88-CE
MATHS
PAPER I

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1988

數學 試卷一
MATHEMATICS PAPER I

8.30 am–10.30 am (2 hours)
This paper must be answered in English

Attempt **ALL** questions in Section A and any **FIVE** questions in Section B.

Full marks will not be given unless the method of solution is shown.

FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area × height
PYRAMID	Volume	=	$\frac{1}{3}$ × base area × height

SECTION A Answer ALL questions in this section.
There is no need to start each question on a fresh page.
Geometry theorems need not be quoted when used.

1. Factorize $a^2 - a - 6$ and $a^3 + 8$.

   Hence find their L.C.M.

   (5 marks)

2. Simplify

   (a) $\dfrac{\sin(180° - \theta)}{\sin(90° + \theta)}$,

   (b) $\sin^2(\pi - \phi) + \sin^2\left(\dfrac{3\pi}{2} + \phi\right)$.

   (5 marks)

3. Solve the inequality $2x^2 \geqslant 5x$.

   (5 marks)

4. The quadratic equation

   $9x^2 - (k + 1)x + 1 = 0$ ....(*)

   has equal roots.

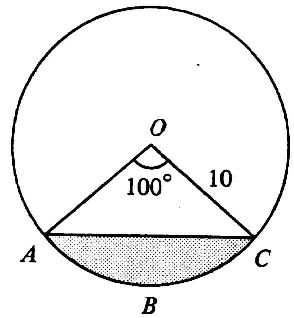
   (a) Find the two possible values of the constant k.

   (b) If k takes the negative value obtained, solve equation (*).

   (6 marks)

5.

Figure 1



In Figure 1, ABC is a circle with centre O and radius 10. $\angle AOC = 100°$. Calculate, correct to 2 decimal places,

(a) the area of sector $OABC$,

(b) the area of $\triangle OAC$,

(c) the area of segment ABC.

(6 marks)

6. Given that $\log 2 = r$ and $\log 3 = s$, express the following in terms of r and s:

   (a) $\log 18$,

   (b) $\log 15$.

   [Note: In this question, all logarithms are to the base 10.]

   (6 marks)

7.

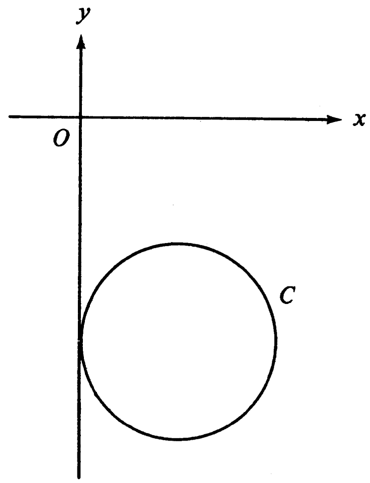


Figure 2

In Figure 2, the circle C has equation

$$x^2 + y^2 - 4x + 10y + k = 0 ,$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(b) If C touches the y-axis, find the radius of C and the value of k.

(6 marks)

SECTION B **Answer any FIVE questions from this section. Each question carries 12 marks.**

8. (a) P is a point inside a square $ABCD$ such that PBC is an equilateral triangle. AP is produced to meet CD at Q.

(i) Draw a diagram to represent the above information.

(ii) Calculate $\angle PAB$ and $\angle PQC$.

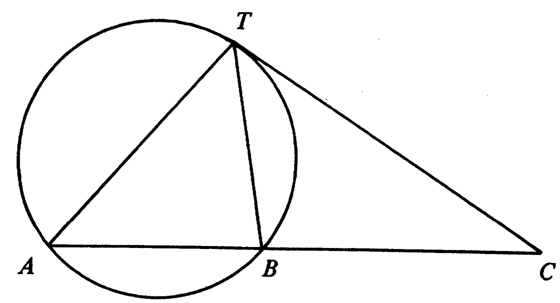
(7 marks)

(b) In Figure 3, CT is tangent to the circle ABT.

(i) Find a triangle similar to ΔACT and give reasons.

(ii) If $CT = 6$ and $BC = 5$, find AB.

Figure 3



(5 marks)

9. (a) Write down the smallest and the largest multiples of 7 between 100 and 999.

(2 marks)

(b) How many multiples of 7 are there between 100 and 999?

