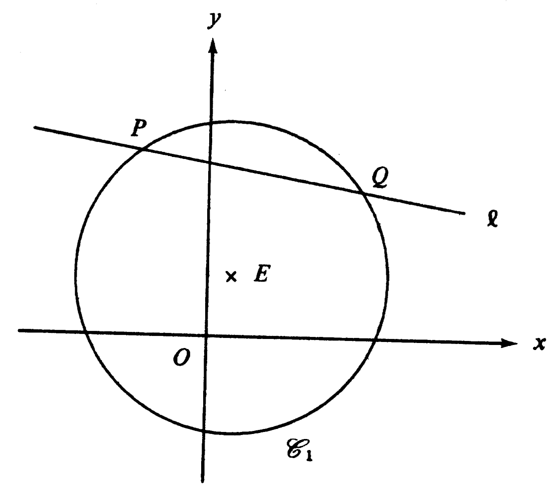

Figure 2

Let E be the centre of the circle $\mathscr{C}_1 : x^2 + y^2 - 2x - 4y - 20 = 0$. The line $\ell : x + 7y - 40 = 0$ cuts $\mathscr{C}_1$ at the points P and Q as shown in Figure 2.

(a) Find the coordinates of E.

(1 mark)

(b) Find the coordinates of P and Q.

(4 marks)

(c) Find the equation of the circle $\mathscr{C}_2$ with PQ as diameter.

(3 marks)

(d) Show that $\mathscr{C}_2$ passes through E.

Hence, or otherwise, find $\angle EPQ$.

(4 marks)

9. The positive numbers 1 , k , $\frac{1}{2}$, . . . are in geometric progression.

(a) Find the value of k , leaving your answer in surd form. (2 marks)

(b) Express the nth term $T(n)$ in terms of n . (2 marks)

(c) Find the sum to infinity, expressing your answer in the form $p + \sqrt{q}$, where p and q are integers. (4 marks)

(d) Express the product $T(1) \times T(3) \times T(5) \times \ldots \times T(2n-1)$ in terms of n . (4 marks)

10. **Answers in this question should be given correct to at least 3 significant figures or in surd form.**

In Figure 3, a triangular board ABC , right-angled at A with $AB = AC = 10$ m, is placed with the vertex A on the horizontal ground. AB and AC make angles of 45° and 30° with the horizontal respectively. The sun casts a shadow $AB'C'$ of the board on the ground such that B' and C' are vertically below B and C respectively.

(a) Find the lengths of AB' and AC' . (2 marks)

(b) Find the lengths of BC , BB' and CC' . (3 marks)

(c) Using the results of (b), or otherwise, find the length of $B'C'$. (3 marks)

(d) Find $\angle B'AC'$.

Hence find the area of the shadow. (4 marks)

10. (Cont'd)

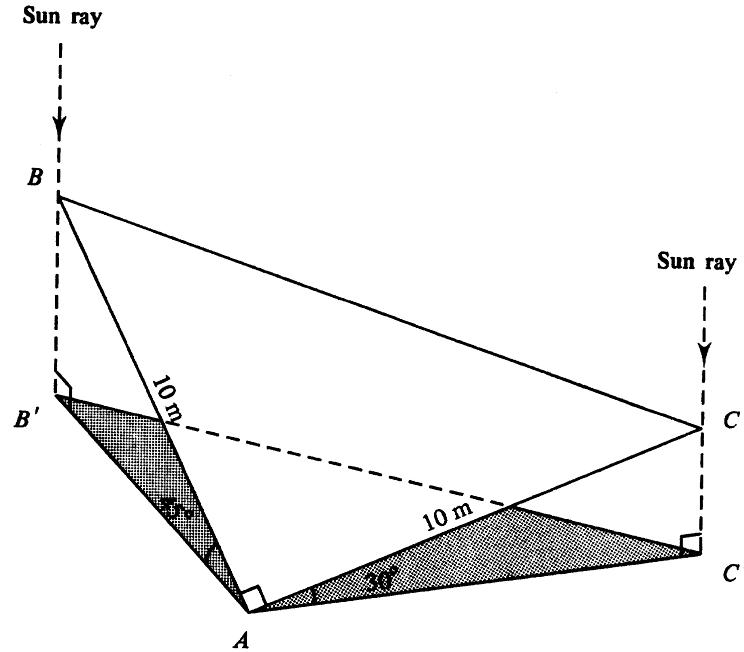

Figure 3

11. Figure 4a shows a rectangular swimming pool 50 m long and 20 m wide. The floor of the pool is an inclined plane. The depth of water is 10 m at one end and 2 m at the other.

