香 港 考 試 局

HONG KONG EXAMINATIONS AUTHORITY

一 九 八 三 年 香 港 中 學 會 考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1983

數學(課程甲) 試卷一

二小時完卷

上午八時三十分至上午十時三十分

本試卷必須用英文作答

MATHEMATICS (SYLLABUS A) PAPER I

Two hours

8.30 a.m. – 10.30 a.m.

This paper must be answered in English

Attempt ALL questions in Section A and any FIVE questions in Section B.
Full marks will not be given unless the method of solution is shown.

FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi r h$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	$\pi r l$
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area × height
PYRAMID	Volume	=	$\frac{1}{3}$ × base area × height

SECTION A **Answer ALL questions in this section.**
There is no need to start each question in this section on a fresh page.
Geometry theorems need not be referred to when used.

1. Factorise $(x^2 + 4x + 4) - (y - 1)^2$. (5 marks)

2. In Figure 1, O is the centre of the circle. A and B are two points on the circle such that OAB is an equilateral triangle. OA is produced to C such that $OA = AC$.

   (a) Find $\angle ABC$.

   (b) Is CB a tangent to the circle at B? Give a reason for your answer.

   (5 marks)

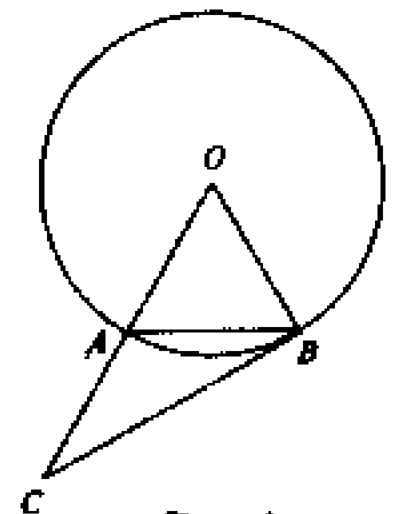

Figure 1

3. Given five real numbers $a - 6$, a, $a + 2$, $a + 3$, $a + 6$,
   find (a) the mean,
   (b) the standard deviation.
   (5 marks)

4. If $a : b = 3 : 4$ and $a : c = 2 : 5$,
   find (a) $a : b : c$,
   (b) the value of $\frac{ac}{a^2 + b^2}$.
   (6 marks)

5. In Figure 2, O is the centre of the sector OAB. $OA = 30$, $CB = 15$ and $AC \perp OB$.

   Find (a) $\angle AOC$,

   (b) the length of the arc AB in terms of π.

   (6 marks)

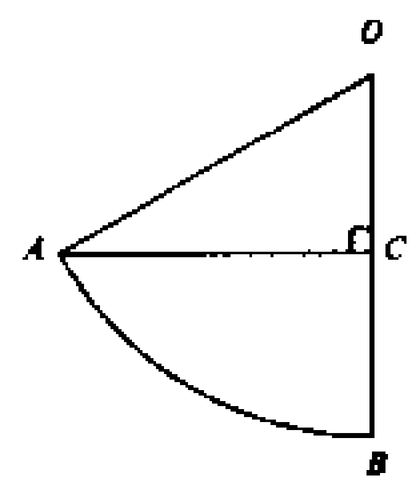

Figure 2

6. The compound interest on \$1000 at 10% per annum for 3 years, compounded yearly, equals the simple interest on another \$1000 at r% per annum for the same period of time. Calculate r to 2 decimal places.
   (6 marks)

7. Find all the values of θ, where $0° \leqslant \theta \leqslant 360°$, such that

$$2\cos^2\theta + 5\sin\theta + 1 = 0$$

   (6 marks)

SECTION A **Answer ALL questions in this section.**
There is no need to start each question in this section on a fresh page.
Geometry theorems need not be referred to when used.

1. Factorise $(x^2 + 4x + 4) - (y - 1)^2$. (5 marks)

2. In Figure 1, O is the centre of the circle. A and B are two points on the circle such that OAB is an equilateral triangle. OA is produced to C such that $OA = AC$.
   (a) Find $\angle ABC$.
   (b) Is CB a tangent to the circle at B? Give a reason for your answer.
   (5 marks)

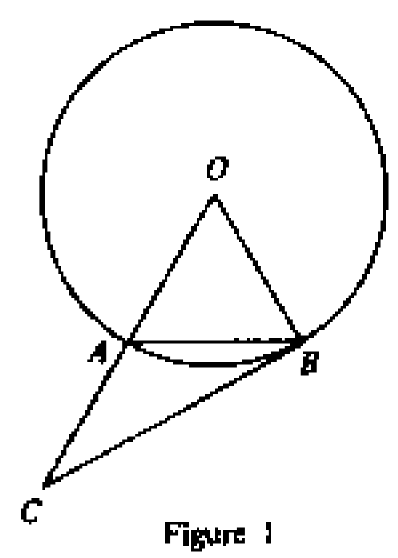

Figure 1

3. The following table shows the distribution of the marks of 1000 students in a mathematics test:

Class of Marks	Number of Students
40 – 49	100
50 – 59	300
60 – 69	400
70 – 79	200

