Form 5

HKCEE 1986
Mathematics II

86 1. If $r = \sqrt[3]{h^3 - 7r^3}$, then the ratio $r : h$ is

A. 1 : 8 .
B. $1 : 2\sqrt{2}$.
C. 1 : 2 .
D. $1 : \sqrt{2}$.
E. $1 : \sqrt[3]{2}$.

86 2. $\dfrac{\frac{x^2}{3} - 3y^2}{\frac{3}{2}(x - 3y)} =$

A. $\frac{1}{2}(x - 3y)$.
B. $\frac{2}{9}(x - 3y)$.
C. $2(x + 3y)$.
D. $\frac{1}{2}(x + 3y)$.
E. $\frac{2}{9}(x + 3y)$.

86 3. If $1 - \dfrac{x+y}{y-x} = a$ $(a \neq 0)$, then $y =$

A. x .
B. $\frac{x}{a}(a - 2)$.
C. $\frac{x}{a}(a - 1)$.
D. $\frac{x}{a}(2 - a)$.
E. $\frac{x}{a}(1 - a)$.

86 4. Which of the following is an identity/are identities?

I. $(x + 1)(x - 1) = x^2 + 1$,
II. $x^2 - 2x + 1 = 0$,
III. $(x - 2)^2 = (2 - x)^2$.

A. I only
B. II only
C. III only
D. I and II only
E. I and III only

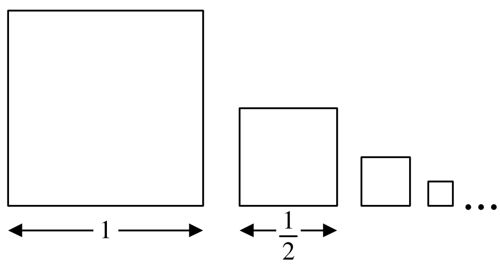
86 5. Given the identity
$$\frac{2}{x-1} + \frac{x+1}{(x-1)^2} + \frac{a}{(1-x)^2} = \frac{bx-2}{(x-1)^2},$$
find the values of the constants a and b.

A. $a = 1, b = 3$.
B. $a = 3, b = 1$.
C. $a = 1, b = -3$.
D. $a = 3, b = -1$.
E. $a = -1, b = 3$.

86 6. If α and β are the roots of the equation $2x^2 + x + 3 = 0$, find the value of
$$\alpha - \frac{\alpha^2}{\alpha + \beta}$$

A. –3 .
B. –2 .
C. 2 .
D. 3 .
E. It cannot be determined.

86 7.



The figure shows an infinite number of squares. The length of a side of the first square is 1. The side of the first square is equal to half of the side of the preceding one. Find the sum of the areas of the infinite number of squares.

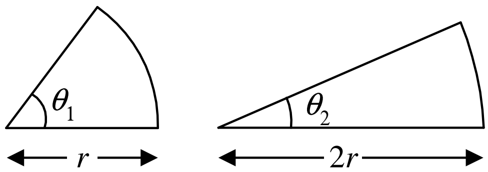
A. 4 .
B. 2 .
C. $\frac{5}{3}$.
D. $\frac{3}{2}$.
E. $\frac{4}{3}$.

86 8. Find the real value of x such that

$$\begin{cases} x^2 + x + 1 = k \\ x - 1 = \dfrac{7}{k} \end{cases}$$, where k is a constant.

A. 3 .
B. 2 .
C. 1 .
D. −1 .
E. −3 .

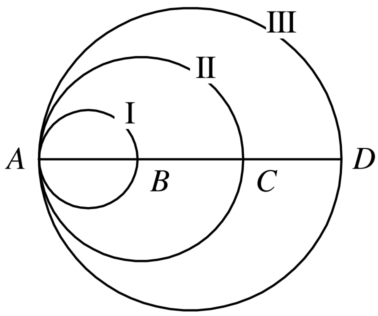
86 9.



The figure shows two sectors with radii r and $2r$. If these two sectors are equal in area, then $\theta_1 : \theta_2 =$

A. 2 : 1 .
B. 3 : 1 .
C. 4 : 1 .
D. 5 : 1 .
E. 6 : 1 .

86 10.