(a) Find the volume of water in the pool in m^3 . (2 marks)

(b) Water in the pool is now pumped out through a pipe of internal radius 0.125 m. Water flows in the pipe at a constant speed of 3 m/s.

(i) Find the volume of water, in m^3, REMAINING in the pool when the depth of water is 8 m at the deeper end.

(ii) Find the volume of water pumped out in 8 hours, correct to the nearest m^3 .

(iii) Let h metres be the depth of water at the deeper end after 8 hours (see Figure 4b). Find the value of h , correct to 1 decimal place. (10 marks)

11. (Cont'd)

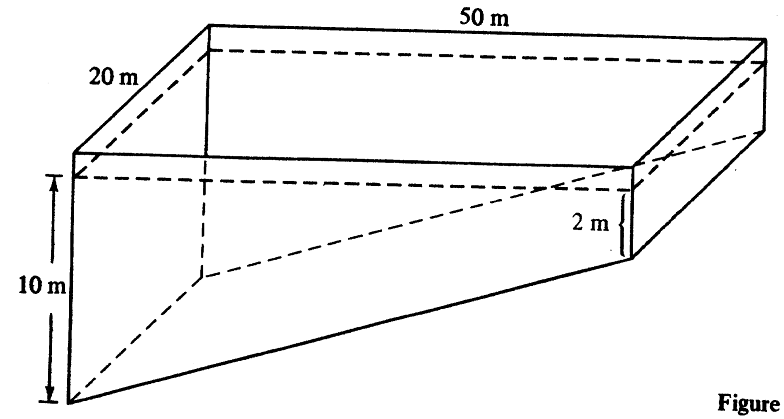

Figure 4a

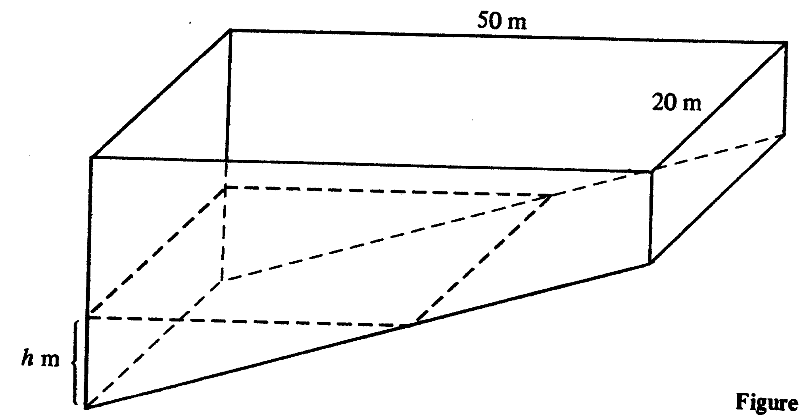

Figure 4b

12.

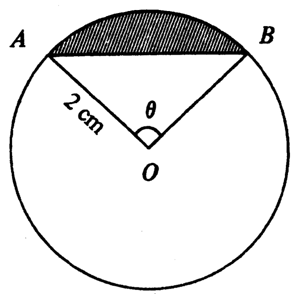

Figure 5

In Figure 5, O is the centre of a circle of radius 2 cm. A and B are two points on the circle such that $\angle AOB = \theta$ radians, where $0 < \theta < \pi$.

(a) (i) Find the area of ΔOAB in terms of θ.

(ii) Find the value of θ for which the area of ΔOAB is the greatest.

(2 marks)

(b) If the area of the shaded segment is 2 cm^2, show that

$$\theta - \sin\theta - 1 = 0 .$$

(3 marks)

(c) Let $f(\theta) = \theta - \sin\theta - 1$ and α be the root of $f(\theta) = 0$. Show that α lies between 0 and 3.

(2 marks)

(d) Using the method of bisection, find the value of α correct to one decimal place.

(5 marks)

13. (a) Bag A contains a number of balls. Some are black and the rest are white. A ball is drawn at random from bag A. Let p be the probability that the ball drawn is black and q be the probability that the ball drawn is white. If $p = 3q$, find q.

(2 marks)

(b) Bag C contains 10 balls of which n $(2 \leqslant n \leqslant 10)$ balls are black.

(i) If two balls are drawn at random from bag C, find the probability, in terms of n, that both balls are black.

(ii) If the probability obtained in (i) is greater than $\frac{1}{3}$, find the possible values of n.

(7 marks)

(c) Bag M contains 1 red and 1 green ball. Bag N contains 3 red and 2 green balls. A ball is drawn at random from bag M and put into bag N; then a ball is drawn at random from bag N. Find the probability that the ball drawn from bag N is red.