   (a) Find the mid-value of the class 50 – 59.
   (b) Estimate the mean of the above distribution of marks.
   (5 marks)

4. If $a : b = 3 : 4$ and $a : c = 2 : 5$,
   find (a) $a : b : c$,
   (b) the value of $\dfrac{ac}{a^2 + b^2}$.
   (6 marks)

5. In Figure 2, O is the centre of the sector OAB. $OA = 30$, $CB = 15$ and $AC \perp OB$.
   Find (a) $\angle AOC$,
   (b) the length of the arc AB in terms of π.
   (6 marks)

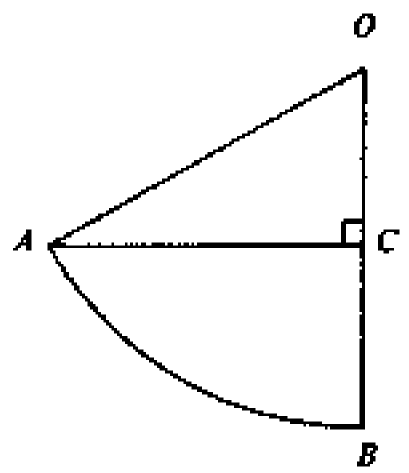

Figure 2

6. The compound interest on \$1000 at 10% per annum for 3 years, compounded yearly, equals the simple interest on another \$1000 at r% per annum for the same period of time. Calculate r to 2 decimal places.
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7. Find all the values of θ, where $0° \leqslant \theta \leqslant 360°$, such that
   $$2\cos^2\theta + 5\sin\theta + 1 = 0.$$
   (6 marks)

SECTION B **Answer FIVE questions in this section.**
Each question carries 12 marks.

8.

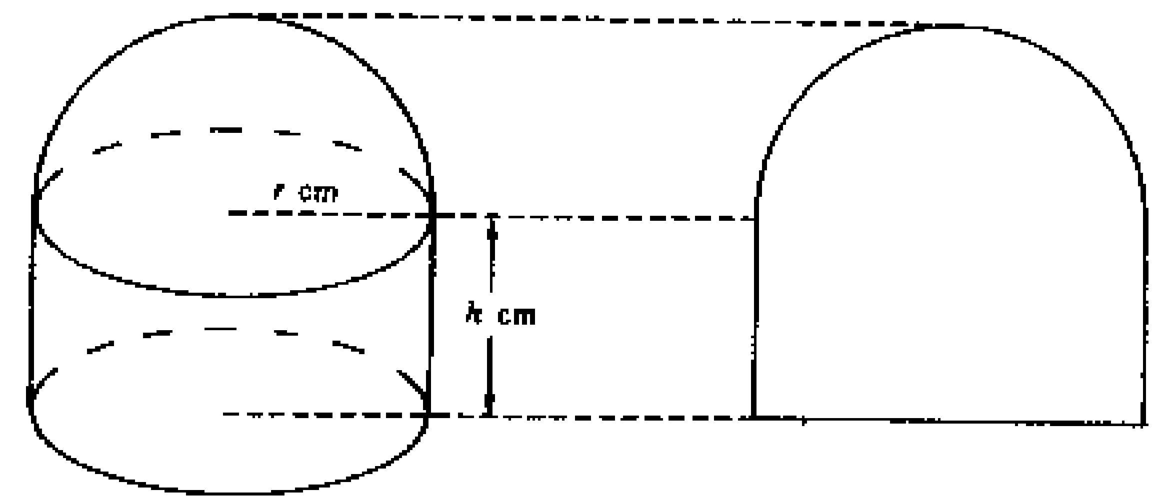

Figure 3(a) Figure 3(b)

The solid in Figure 3(a) is made up of two parts. The lower part is a right circular cylinder of height h cm and radius r cm; the upper part is a hemisphere of the same radius r cm. The two parts are of the same volume.

(a) Find the ratio $r : h$. (3 marks)

(b) Figure 3(b) shows a section of the solid through the axis of the cylinder. The perimeter of this section is 136 cm.
   (i) Calculate r to 2 significant figures.
   (ii) Calculate the total external surface area (including the base) of the solid in cm^2 to 1 significant figure.
   (9 marks)

9. In Figure 4, O is the origin and A is the point (8, 2).
   (a) B is a point on the x-axis such that the slope of AB is 1. Find the coordinates of B. (2 marks)
   (b) C is another point on the x-axis such that $AB = AC$. Find the coordinates of C. (2 marks)
   (c) Find the equation of the straight line AC. If the line AC cuts the y-axis at D, find the coordinates of D. (3 marks)