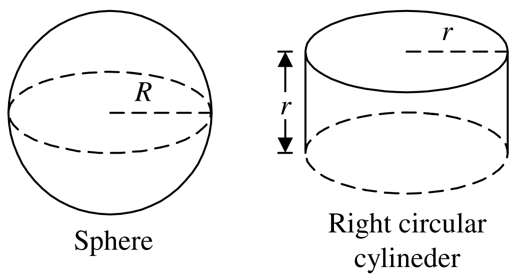
In the figure, $ABCD$ is a straight line with $AB = BC = CD$. Three circles I, II and III are drawn respectively on AB, AC and AD as diameters. Areas of circle I : Area of circle II : Area of circle III =

A. 1 : 2 : 3 .
B. 1 : 2 : 4 .
C. 1 : 4 : 9 .
D. 1 : 4 : 16 .
E. 1 : 8 : 27 .

86 11. A man drives a car at 45 km/h for 3 hours and then at 50 km/h for 2 hours. His average speed for the whole journey is

A. 47 km/h .
B. 47.5 km/h .
C. 48 km/h .
D. 48.5 km/h .
E. 49 km/h .

86 12.



In the figure, if $\dfrac{\text{Volume of the sphere}}{\text{Volume of the right circular cylinder}} = \dfrac{9}{2}$, then $\dfrac{R}{r} =$

A. $\frac{3}{2}$.
B. $\frac{3}{\sqrt{2}}$.
C.
D. $\frac{\sqrt[3]{9}}{\sqrt[3]{2}}$.
E. $\frac{9}{2}$.

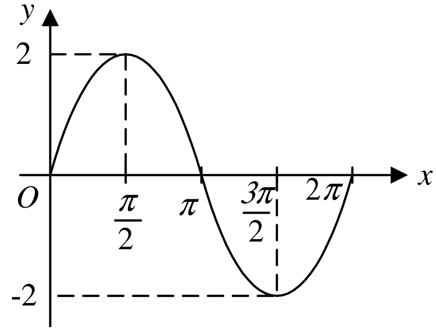
86
13. If the compound interest on \$1000 for two years at 9% p.a., pay half-yearly is \$$x$, find x.

A. $1000 \times \frac{9}{100} \times 2$.
B. $1000(1 + \frac{9}{100})^4$.
C. $1000(1 + \frac{4.5}{100})^4$.
D. $1000(1 + \frac{9}{100})^2 - 1000$.
E. $1000(1 + \frac{4.5}{100})^4 - 1000$.

86
14. If $\sin\theta\cos\theta = \frac{1}{4}$, then $(\sin\theta + \cos\theta)^2$ =

A. 2 .
B. $\frac{3}{2}$.
C. 1 .
D. $\frac{1}{2}$.
E. $\frac{1}{4}$.

86
15.



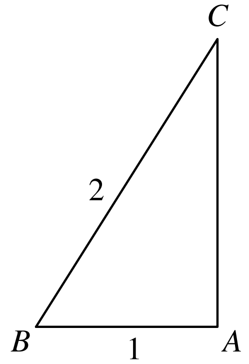
Which of the following functions may be represented by the above graph in the interval 0 to 2π?

A. $y = \cos 2x$.
B. $y = 2\cos x$.
C. $y = \frac{1}{2}\cos 2x$.
D. $y = \sin 2x$.
E. $y = 2\sin 2x$.

86
16. $\sin^4\theta - \cos^4\theta =$

A. -1 .
B. $1 - 2\cos^4\theta$.
C. $\sin\theta - \cos\theta$.
D. $\sin^2\theta - \cos^2\theta$.
E. $2\sin^4\theta - 1$.

86
17.

