SECTION A Answer ALL questions in this section.
There is no need to start each question in this section on a fresh page.
Geometry theorems need not be quoted when used.

1. Simplify $\frac{2+i}{1-3i}$.

Express your answer in the form $a + bi$ where a and b are real numbers. (5 marks)

2. If $\begin{cases} 4^{x-y} = 4 \\ 4^{x+y} = 16 \end{cases}$,

solve for x and y. (5 marks)

3. Solve $2x^2 - x < 36$. (5 marks)

4. In Figure 1, the circle, centre O and radius 6, touches the straight line BC at C. $BC = 2\sqrt{3}$. OAB is a straight line. Find the area of the shaded sector in terms of π.

(6 marks)

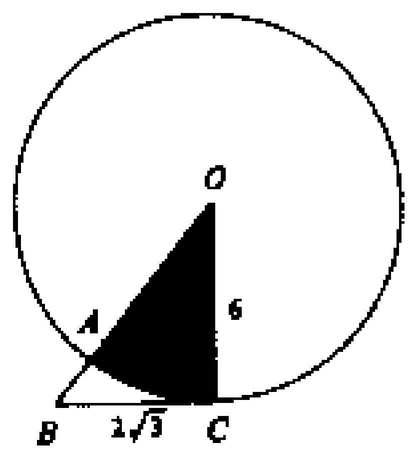

Figure 1

5. Solve $2\sin^2\theta + 5\sin\theta - 3 = 0$ for θ, where $0° \leq \theta < 360°$. (6 marks)

6. If two dice are thrown once, find the probability that the sum of the numbers on the dice is

(a) equal to 4,

(b) less than 4,

(c) greater than 4. (6 marks)

7. In a certain school, the numbers of students living on Hong Kong Island, in Kowloon and the New Territories are in the ratios $2 : 7 : 3$. The pie-chart in Figure 2 shows the distribution.

(a) Find x, y and z.

(b) If the number of students living on Hong Kong Island is 240, find the total number of students in the school.

(6 marks)

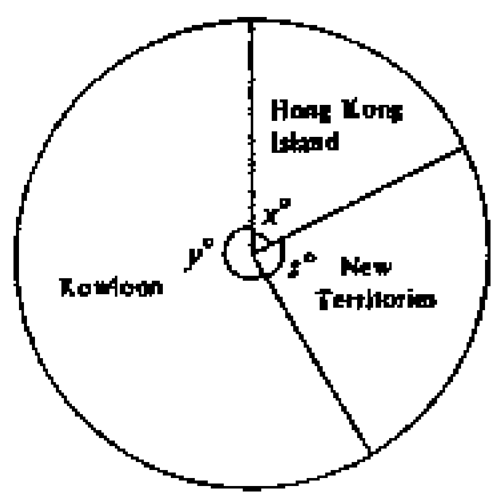

Figure 2

SECTION B Answer FIVE questions in this section.
Each question carries 12 marks.

8. Figure 3 represents the framework of a cuboid made of iron wire. It has a square base of side x cm and a height of y cm. The length of the diagonal AB is 9 cm. The total length of wire used for the framework (including the diagonal AB) is 69 cm.

(a) Find all the values of x and y. (10 marks)

(b) Hence calculate $\angle ABC$ to the nearest degree for the case in which $y > x$. (2 marks)

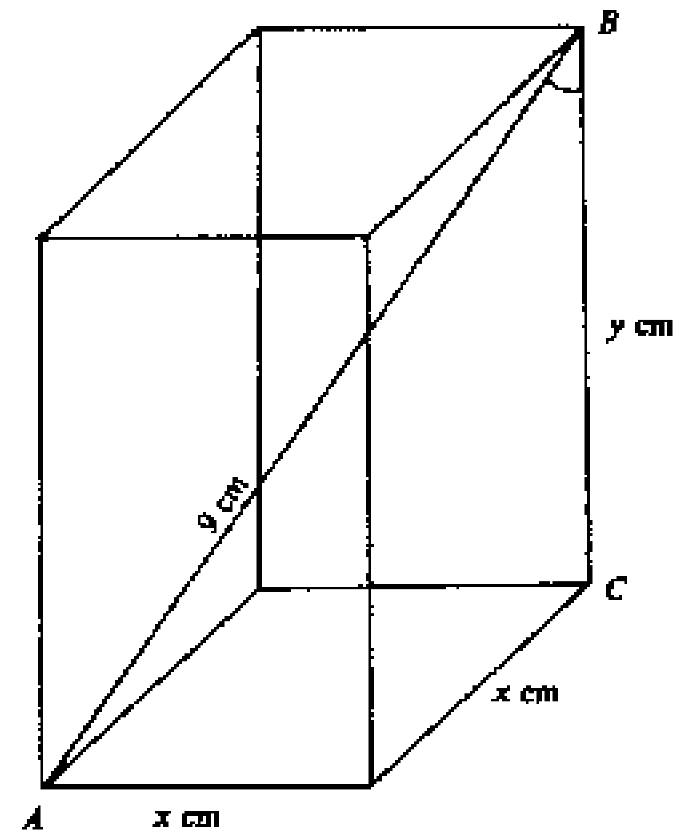

Figure 3
(Figure not drawn to scale.)

9.