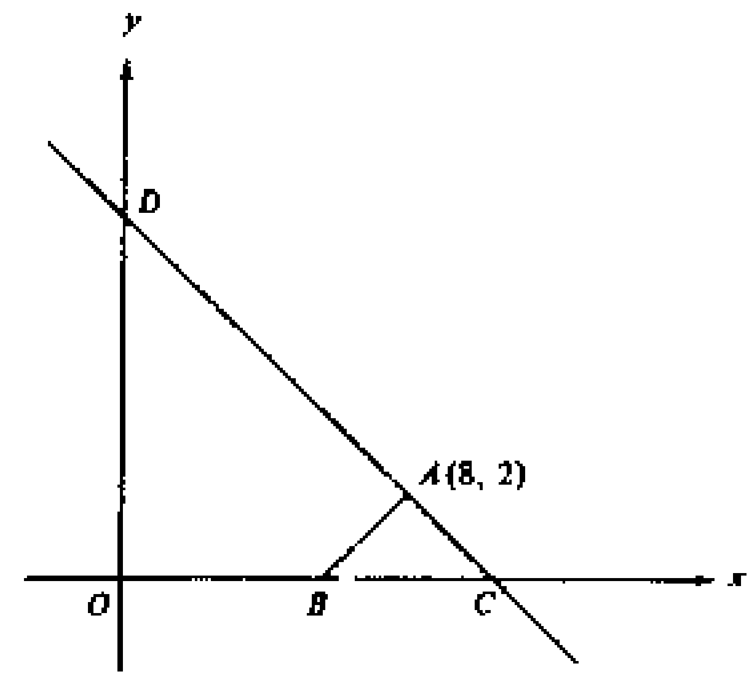

Figure 4

   (d) Find the equation of the circle passing through the points O, B and D. Show that this circle passes through A.
   (5 marks)

SECTION B Answer FIVE questions in this section.
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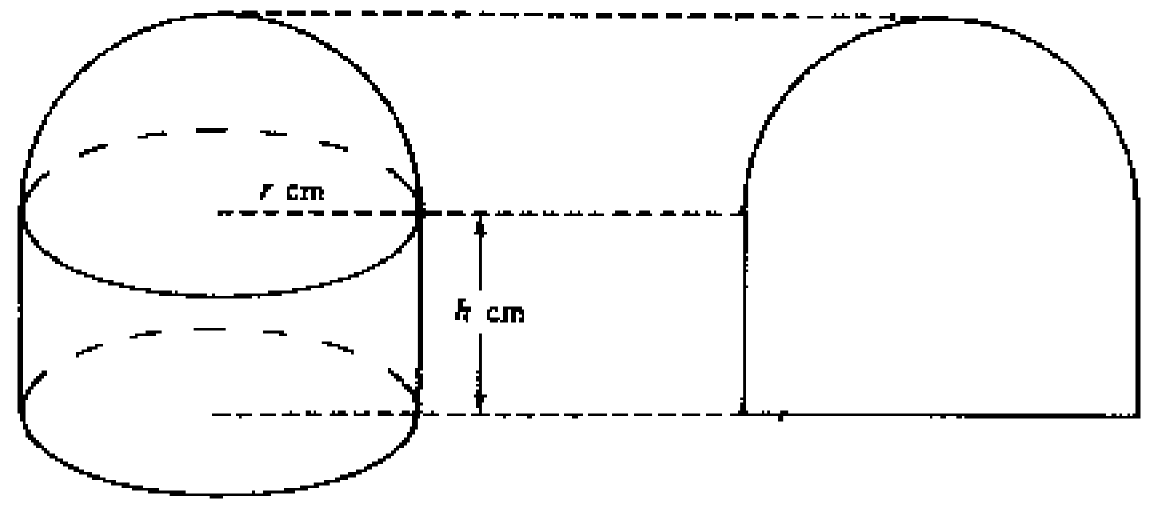

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(a) B is a point on the x-axis such that the slope of AB is 1. Find the coordinates of B.
(2 marks)

(b) C is another point on the x-axis such that $AB = AC$. Find the coordinates of C.
(2 marks)

(c) Find the equation of the straight line AC. If the line AC cuts the y-axis at D, find the coordinates of D.
(3 marks)

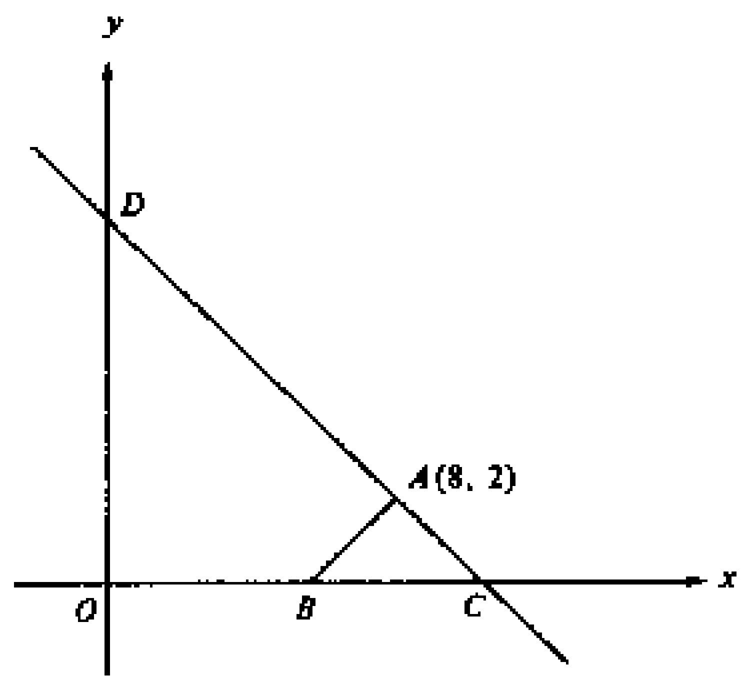


Figure 4

(d) Find the equation of the circle passing through the points O, B and D. Show that this circle passes through A.
(5 marks)