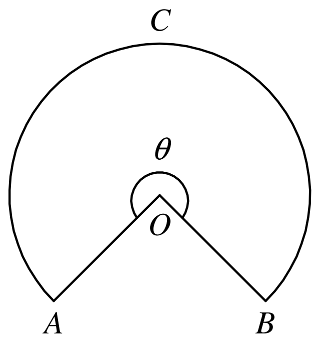


In the figure, $\angle A : \angle B : \angle C =$

A. $2 : \sqrt{3} : 1$.
B. $4 : 3 : 1$.
C. $3 : 2 : 1$.
D. $\sqrt{3} : \sqrt{2} : 1$.

E. $1 : 2 : \sqrt{3}$

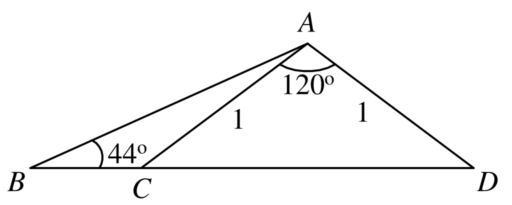
86 18.



In the figure, if the area of the sector is x, then arc ACB =

A. $\dfrac{2x}{r}$.
B. $\dfrac{x}{r}$.
C. $\dfrac{2x}{r^2}$.
D. $\dfrac{\pi x}{90r}$.
E. $\dfrac{90x}{\pi r}$.

86 19.



In the figure, $AC = AD = 1$. $\angle ABD = 20^\circ$ and $\angle CAD = 120^\circ$, find AB.

A. $2\cos 20^\circ$.
B. $\dfrac{1}{2\sin 20^\circ}$.
C. $\dfrac{\sqrt{3}}{2\sin 20^\circ}$.
D. $\sqrt{3}\cos 20^\circ$.
E. $2\sin 20^\circ$.

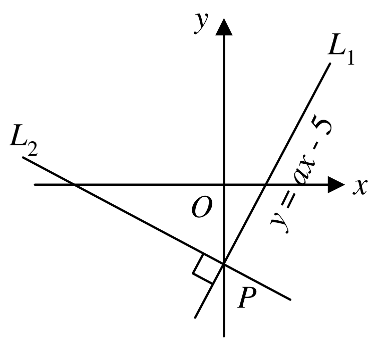
86 20. The bearing of a lighthouse as observed from an ocean liner is N37oE, the bearing of the ocean linear as observed from the light house is

A. N37°E .
B. N53°W .
C. S37°E .
D. S37°W .
E. S53°W .

86 21. Which of the following represents a circle?

A. $2x^2 - 8y + 5 = 0$.
B. $2x^2 + y^2 - 4x - 3y = 0$.
C. $3x^2 + 3y^2 - 5x - 7 = 0$.
D. $x^2 - y^2 - 7x + 6y + 1 = 0$.
E. $x^2 + y^2 + 2xy + 7y - 1 = 0$.

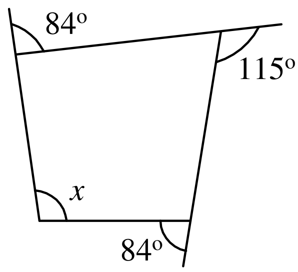
86 22.



In the figure, L_1 and L_2 are two straight lines perpendicular to each other and intersecting at P on the y-axis. If the equation of L_1 is $y = ax - 5$, then the equation of L_2 is

A. $y = -\dfrac{1}{a}x - 5$.
B. $y = -\dfrac{1}{a} + 5$.
C. $y = -ax - 5$.
D. $y = -ax + 5$.
E. $y = -\dfrac{1}{a}x$.

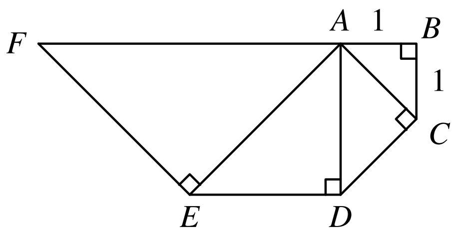
86 23.



In the figure, $x =$

A. 77° .
B. 84° .
C. 96° .
D. 103° .
E. 115° .

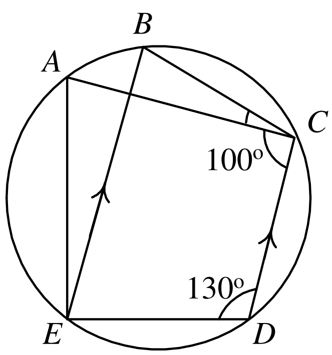
86 24.