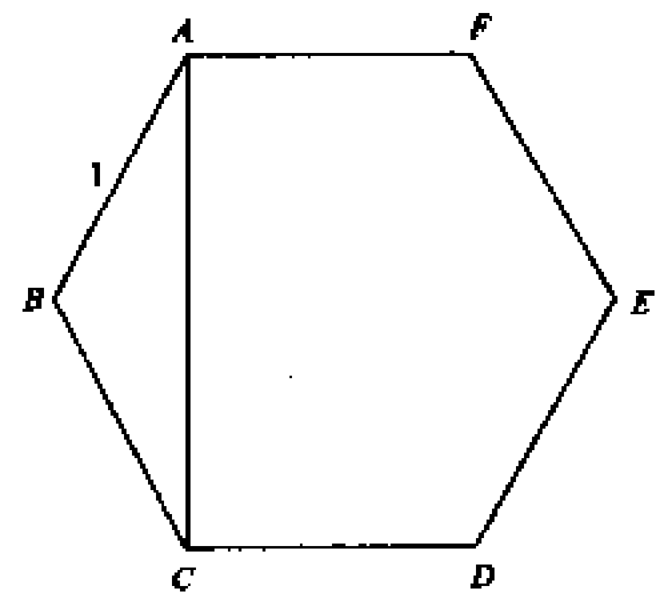

Figure 4(a)

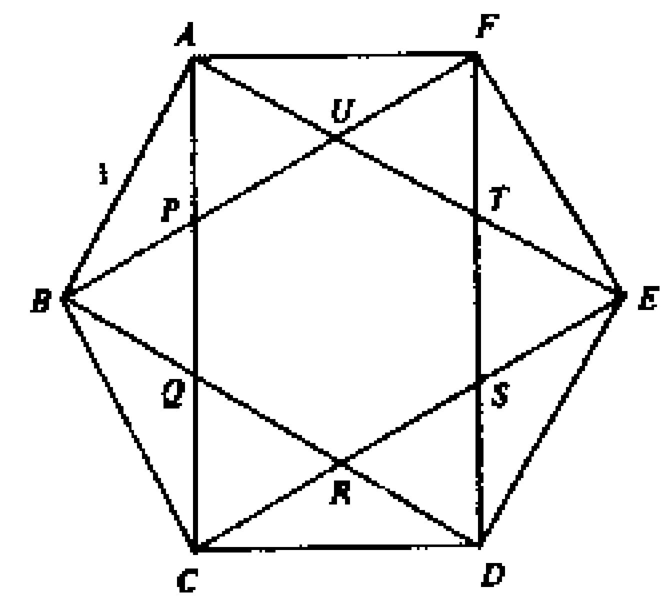

Figure 4(b)

(In this question, answers should be given in surd form.)

In Figures 4(a) and 4(b), $ABCDEF$ is a regular hexagon with $AB = 1$.

(a) Calculate the area of the hexagon in Figure 4(a) and the length of its diagonal AC. (6 marks)

(b) In Figure 4(b), $PQRSTU$ is another regular hexagon formed by the diagonals of $ABCDEF$.

(i) Calculate the length of PQ.

(ii) Calculate the area of the hexagon $PQRSTU$.

(6 marks)

10. (a) (i) Find the sum of all the multiples of 3 from 1 to 1000 .

(ii) Find the sum of all the multiples of 4 from 1 to 1000 (including 1000) .

(6 marks)

(b) Hence, or otherwise, find the sum of all the integers from 1 to 1000 (including 1 and 1000) which are neither multiples of 3 nor multiples of 4 .

(6 marks)

11.

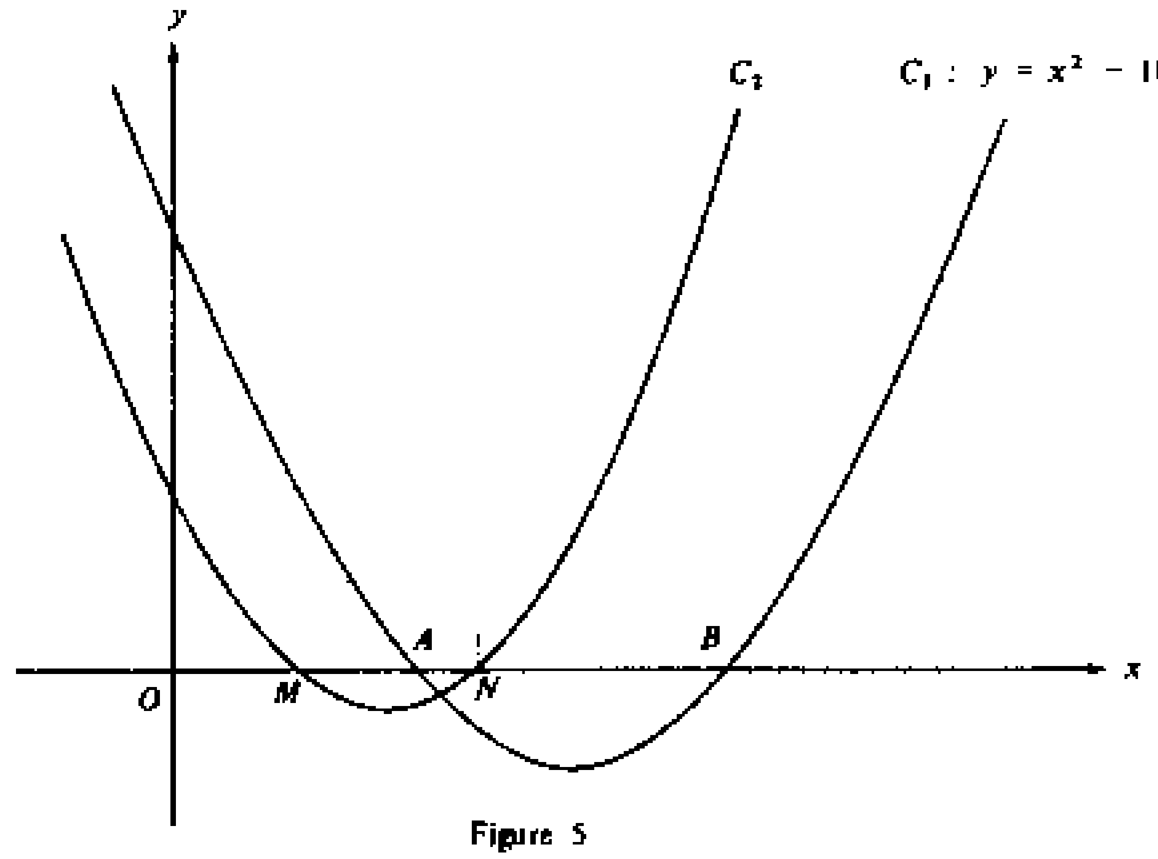

Figure 5

In Figure 5, O is the origin. The curve

$C_1 : \quad y = x^2 - 10x + k$ (where k is a fixed constant)

intersects the x-axis at the points A and B .