10. A ball is dropped vertically from a height of 10 m, and when it reaches the ground, it rebounds to a height of $10 \times \frac{3}{4}$ m. The ball continues to fall and rebound again and again, each time rebounding to $\frac{3}{4}$ of the height from which it previously fell (see Figure 5).

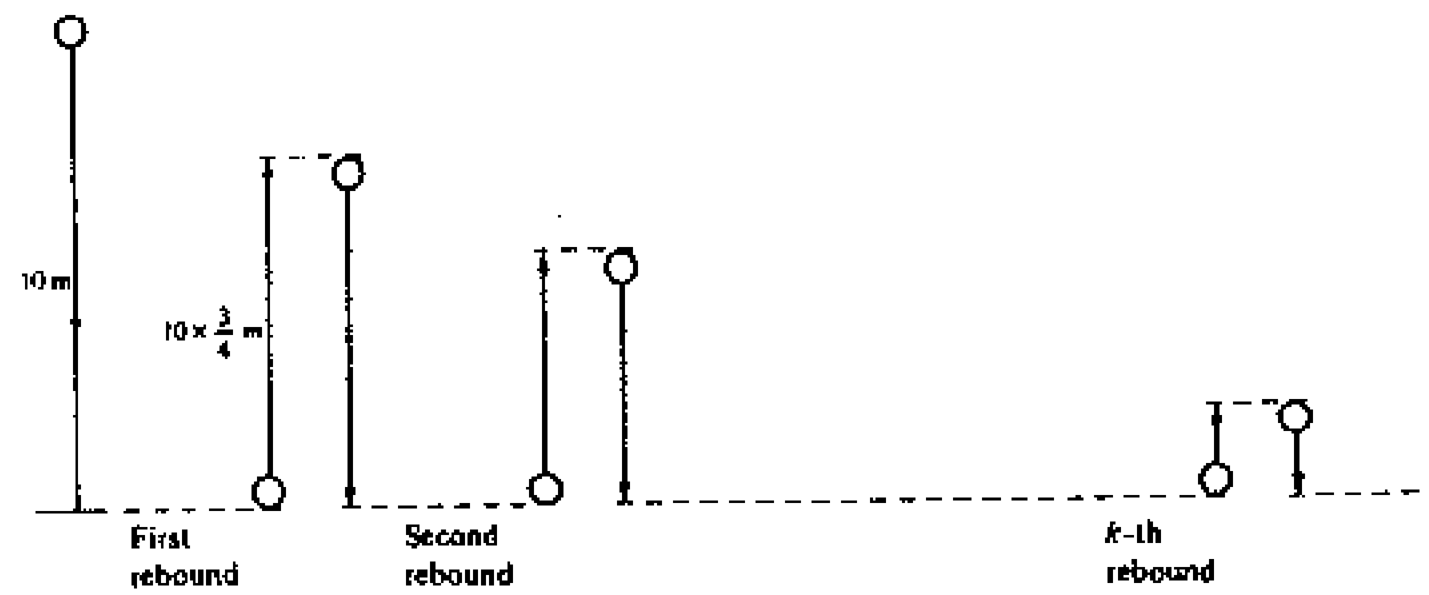


Figure 5

(a) Find the total distance travelled by the ball just before it makes its second rebound. (3 marks)

(b) Find, in terms of k, the total distance travelled by the ball just before it makes its $(k + 1)$th rebound. (6 marks)

(c) Find the total distance travelled by the ball before it comes to rest. (3 marks)

11. In a short test, there are 3 questions. For each question, 1 mark will be awarded for a correct answer and no marks for a wrong answer. The probability that John correctly answers a question in the test is 0.6. Find the probability that

(a) John gets 3 marks in the test. (3 marks)

(b) John gets no marks in the test, (3 marks)

(c) John gets 1 mark in the test, (4 marks)

(d) John gets 2 marks in the test. (2 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

12. **If you attempt this question, fill in the details in the first three boxes above and tie this sheet into your answer book.**

(a) On the graph paper provided below, draw the following straight lines:

$$y = 2x,$$
$$x + y = 30,$$
$$2x + 3y = 120.$$

(3 marks)

(b) On the same graph paper, shade the region that satisfies all the following inequalities:

$$\begin{cases} y \geq 0, \\ y \leq 2x, \\ x + y \geq 30, \\ 2x + 3y \leq 120. \end{cases}$$