Find the sum of these multiples.

(6 marks)

(c) Find the sum of all positive three-digit integers which are **NOT** divisible by 7.

(4 marks)

10. A variable quantity y is the sum of two parts. The first part varies directly as another variable x, while the second part varies directly as x^2. When $x = 1$, $y = -5$; when $x = 2$, $y = -8$.

(a) Express y in terms of x.

Hence find the value of y when $x = 6$.

(8 marks)

(b) Express y in the form $(x - p)^2 - q$, where p and q are constants.

Hence find the least possible value of y when x varies.

(4 marks)

11. Figure 4 shows the cumulative frequency curve of the marks of 600 students in a mathematics contest.

(a) From the curve, find

(i) the median, and

(ii) the interquartile range of the distribution of marks.

(4 marks)

(b) A student with marks greater than or equal to 100 will be awarded a prize.

(i) Find the number of students who will be awarded prizes.

(ii) If one student is chosen at random from the 600 students, find the probability that the student is a prize-winner.

(iii) If two students are chosen at random, find the probability that

(1) both of them are prize-winners,

(2) at least one of them is a prize-winner.

(8 marks)

11. (cont'd)

Candidates need NOT hand in this graph.

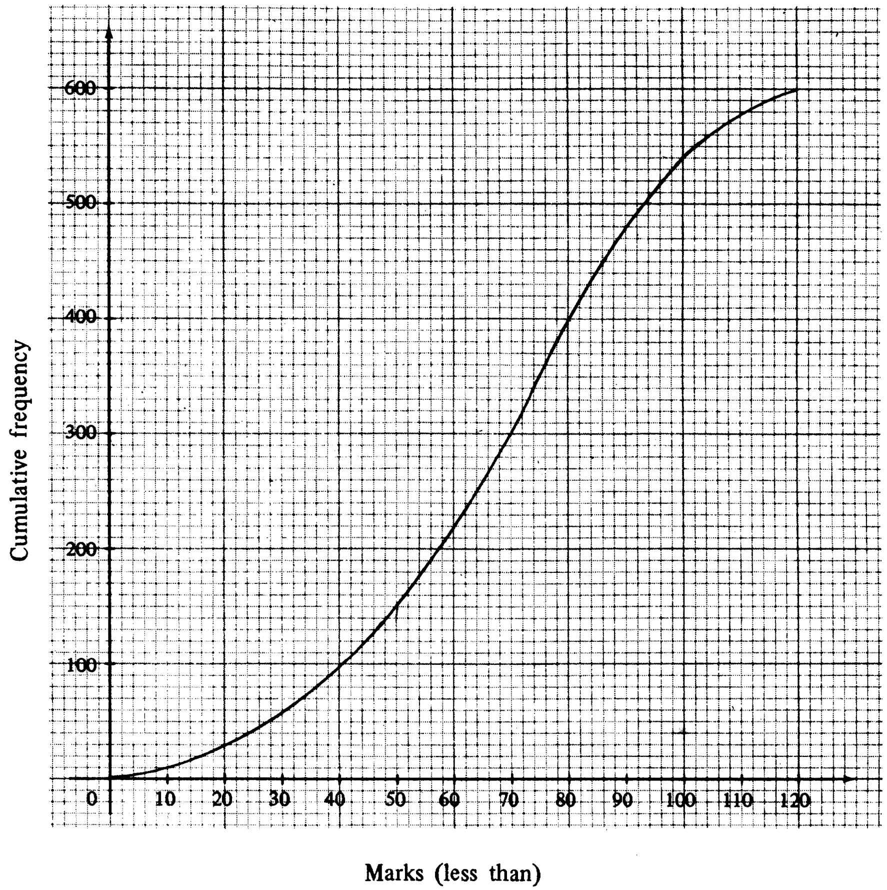


Figure 4

12. In Figure 5, L_1 is the line $x = 3$ and L_2 is the line $y = 4$. L_3 is the line passing through the points (3, 0) and (0, 4).

(a) Find the equation of L_3 in the form $ax + by = c$, where a, b and c are integers.

(2 marks)

(b) Write down the three constraints which determine the shaded region, including the boundary.

(3 marks)

(c) Let $P = x + 4y$. If (x, y) is any point satisfying all the constraints in (b), find the greatest and the least values of P.