In the figure, *ABC*, *ACD*, *ADE* and *AEF* are right angled isosceles triangles. If $AB = BC = 1$, how long is *AF*?

A. $2\sqrt{5}$.
B. 4 .
C. $2\sqrt{3}$.
D. 3 .
E. $\sqrt{5}$.

86 25.



In the figure, *A*, *B*, *C*, *D* and *E* lie on a circle. *AC* intersects *BE* at *K*. $\angle ACD = 100^\circ$ and $\angle CDE = 130^\circ$. If $BE \mathbin{//} CD$, then $\angle ACB =$

A. 25° .
B. 30° .
C. 36° .
D. 40° .
E. 42° .

86 26. In a shooting game, the probabilities for John and Mary to hit a target are $\frac{4}{5}$ and $\frac{3}{5}$ respectively. When both shoot at the target, what is the probability that they both miss.

A. $\frac{2}{25}$
B. $\frac{3}{25}$
C. $\frac{8}{25}$
D. $\frac{12}{25}$
E. $\frac{13}{25}$

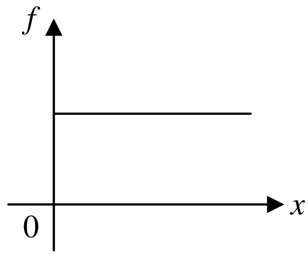
86 27. Given two groups of numbers

$a + 1, a + 2, a + 3$ and
$b + 1, b + 2, b + 3$,

where $a > b$. m_1 and m_2 are respectively the means of the two groups, and s_1 and s_2 are respectively their standard deviations. Which of the following is true?

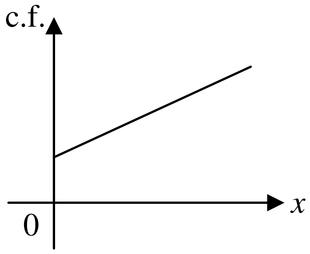
A. $m_1 > m_2$ and $s_1 > s_2$.
B. $m_1 > m_2$ and $s_1 = s_2$.
C. $m_1 = m_2$ and $s_1 > s_2$.
D. $m_1 = m_2$ and $s_1 = s_2$.
E. $m_1 > m_2$ and $s_1 < s_2$.

86 28.

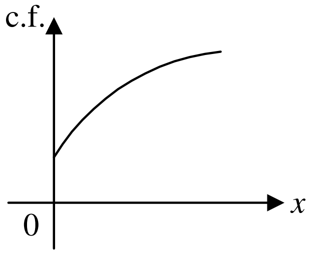


The figure shows the frequency curve of a certain distribution. Which of the following can be the distribution's cummulative frequency curve?

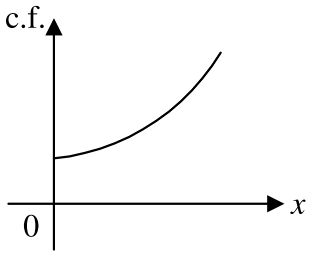
A.



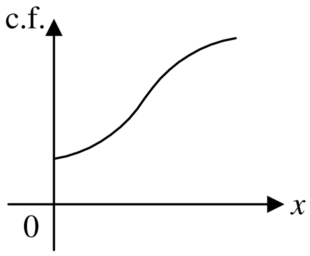
B.



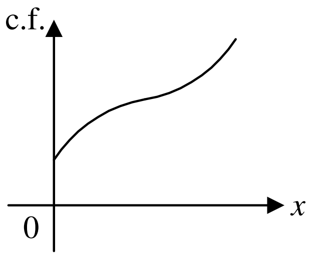
C.



D.



E.