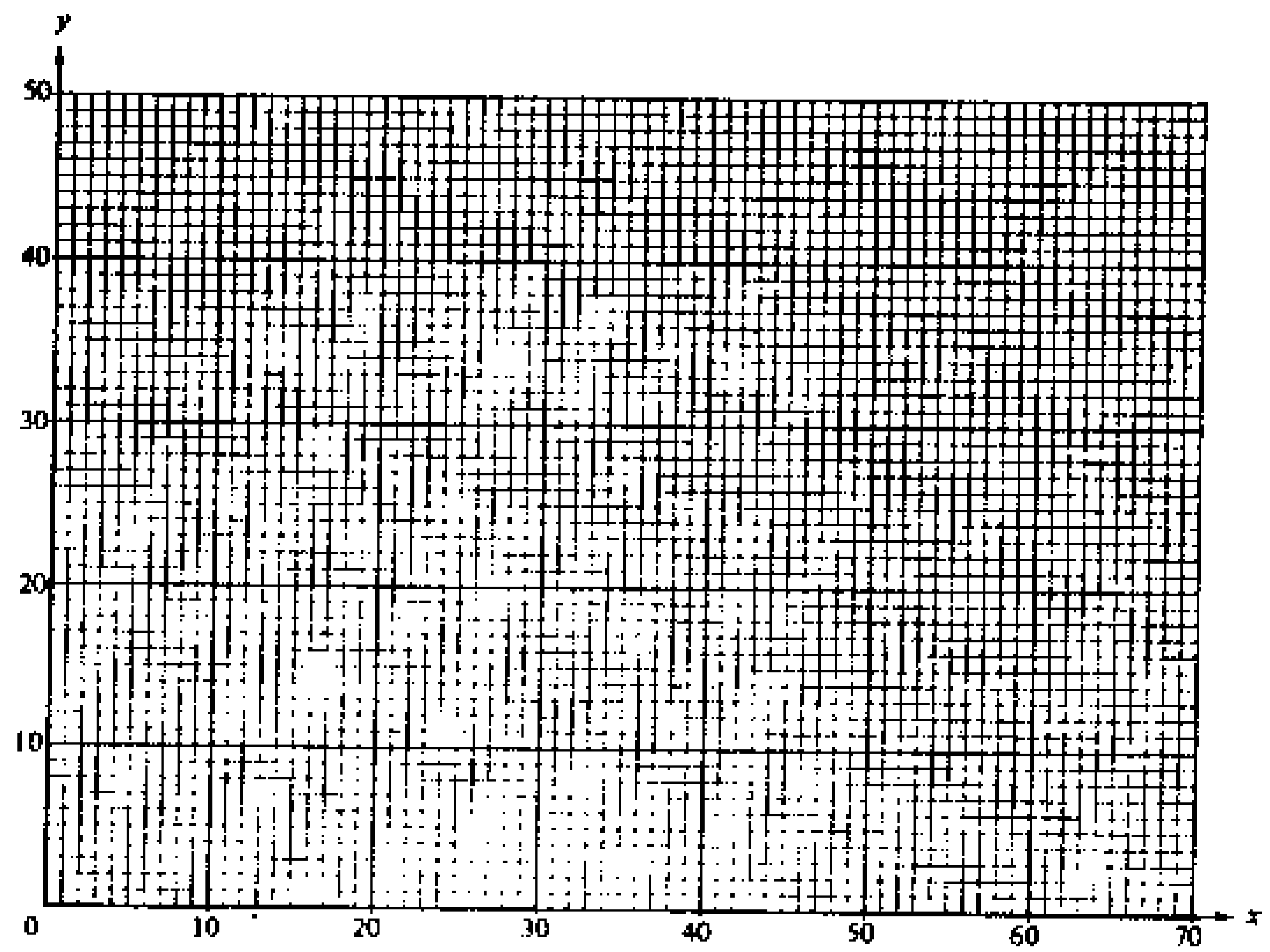
(3 marks)

(c) It is given that $P = 3x + 2y$.
Under the constraints given by the inequalities in (b),

(i) find the maximum and minimum values of P, and

(ii) find the maximum and minimum values of P if there is the additional constraint $x \leq 45$.

(6 marks)



13.