(a) By considering the sum and the product of the roots of $x^2 - 10x + k = 0$, or otherwise,

(i) find $OA + OB$,

(ii) find $OA \times OB$ in terms of k .

(4 marks)

(b) M and N are the mid-points of OA and OB respectively (see Figure 5).

(i) Find $OM + ON$.

(ii) Find $OM \times ON$ in terms of k .

(4 marks)

(c) Another curve

$C_2 : \quad y = x^2 + px + r$ (where p and r are fixed constants)

passes through the points M and N .

(i) Using the results in (b) or otherwise, find the value of p and express r in terms of k .

(ii) If $OM = 2$, find k .

(4 marks)

12. The price of a certain monthly magazine is x dollars per copy. The total profit on the sale of the magazine is P dollars. It is given that $P = Y + Z$, where Y varies directly as x and Z varies directly as the square of x . When x is 20 , P is 80 000 ; when x is 35 , P is 87 500 .

(a) Find P when $x = 15$. (7 marks)

(b) Using the method of completing the square, express P in the form $P = a - b(x - c)^2$ where a , b and c are constants. Find the values of a , b and c . (3 marks)

(c) Hence, or otherwise, find the value of x when P is a maximum. (2 marks)

13.

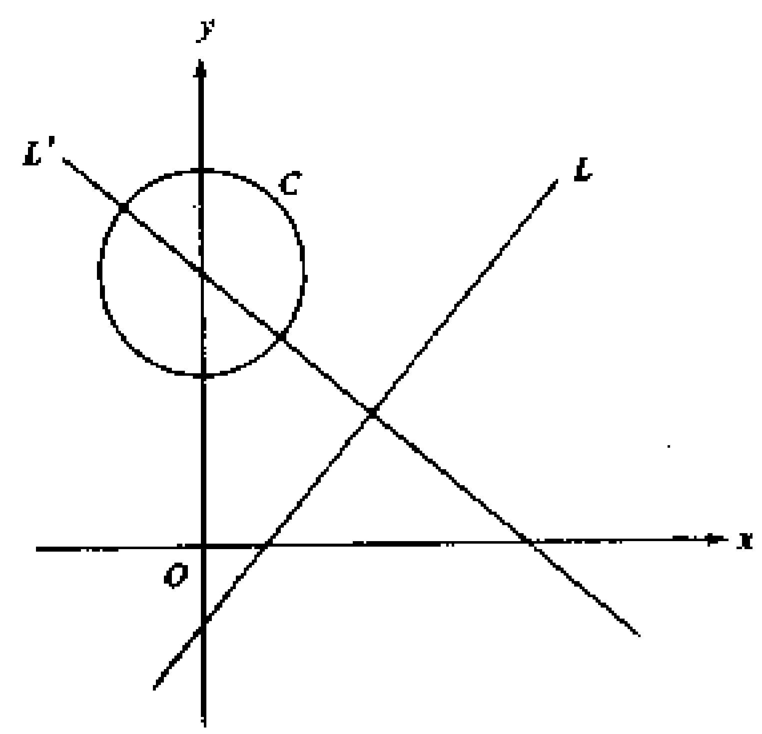

Figure 6

In Figure 6, C is the circle $x^2 + y^2 - 14y + 40 = 0$ and L is the line $4x - 3y - 4 = 0$.

(a) Find the radius and the coordinates of the centre of the circle C . (3 marks)

(b) The line L' passes through the centre of the circle C and is perpendicular to the given line L . Find the equation of the line L' . (3 marks)

(c) Find the coordinates of the point of intersection of the line L and the line L' . (3 marks)

(d) Hence, or otherwise, find the shortest distance between the circle C and the line L . (3 marks)

14.