86 29. If $(10^x)^y = (2^z)(5^z)$, then which of the following must be true

A. $xy = z$.
B. $xy = 2z$.
C. $xy = z^2$.
D. $x^y = z$.
E. $x^y = 2z$.

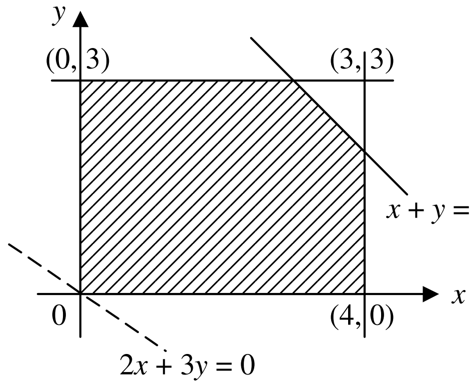
86 30. $\left(\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}}\right)^2 =$

A. $\frac{(x+y)^2}{xy}$.
B. $\frac{x^2+y^2}{xy}$.
C. $\frac{x+y+2}{xy}$.
D. $\frac{x+y}{xy}$.
E. 1 .

86 31. The L.C.M. of $12a^2b$ and $18ab^3c$ is

A. $6ab$.
B. $6a^2b^3c$.
C. $36ab$.
D. $36a^2b^3c$.
E. $216a^3b^4c$.

86 32.



Let $p = 2x + 3y$. Under the following constraints

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x \leq 4 \\ y \leq 3 \\ x + y \leq 6 \end{cases}$$

what is the greatest value of p?

A. 8 .
B. 14 .
C. 15 .
D. 16 .
E. 17 .

86 33. If $\log x^2 + \log y^2 = \log z^2$, where x, y and z are positive numbers, which of the following must be true?

I. $x^2 + y^2 = z^2$.
II. $\log x + \log y = \log z$
III. $x^2 y^2 = z^2$

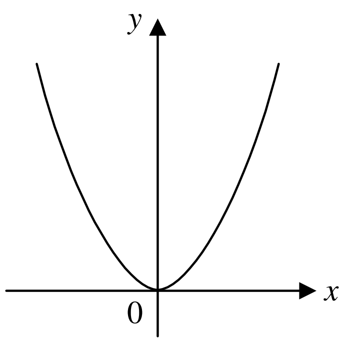
A. I only
B. II only
C. III only
D. I and II only
E. II and III only

86 34. Let $F(x) = 2x^3 + 3x^2 - 11x - 6$. Given that $F(2) = 0$ and $F(-3) = 0$, then $F(x)$ can be factorized as

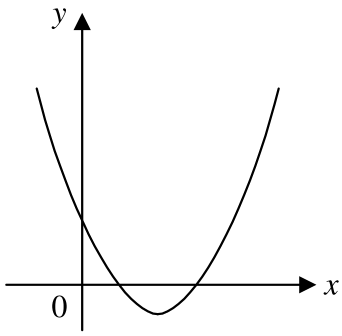
A. $(x + 2)(x - 3)(2x + 1)$.
B. $(x + 2)(x - 3)(2x - 1)$.
C. $(x - 2)(x + 3)(2x + 1)$.
D. $(x - 2)(x - 3)(2x + 1)$.
E. $(x - 2)(x + 3)(2x - 1)$.

86 35. If a, b and c are positive numbers, which of the following is a possible graphical representation of $y = ax^2 + bx + c$

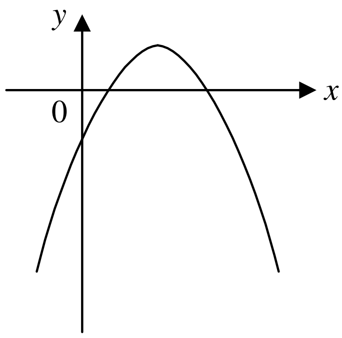
A.



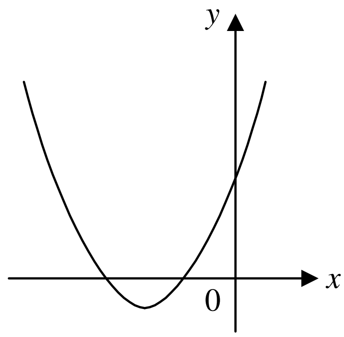
B.