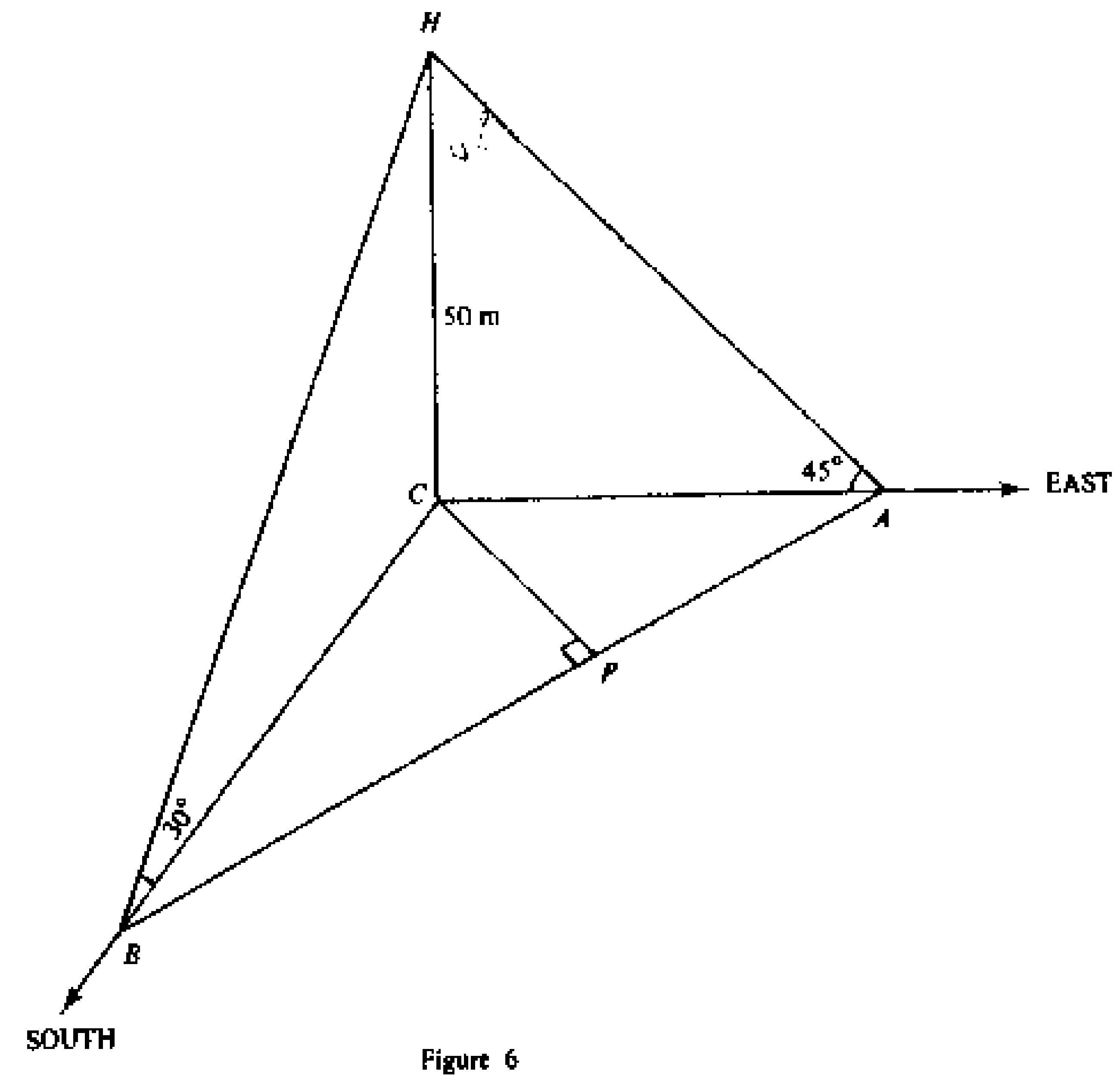


Figure 6

In Figure 6, A, B and C are three points on the same horizontal ground. HC is a vertical tower 50 m high. A and B are respectively due east and due south of the tower. The angles of elevation of H observed from A and B are respectively $45°$ and $30°$.

(a) Find the distance between A and B. (6 marks)

(b) P is a point on AB such that $CP \perp AB$.

(i) Find the distance between C and P to the nearest metre.

(ii) Find the angle of elevation of H observed from P to the nearest degree.

(6 marks)

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14. If you attempt this question, fill in the details in the first three boxes above and tie this sheet into your answer book.

Equal squares each of side k cm are cut from the four corners of a square sheet of paper of side 7 cm (see Figure 7(a)). The remaining part is folded along the dotted lines to form a rectangular box as shown in Figure 7(b).

(a) Show that the volume V of the rectangular box, in cm^3, is $V = 4k^3 - 28k^2 + 49k$.

(3 marks)

(b) Figure 7(c) shows the graph of $y = 4x^3 - 28x^2 + 49x$ for $0 \leq x \leq 5$. Draw a suitable straight line in Figure 7(c) and use it to find all the possible values of x such that $4x^3 - 28x^2 + 49x - 20 = 0$. (Give the answers to 1 decimal place.)

(4 marks)

(c) Using the results of (a) and (b), deduce the values of k such that the volume of the box is 20 cm^3. (Give the answers to 1 decimal place.)

(2 marks)

(d) By the method of magnification, find the smaller value of k in (c) to two decimal places.