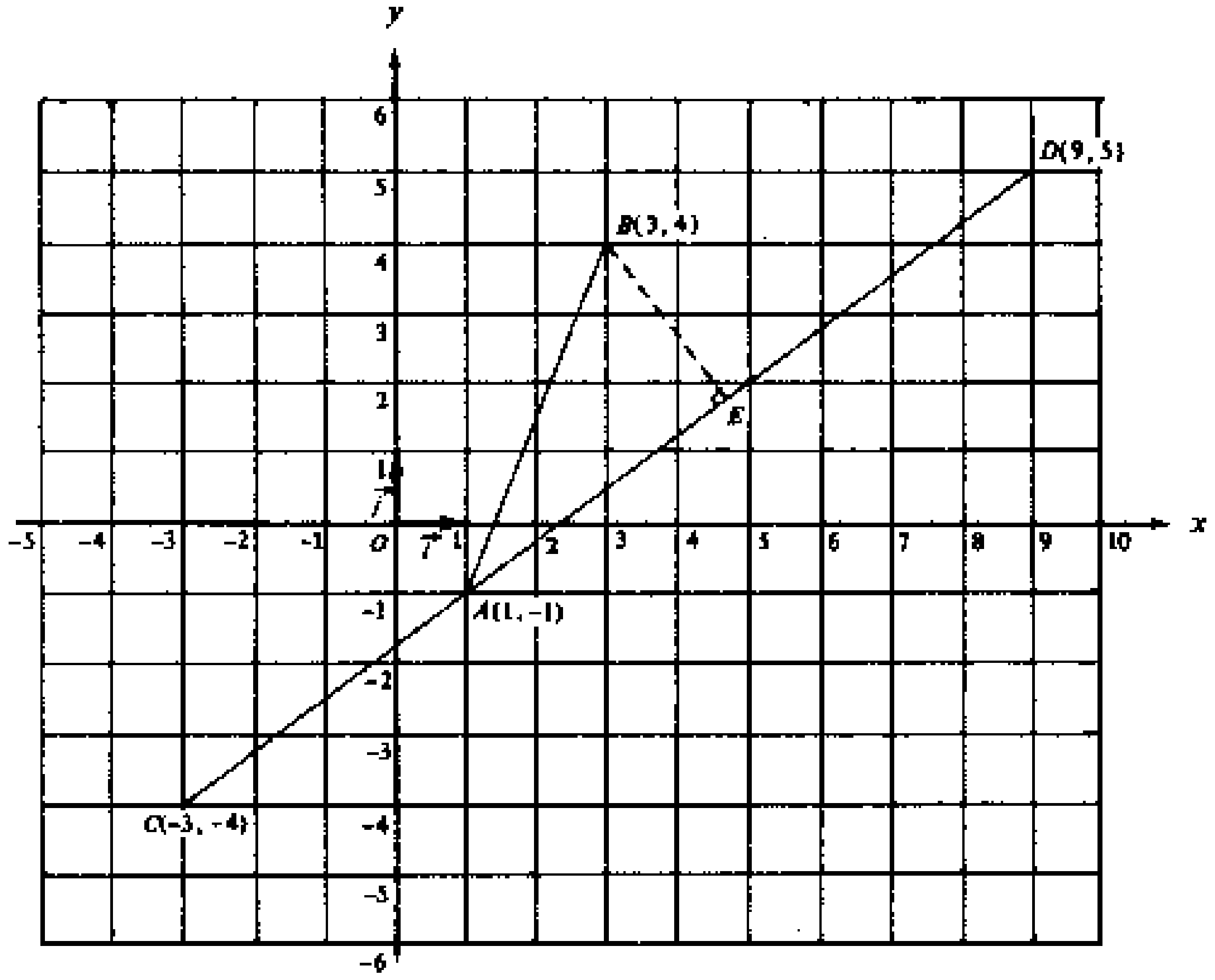

Figure 7

In Figure 7, O is the origin, $\vec{i}$ and $\vec{j}$ are perpendicular unit vectors along the x-axis and the y-axis respectively. CAD is a straight line.

(a) Express $\overrightarrow{AB}$ and $\overrightarrow{CD}$ in terms of $\vec{i}$ and $\vec{j}$. (4 marks)

(b) If $\vec{u}$ is a unit vector along $\overrightarrow{CD}$, express $\vec{u}$ in terms of $\vec{i}$ and $\vec{j}$. (4 marks)

(c) (i) Find the value of $\overrightarrow{AB} \cdot \vec{u}$.

(ii) E is a point on CD such that $BE \perp CD$. Find the length of the vector $\overrightarrow{AE}$.

(4 marks)

END OF PAPER

香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九八二年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1982

數學（課程二）
試卷一

二小時完卷
上午八時三十分至上午十時三十分
本試卷必須用英文作答

MATHEMATICS (SYLLABUS 2)
PAPER 1

Two hours
8.30 a.m.—10.30 a.m.
This paper must be answered in English

Attempt ALL questions in Section A and any FIVE questions in Section B.
Full marks will not be given unless the method of solution is shown.

FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area × height
PYRAMID	Volume	=	$\frac{1}{3}$ × base area × height

SECTION A Answer ALL questions in this section.
There is no need to start each question in this section on a fresh page.
Geometry theorems need not be quoted when used.

1. If $a - b = 10$ and $ab = k$, express $a^2 + b^2$ in terms of k. (5 marks)

2. If $\begin{cases} 4^{x-y} = 4 \\ 4^{x+y} = 16 \end{cases}$,
solve for x and y.
(5 marks)

3. Solve $2x^2 - x < 36$. (5 marks)

4. In Figure 1, the circle, centre O and radius 6, touches the straight line BC at C. $BC = 2\sqrt{3}$. OAB is a straight line. Find the area of the shaded sector in terms of π.
(6 marks)

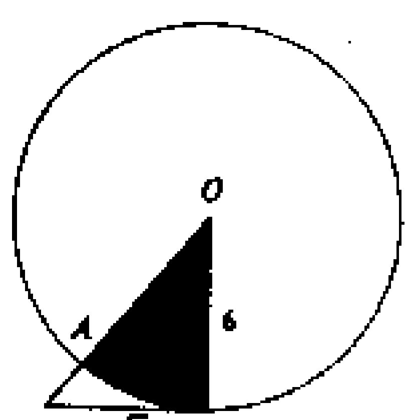

Figure 1

5. Solve $2\sin^2\theta + 5\sin\theta - 3 = 0$ for θ, where $0° \leqslant \theta < 360°$. (6 marks)

6. In Figure 2, O is the centre of the circle BAD. BOC and ADC are straight lines. If $\angle ADO = 50°$ and $\angle ACB = 20°$, find x, y and z.
(6 marks)

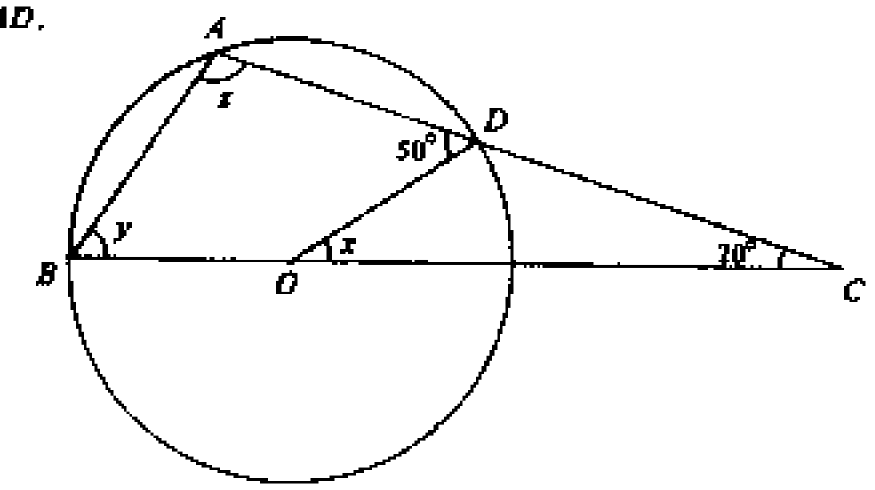

Figure 2

7. Solve $x - \sqrt{x+1} = 5$. (6 marks)

SECTION B Answer FIVE questions in this section.
Each question carries 12 marks.