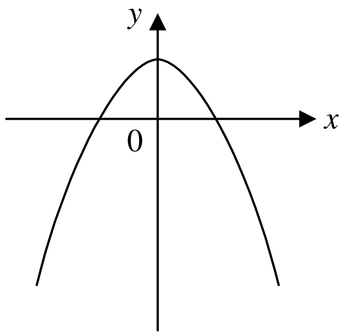
C.



D.



E.



86 36. If $a > 0$ and $b < 0$, which of the following is/are negative?

I. $\frac{1}{a} - \frac{1}{b}$

II. $\frac{a}{b} + \frac{b}{a}$

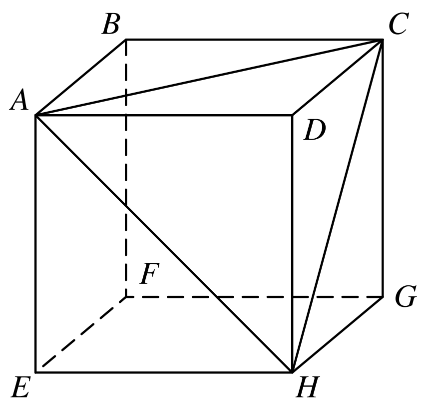
III. $\frac{a^2}{b} - \frac{b^2}{a}$

A. I only
B. III only
C. I and II only
D. I and III only
E. II and III only

86 37. If $2 < x < 3$ and $3 < y < 4$, then the range of values of $\frac{x}{y}$ is

A. $\frac{1}{2} < \frac{x}{y} < \frac{3}{4}$.
B. $\frac{1}{2} < \frac{x}{y} < 1$.
C. $\frac{2}{3} < \frac{x}{y} < \frac{3}{4}$.
D. $\frac{2}{3} < \frac{x}{y} < 1$.
E. $\frac{4}{3} < \frac{x}{y} < \frac{3}{2}$.

86 38.



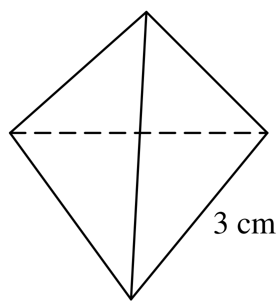
ABCDEFGH is a cube of side 3 cm. A tetrahedron *DACH* is cut away along the plane *ACH*. The volume of the remaining solid is

A. 6 cm^3 .
B. 9 cm^3 .
C. 13.5 cm^3 .
D. 18 cm^3 .
E. 22.5 cm^3 .

86 39. The marked price of an article is originally *P*. The marked price is then increased so that when a discount of 10% is made on the new marked price, the selling price is still *P*. What is the new marked price?

A. $\frac{9}{10}P$.
B. $\frac{109}{100}P$.
C. $\frac{11}{10}P$.
D. $\frac{111}{100}P$.
E. $\frac{10}{9}P$.

86 40.



The total surface area of a regular tetrahedron of side 3 cm is

A. $\frac{9\sqrt{3}}{4}$ cm^2 .
B. 9 cm^2 .
C. $\frac{27\sqrt{3}}{4}$ cm^2 .
D. $9\sqrt{3}$ cm^2 .

E. $12\sqrt{3}\ \text{cm}^2$.

86 41. Ten litres of a mixture contain 60% of alcohol and 40% of water by volume. How many litres of water should be added so that it contains 30% of alcohol by volume?

A. 5 .
B. 10 .
C. 15 .
D. 20 .
E. 30 .

86 42. If A, B, and C can finish running the same distance in 3, 4 and 5 minutes respectively, then A's speed : B's peed : C's speed =

A. 3 : 4 : 5
B. 5 : 4 : 3
C. 9 : 8 : 7
D. 20 : 15 : 12
E. 25 : 16 : 9

86 43. If the five interior angles of a convex pentagon from an A.P. with a common difference of 10°, then the smallest interior angle of the pentagon is

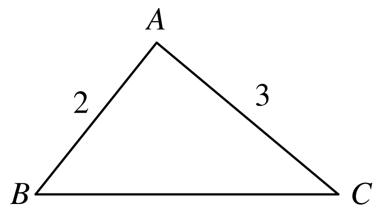
A. 52°
B. 72°
C. 88°
D. 98°
E. 108°

86 44. Let p be a positive constant such that $p\ \sin\ \theta = \sqrt{3}$ and $p\ \sin\ \theta = 1$. Find all the values of θ in the interval 0 to 2π.