(3 marks)

14\. (a) In Figure 6, draw and shade the region that satisfies the following inequalities:

$$\begin{cases} y \geqslant 20 \\ 2x - y \geqslant 40 \\ x + y \leqslant 100 \end{cases}$$

(4 marks)

(b) The vitamin content and the cost of three types of food X , Y and Z are shown in the following table:

	Food X	Food Y	Food Z
Vitamin A (units/kg)	400	600	400
Vitamin B (units/kg)	800	200	400
Cost (dollars/kg)	6	5	4

A man wants to produce 100 kg of a mixture by mixing these three types of food. Let the amount of food X , food Y and food Z used be x , y and z kilograms respectively.

(i) Express z in terms of x and y .

(ii) Express the cost of the mixture in terms of x and y .

(iii) Suppose the mixture must contain at least 44 000 units of vitamin A and 48 000 units of vitamin B. Show that

$$\begin{cases} y \geqslant 20 \\ 2x - y \geqslant 40 \\ x + y \leqslant 100 \end{cases}$$

(iv) Using the result in (a), determine the values of x , y and z so that the cost is the least.

(8 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page

14\. (Cont'd)

If you attempt Question 14, fill in the details in the first three boxes above and tie this sheet inside your answer book.

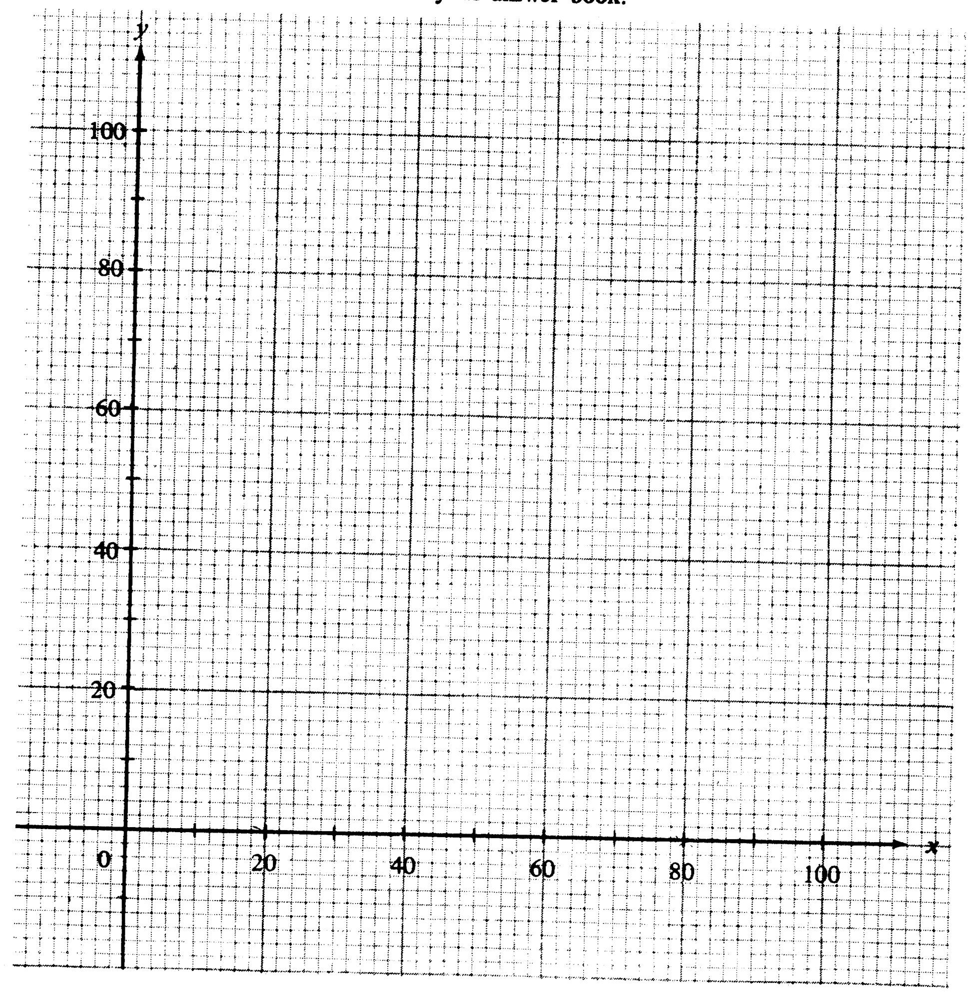

Figure 6

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