8. Figure 3 represents the framework of a cuboid made of iron wire. It has a square base of side x cm and a height of y cm. The length of the diagonal AB is 9 cm. The total length of wire used for the framework (including the diagonal AB) is 69 cm.

(a) Find all the values of x and y. (10 marks)

(b) Hence calculate $\angle ABC$ to the nearest degree for the case in which $y > x$. (2 marks)

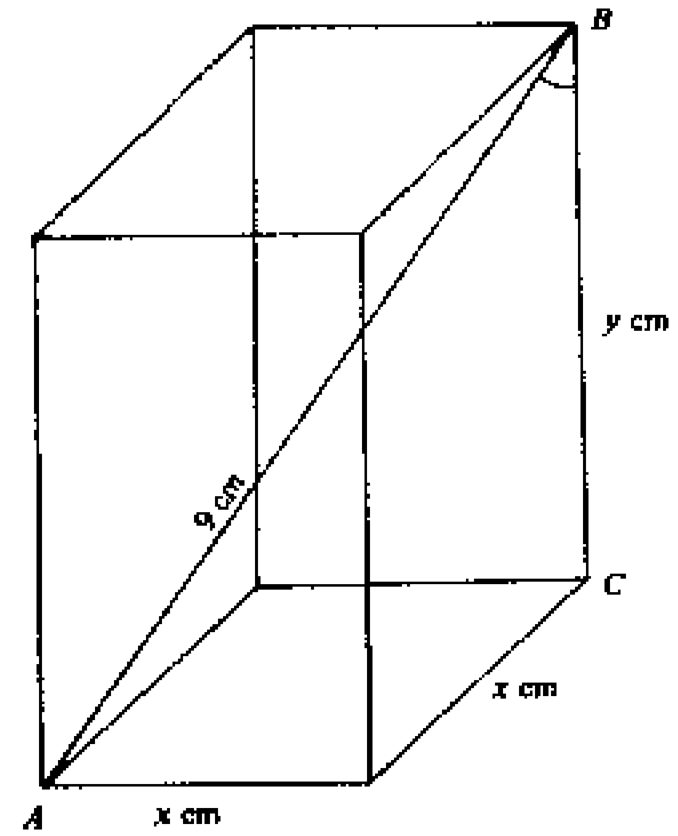

Figure 3
(Figure not drawn to scale.)

9.

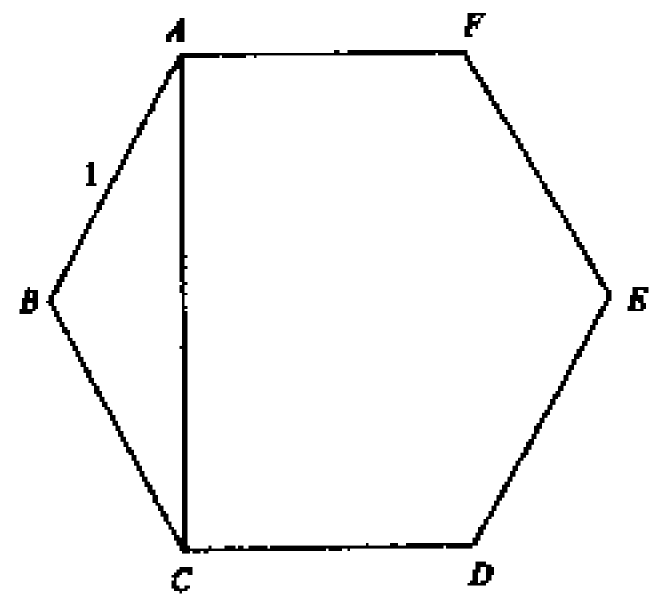

Figure 4(a)

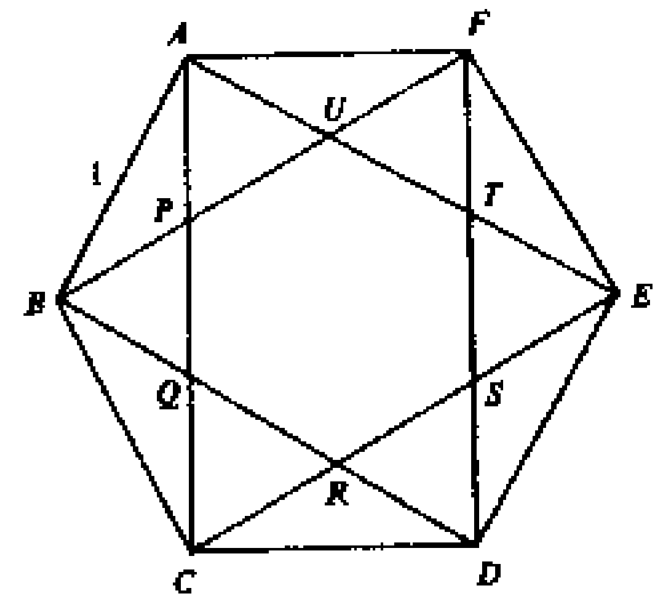

Figure 4(b)

(In this question, answers should be given in surd form.)

In Figures 4(a) and 4(b), $ABCDEF$ is a regular hexagon with $AB = 1$.

(a) Calculate the area of the hexagon in Figure 4(a) and the length of its diagonal AC. (6 marks)

(b) In Figure 4(b), $PQRSTU$ is another regular hexagon formed by the diagonals of $ABCDEF$.

(i) Calculate the length of PQ.

(ii) Calculate the area of the hexagon $PQRSTU$. (6 marks)

10. (a) (i) Find the sum of all the multiples of 3 from 1 to 1000 .

(ii) Find the sum of all the multiples of 4 from 1 to 1000 (including 1000).