A. $\dfrac{\pi}{3}$.
B. $\dfrac{\pi}{6}$.
C. $\dfrac{\pi}{3}, \dfrac{4\pi}{3}$
D. $\dfrac{\pi}{6}, \dfrac{7\pi}{3}$

E. Cannot be determined

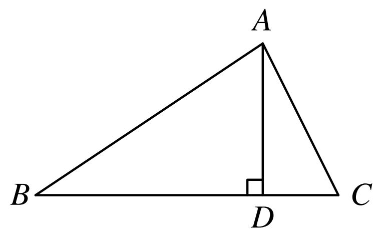
86 45.



In ΔABC, $AB = 2$, $AC = 3$ and $\sin B = \dfrac{3}{4}$, then $\cos^2 C =$

A. $\dfrac{9}{16}$.
B. $\dfrac{9}{13}$.
C. $\dfrac{1}{4}$.
D. $\dfrac{1}{2}$.
E. $\dfrac{3}{4}$.

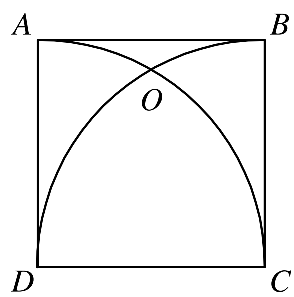
86 46.



In the figure, $BD : DC =$

A. $\sin C : \sin B$.
B. $\cos C : \cos B$.
C. $\tan C : \tan B$.
D. $\sin B : \sin C$.
E. $\cos B : \cos C$.

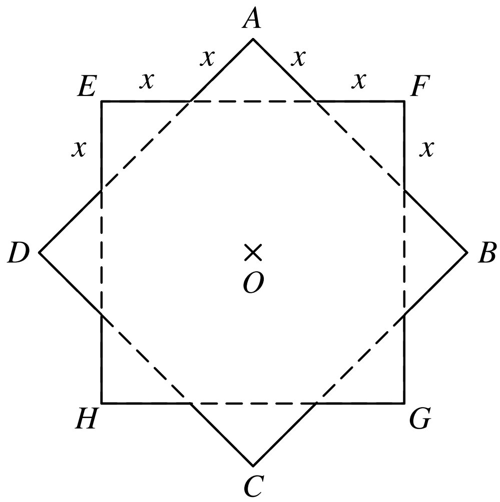
86 47.



In the figure, *ABCD* is a square. Arcs *AC* and *BD* are drawn with centres *D* and *C* respectively, intersecting at *O*. Arc *AO* : Arc *OC* =

A. $1:\sqrt{2}$.
B. $1:\sqrt{3}$.
C. 1 : 2 .
D. 1 : 3 .
E. 2 : 3 .

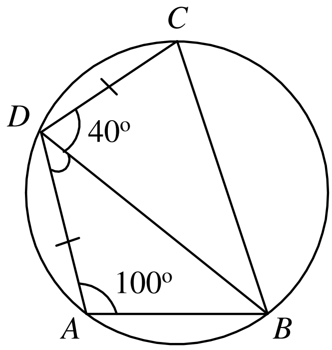
86
48.



In the figure, *ABCD* and *EFGH* are two squares of side 1. They are placed one upon the other with their centres both at *O* to form a star with 16 sides, each of length *x*. Find *x*.

A. $\frac{2}{7}$.
B. $\frac{1}{3}$.
C. $\frac{2}{5}$.
D. $\frac{1}{2+\sqrt{2}}$.
E. $\frac{1}{1+\sqrt{2}}$.

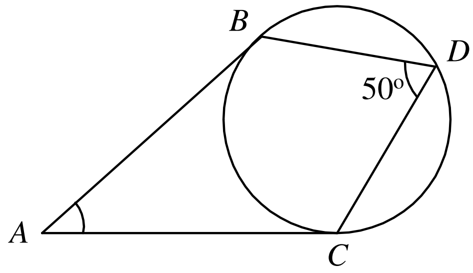
86
49.