(4 marks)

(d) If one more constraint $2x - 3y + 3 \leqslant 0$ is added, shade in Figure 5 the new region satisfying all the four constraints.

For any point (x, y) lying in the new region, find the least value of P defined in (c).

(3 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

12. (cont'd)

If you attempt this question, fill in the details in the first three boxes above and tie this sheet inside your answer book.

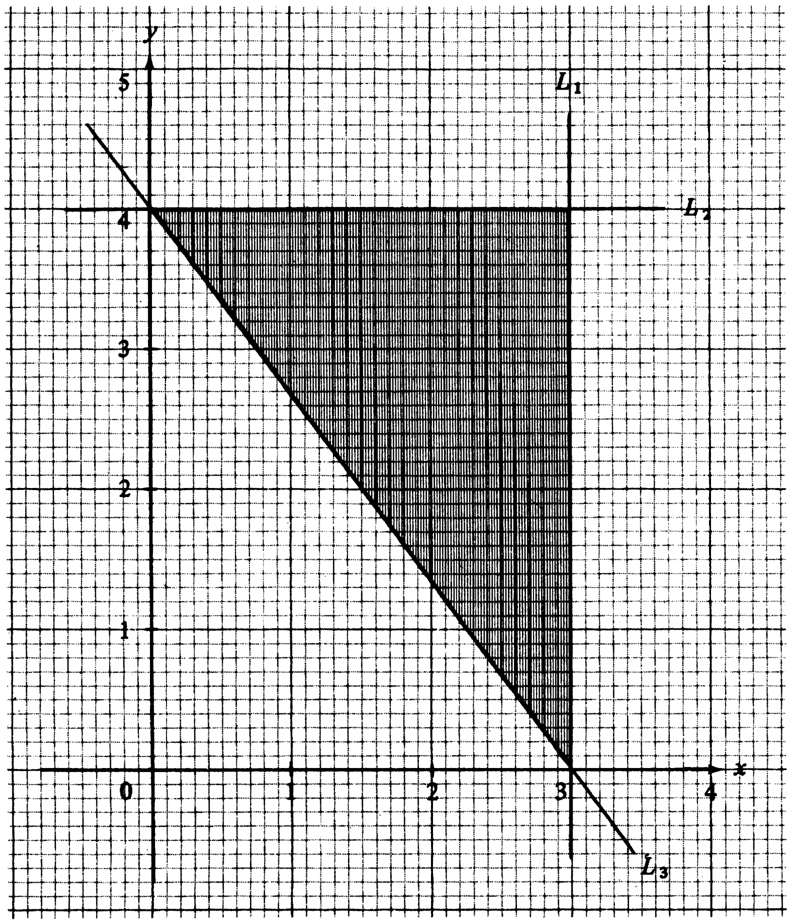


Figure 5

This is a blank page.

13.