(6 marks)

(b) Hence, or otherwise, find the sum of all the integers from 1 to 1000 (including 1 and 1000) which are neither multiples of 3 nor multiples of 4 .

(6 marks)

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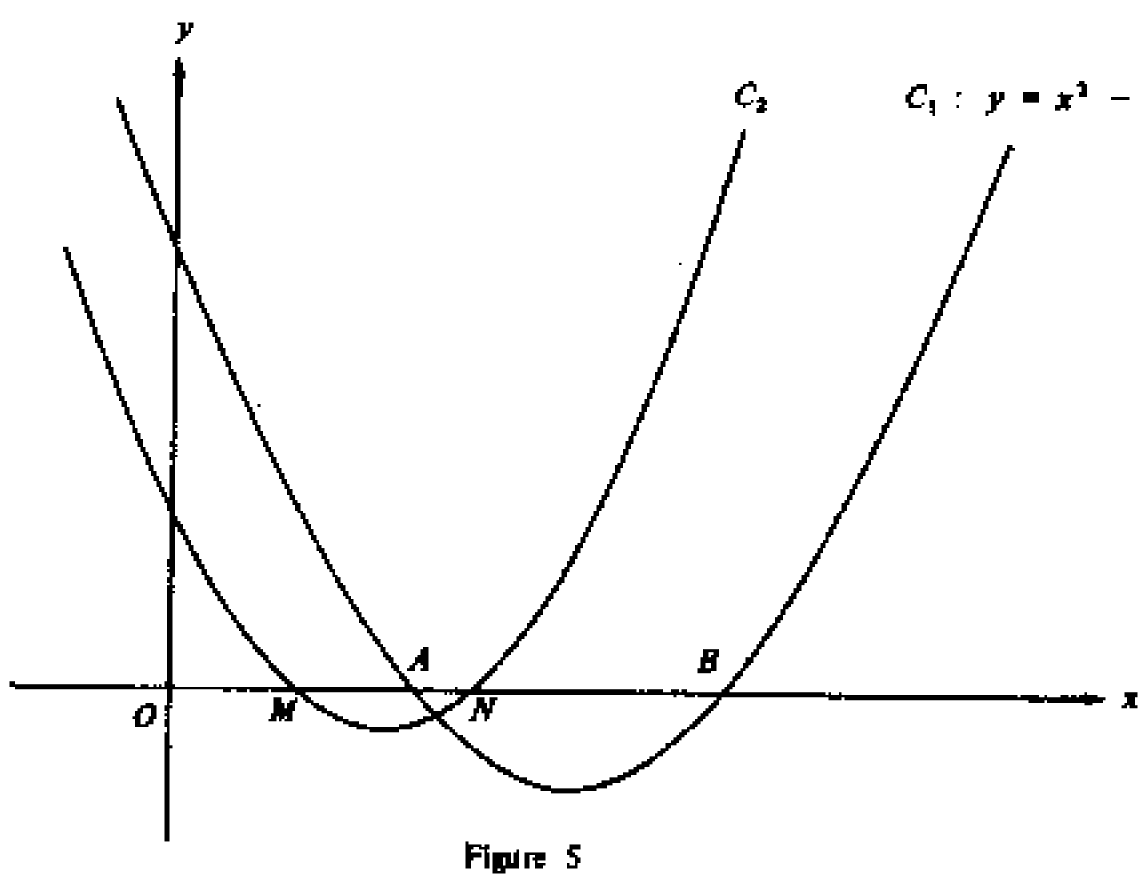

Figure 5

In Figure 5, O is the origin. The curve

$C_1 : \quad y = x^2 - 10x + k$ (where k is a fixed constant)

intersects the x-axis at the points A and B .

(a) By considering the sum and the product of the roots of $x^2 - 10x + k = 0$, or otherwise,

(i) find $OA + OB$,

(ii) find $OA \times OB$ in terms of k .

(4 marks)

(b) M and N are the mid-points of OA and OB respectively (see Figure 5).

(i) Find $OM + ON$.

(ii) Find $OM \times ON$ in terms of k .

(4 marks)

(c) Another curve

$C_2 : \quad y = x^2 + px + r$ (where p and r are fixed constants)

passes through the points M and N .

(i) Using the results in (b) or otherwise, find the value of p and express r in terms of k .

(ii) If $OM = 2$, find k .

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12. The price of a certain monthly magazine is x dollars per copy. The total profit on the sale of the magazine is P dollars. It is given that $P = Y + Z$, where Y varies directly as x and Z varies directly as the square of x . When x is 20 , P is 80 000 ; when x is 35 , P is 87 500 .

(a) Find P when $x = 15$. (7 marks)

(b) Using the method of completing the square, express P in the form $P = a - b(x - c)^2$ where a , b and c are constants. Find the values of a , b and c .

(3 marks)

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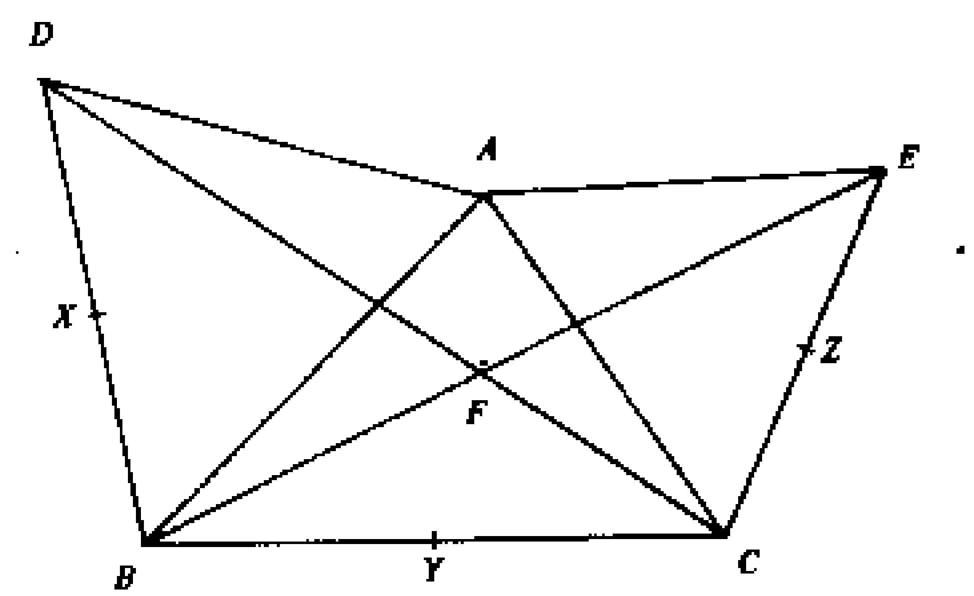

Figure 6

In Figure 6, ΔADB and ΔACE are equilateral triangles. DC and BE intersect at F .