DA and *DC* are equal chords of the circle *ABCD*. $\angle CDB = 40^\circ$ and $\angle DAB = 100^\circ$. $\angle ADB =$

A. 20° .
B. 25° .
C. 30° .
D. 35° .
E. 40° .

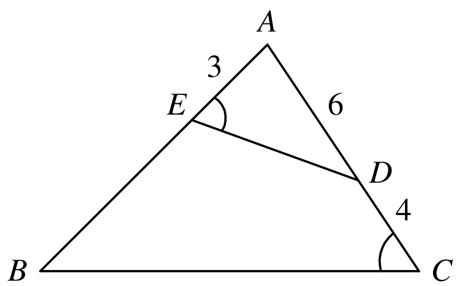
86
50.



In the figure, *AB* and *AC* are tangents to the circle *BCD*. If $\angle BDC = 50^\circ$, then $\angle A =$

A. 130° .
B. 100° .
C. 85° .
D. 80° .
E. 50° .

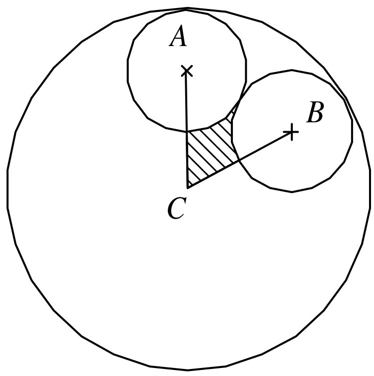
86
51.



In ΔABC, $AP = 3$, $AQ = 6$ and $QC = 4$. If $\angle APQ = \angle ACB$, then $PB =$

A. 7 .
B. 8 .
C. 10 .
D. 17 .
E. 20 .

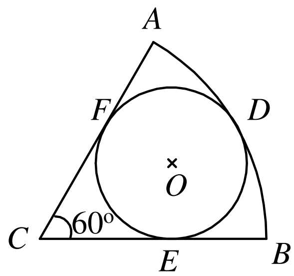
86
52.



Three circles, centres A, B and C touch each other as shown in the figure. The radii of the two circles with centre A and B are both 1 cm and radius of the circle with centre C is 3 cm. Find the area of the shaded part in cm^2.

A. $\sqrt{3} - \dfrac{\pi}{3}$.
B. $\sqrt{3} - \dfrac{\pi}{6}$.
C. $2\sqrt{3} - \dfrac{\pi}{3}$.
D. $2\sqrt{3} - \dfrac{\pi}{6}$.
E. It cannot be determined

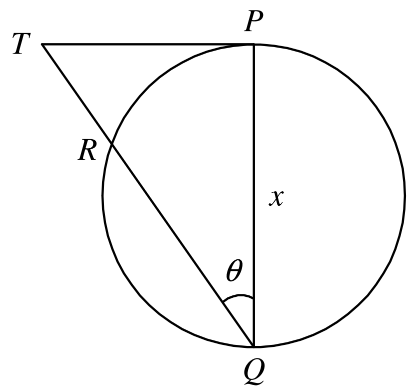
86
53.



A circle, centre O, touches the sector ABC internally at D, E and F. $\angle C = 60^\circ$ and $AC = 18$. Find the radius of the circle.

A. 9 .
B. 6 .
C. 5 .
D. 4 .
E. 3 .

86
54.



In the figure, PQ is a diameter and PT is a tangent of the circle. QT cuts the circle at R. Let $\angle Q = \theta$ and $PQ = x$, then $TR =$

A. $\dfrac{x}{\cos\theta}$.
B. $\dfrac{x}{\sin\theta}$.
C. $\dfrac{x}{\sin\theta\tan\theta}$.
D. $x \sin\theta \tan\theta$.
E. $x \cos\theta \tan\theta$.