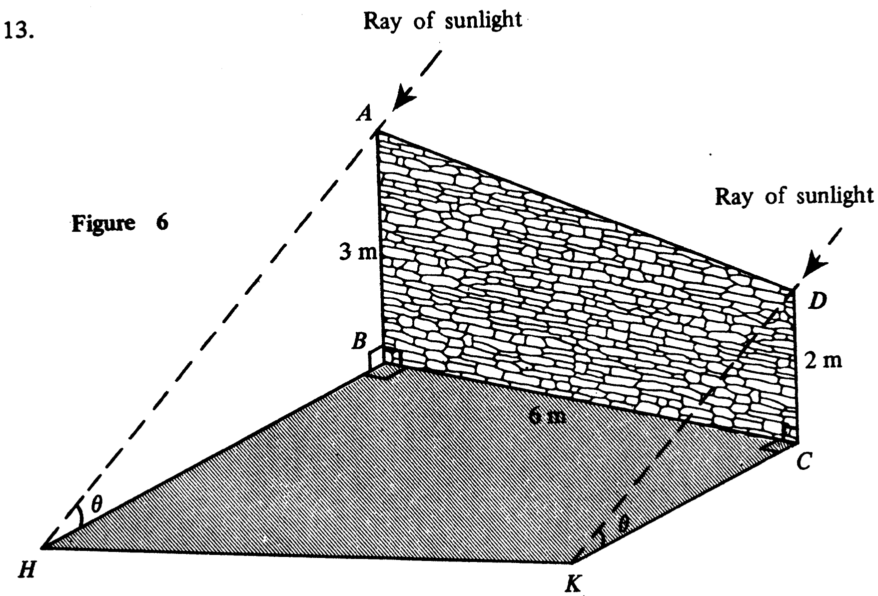


Figure 6

In Figure 6, $ABCD$ is a wall in the shape of a trapezium with AB and DC vertical. Rays of sunlight coming from the back of the wall cast a shadow $HBCK$ on the horizontal ground such that the edges HB and KC of the shadow are perpendicular to BC. Suppose the angle of elevation of the sun is θ, $AB = 3$ m, $CD = 2$ m and $BC = 6$ m.

(a) Express HB and KC in terms of θ.

(3 marks)

(b) (i) Find the area S_1 of the wall.

(ii) Find, in terms of θ, the area S_2 of the shadow.

Hence show that $\frac{S_1}{S_2} = \tan\theta$.

(3 marks)

(c) If $\theta = 30°$, find the length of the edge HK, leaving your answer in surd form.

(6 marks)

14. Figure 7 shows the graph of $y = x^3$ for $x \geqslant 0$.

(a) Let r be the real root of the equation $x^3 - \frac{4}{3}x - 6 = 0$.

(i) By adding a suitable straight line to the figure, find an interval of width 0.1 which contains r.

(ii) Use the method of bisection to find the value of r correct to two decimal places. Show your working in the form of a table.

(9 marks)

(b) Use (a) to find, correct to two decimal places, the real root of the equation $3(t+1)^3 - 4(t+1) - 18 = 0$.

(3 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

14. (cont'd)

If you attempt this question, fill in the details in the first three boxes above and tie this sheet inside your answer book.

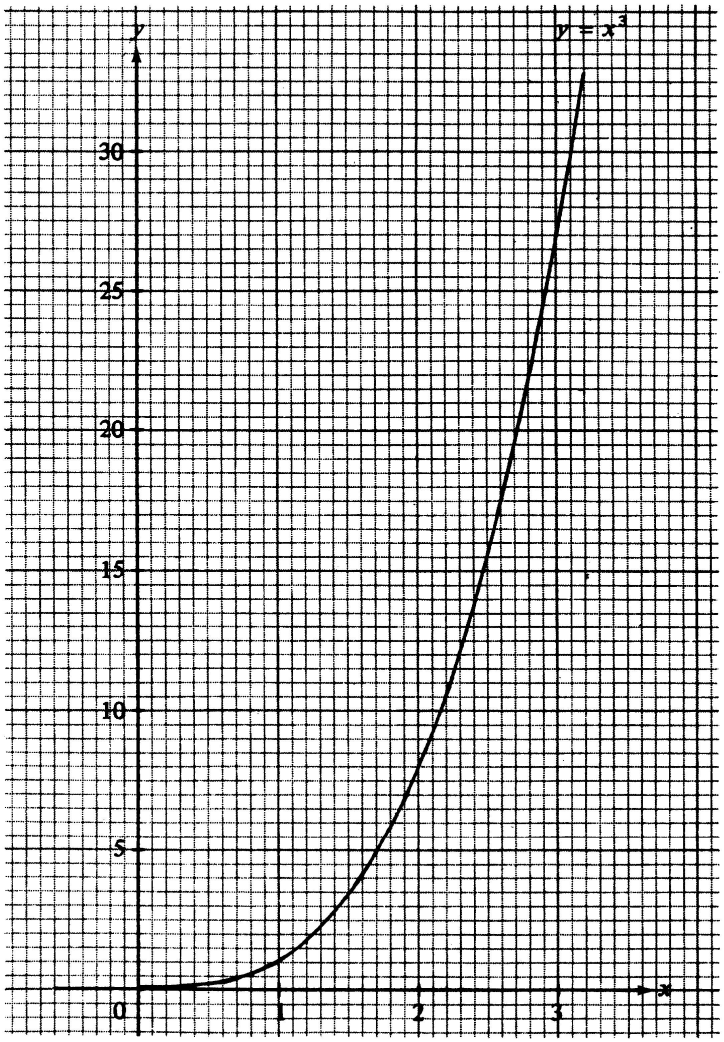


Figure 7

END OF PAPER