(a) Prove that $DC = BE$.
[Hint: Consider ΔADC and ΔABE .]

(4 marks)

(b) (i) Prove that A , D , B and F are concyclic.

(ii) Find $\angle BFD$.

(4 marks)

(c) Let the mid-points of DB , BC and CE be X , Y and Z respectively. Find the angles of ΔXYZ .

(4 marks)

14.

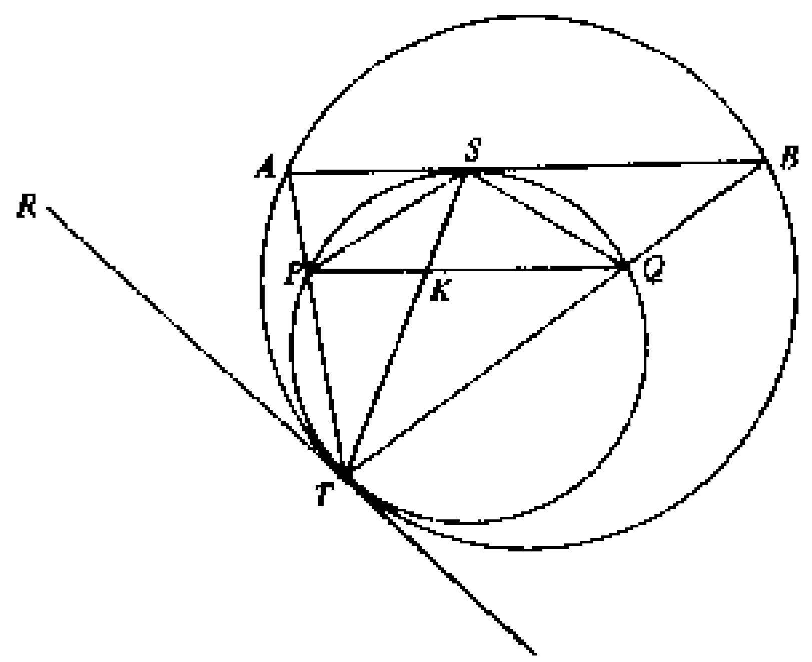

Figure 7

In Figure 7, two circles touch internally at T. TR is their common tangent. AB touches the smaller circle at S. AT and BT cut the smaller circle at P and Q respectively. PQ and ST intersect at K.

(a) Prove that $PQ \parallel AB$. (4 marks)

(b) Prove that ST bisects $\angle ATB$. (4 marks)

(c) ΔSTQ is similar to four other triangles in Figure 7. Write down any three of them. (No proof is required.) (4 marks)

END OF PAPER

香 港 考 試 局

HONG KONG EXAMINATIONS AUTHORITY

一 九 八 二 年 香 港 中 學 會 考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1982

數學(課程三)
試卷一

二小時完卷
上午八時三十分至上午十時三十分
本試卷必須用英文作答

MATHEMATICS (SYLLABUS 3)
PAPER 1

Two hours
8.30 a.m.—10.30 a.m.
This paper must be answered in English

Attempt ALL questions in Section A and any FIVE questions in Section B.
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CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area × height
PYRAMID	Volume	=	$\frac{1}{3}$ × base area × height

SECTION A Answer ALL questions in this section.
There is no need to start each question in this section on a fresh page.
Geometry theorems need not be quoted when used.

1. If $a - b = 10$ and $ab = k$, express $a^2 + b^2$ in terms of k. (5 marks)

2. In Figure 1, $AB = 4$, $AC = 5$ and $BC = 7$. Calculate $\angle A$ to the nearest degree. (5 marks)

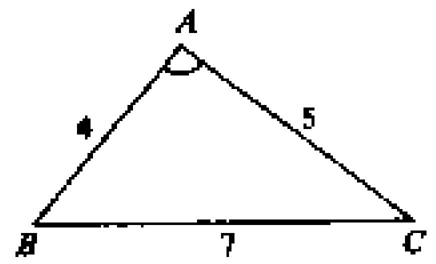

Figure 1

3. Solve $2x^2 - x < 36$. (5 marks)

4. In Figure 2, the circle, centre O and radius 6, touches the straight line BC at C. $BC = 2\sqrt{3}$. OAB is a straight line. Find the area of the shaded sector in terms of π. (6 marks)

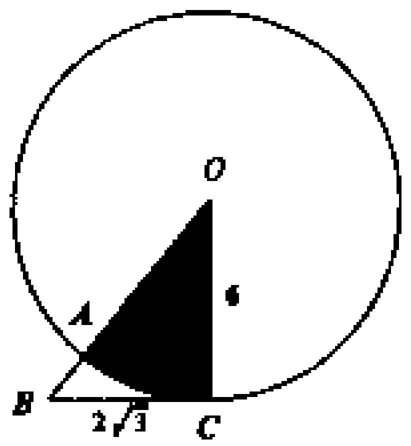

Figure 2

5. Solve $2\sin^2\theta + 5\sin\theta - 3 = 0$ for θ, where $0° \le \theta < 360°$. (6 marks)

6. If two dice are thrown once, find the probability that the sum of the numbers on the dice is

(a) equal to 4,
(b) less than 4,
(c) greater than 4.