(3 marks)

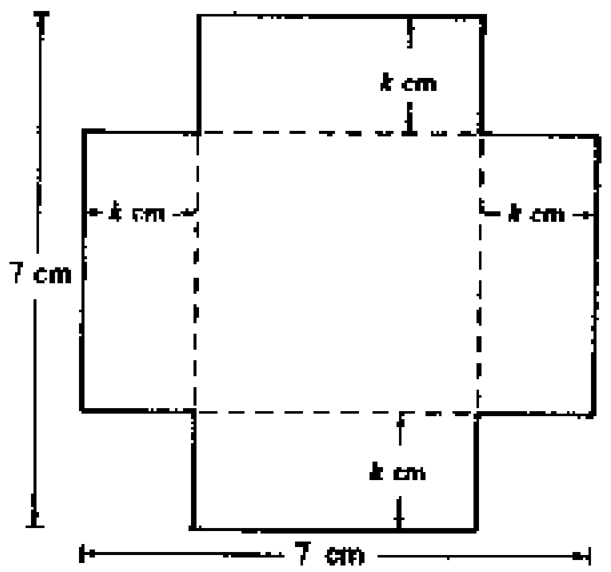

Figure 7(a)

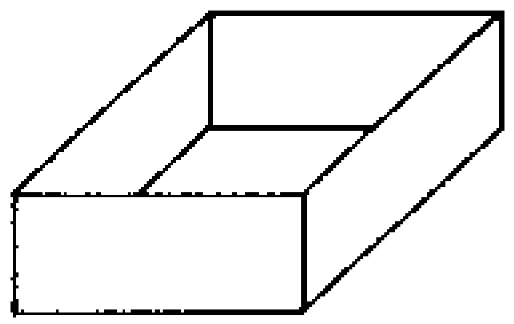

Figure 7(b)

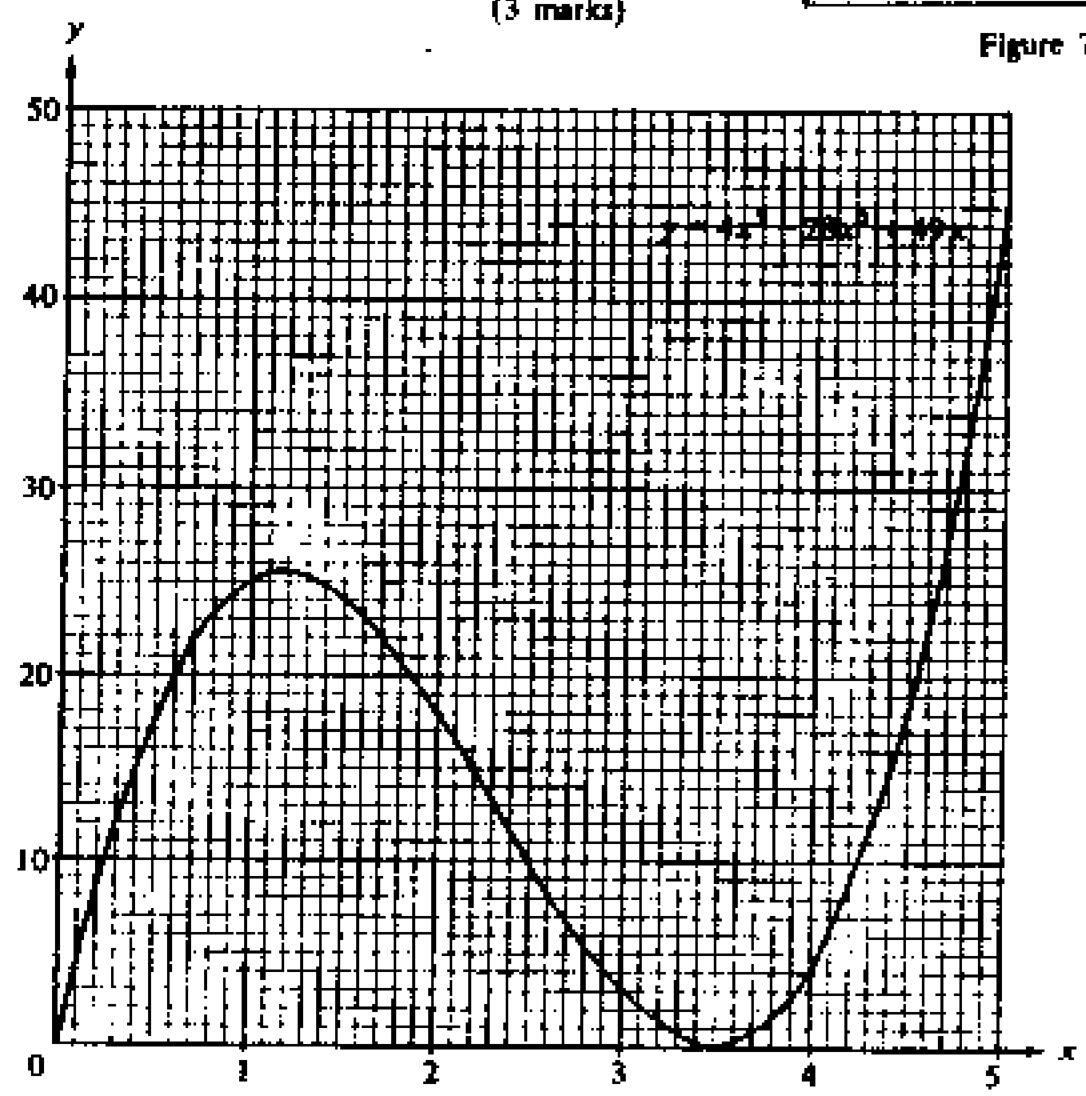

Figure 7(c)

END OF PAPER

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HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1983

數學(課程乙) 試卷一

MATHEMATICS (SYLLABUS B) PAPER I

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	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
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PYRAMID	Volume	=	$\frac{1}{3}$ × base area × height

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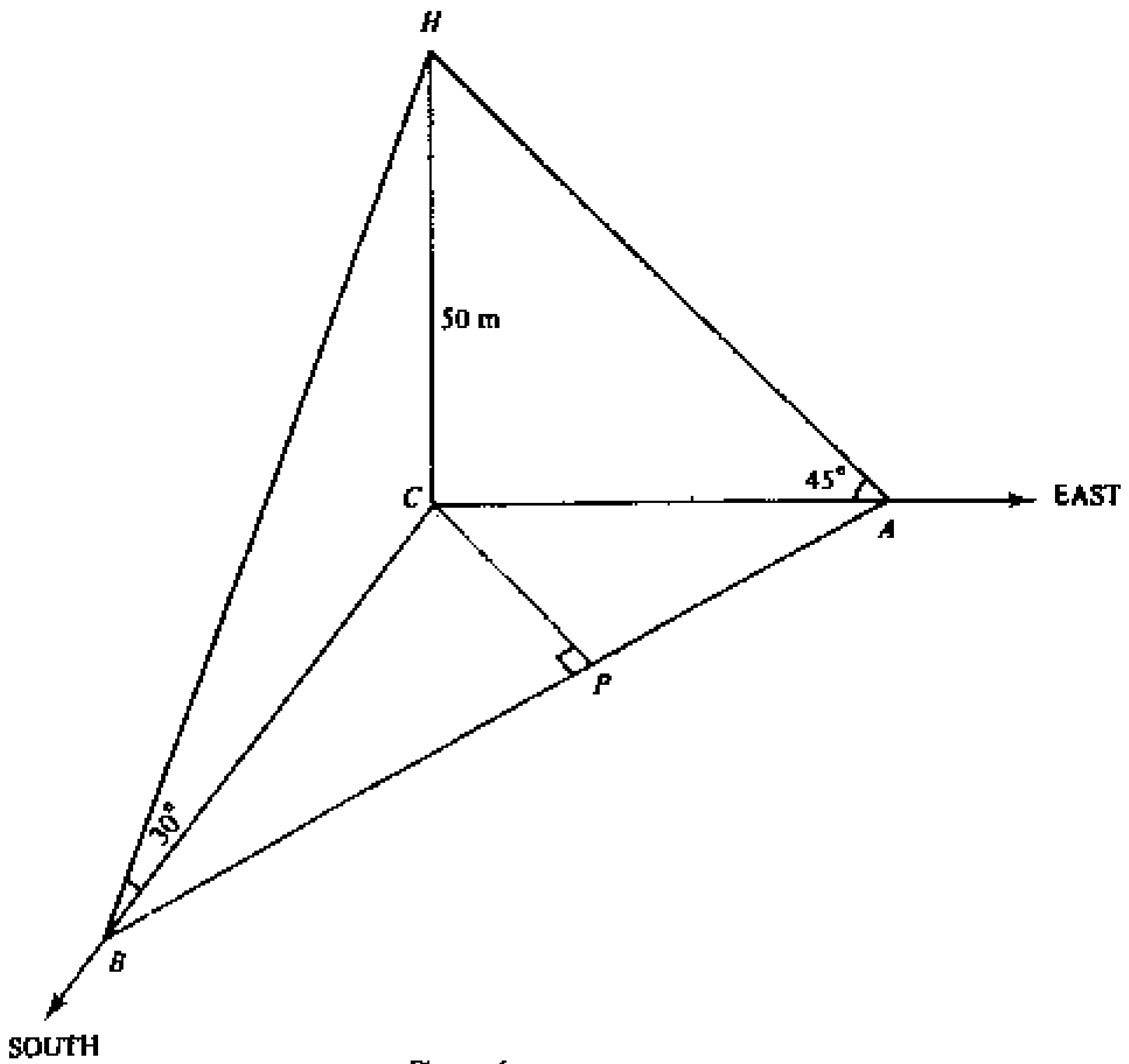

Figure 6

In Figure 6, A, B and C are three points on the same horizontal ground. HC is a vertical tower 50 m high. A and B are respectively due east and due south of the tower. The angles of elevation of H observed from A and B are respectively 45° and 30°.

(a) Find the distance between A and B. (6 marks)

(b) P is a point on AB such that $CP \perp AB$.

(i) Find the distance between C and P to the nearest metre.

(ii) Find the angle of elevation of H observed from P to the nearest degree.

(6 marks)

14. α and β are the roots of the quadratic equation $x^2 - 2mx + n = 0$ where m and n are real numbers.

(a) Find, in terms of m and n,

(i) $(m - \alpha) + (m - \beta)$,

(ii) $(m - \alpha)(m - \beta)$.

(5 marks)

(b) Find, in terms of m and n, the quadratic equation having roots $m - \alpha$ and $m - \beta$.

(3 marks)

(c) If $n = 4$, find the range of values of m such that the equation $x^2 - 2mx + n = 0$ has real roots.

(4 marks)

END OF PAPER