(6 marks)

7. Solve $x - \sqrt{x+1} = 5$. (6 marks)

SECTION B Answer FIVE questions in this section.
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8. Figure 3 represents the framework of a cuboid made of iron wire. It has a square base of side x cm and a height of y cm. The length of the diagonal AB is 9 cm. The total length of wire used for the framework (including the diagonal AB) is 69 cm.

(a) Find all the values of x and y. (10 marks)

(b) Hence calculate $\angle ABC$ to the nearest degree for the case in which $y > x$. (2 marks)

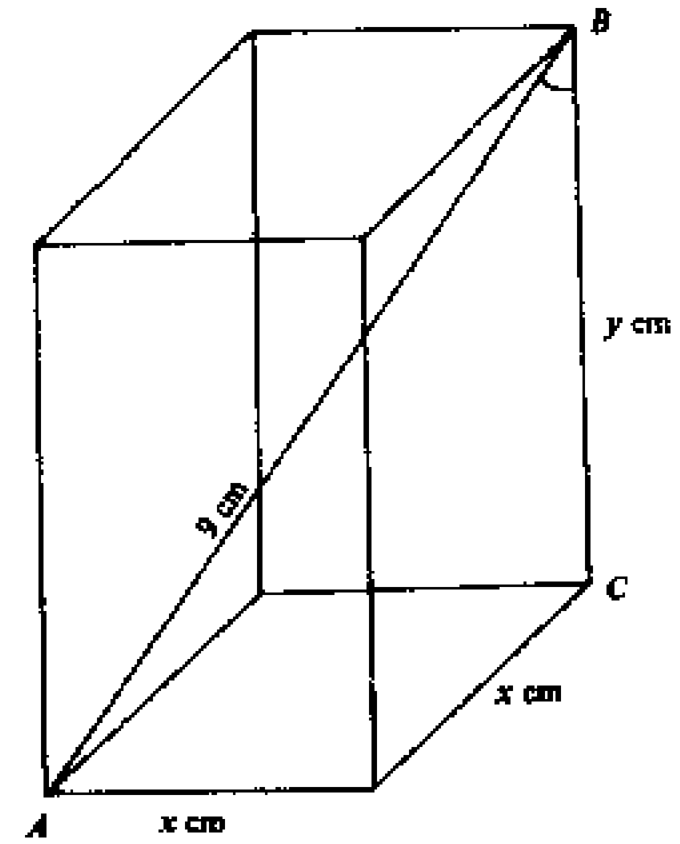


Figure 3
(Figure not drawn to scale.)

9.

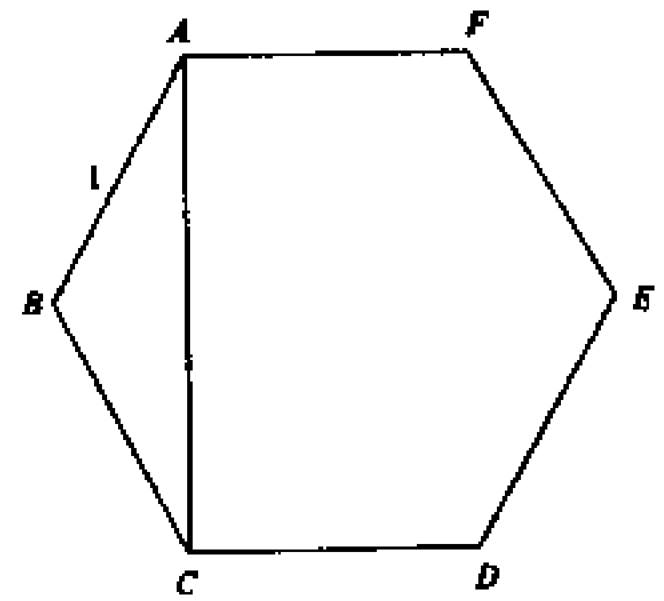


Figure 4(a)

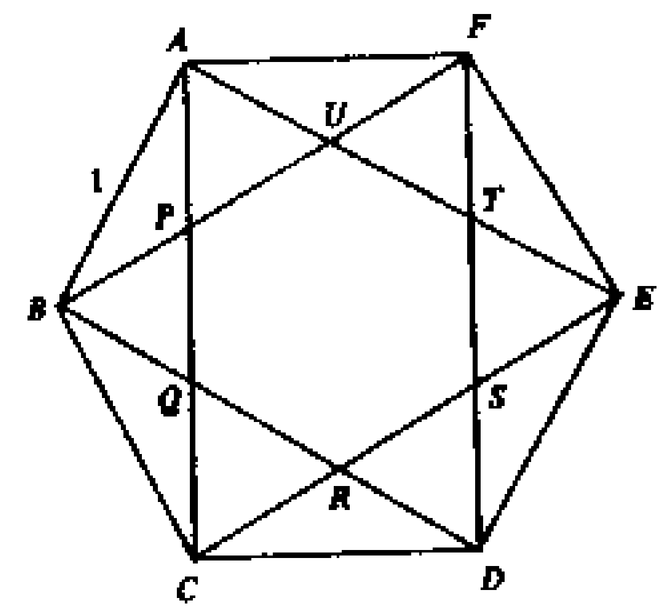


Figure 4(b)

(In this question, answers should be given in surd form.)

In Figures 4(a) and 4(b), $ABCDEF$ is a regular hexagon with $AB = 1$.

(a) Calculate the area of the hexagon in Figure 4(a) and the length of its diagonal AC. (6 marks)

(b) In Figure 4(b), $PQRSTU$ is another regular hexagon formed by the diagonals of $ABCDEF$.

(i) Calculate the length of PQ.

(ii) Calculate the area of the hexagon $PQRSTU$. (6 marks)

(a) (i) Find the sum of all the multiples of 3 from 1 to 1000 .

(ii) Find the sum of all the multiples of 4 from 1 to 1000 (including 1000) .

(6 marks)

(b) Hence, or otherwise, find the sum of all the integers from 1 to 1000 (including 1 and 1000) which are neither multiples of 3 nor multiples of 4 .

(6 marks)

11.

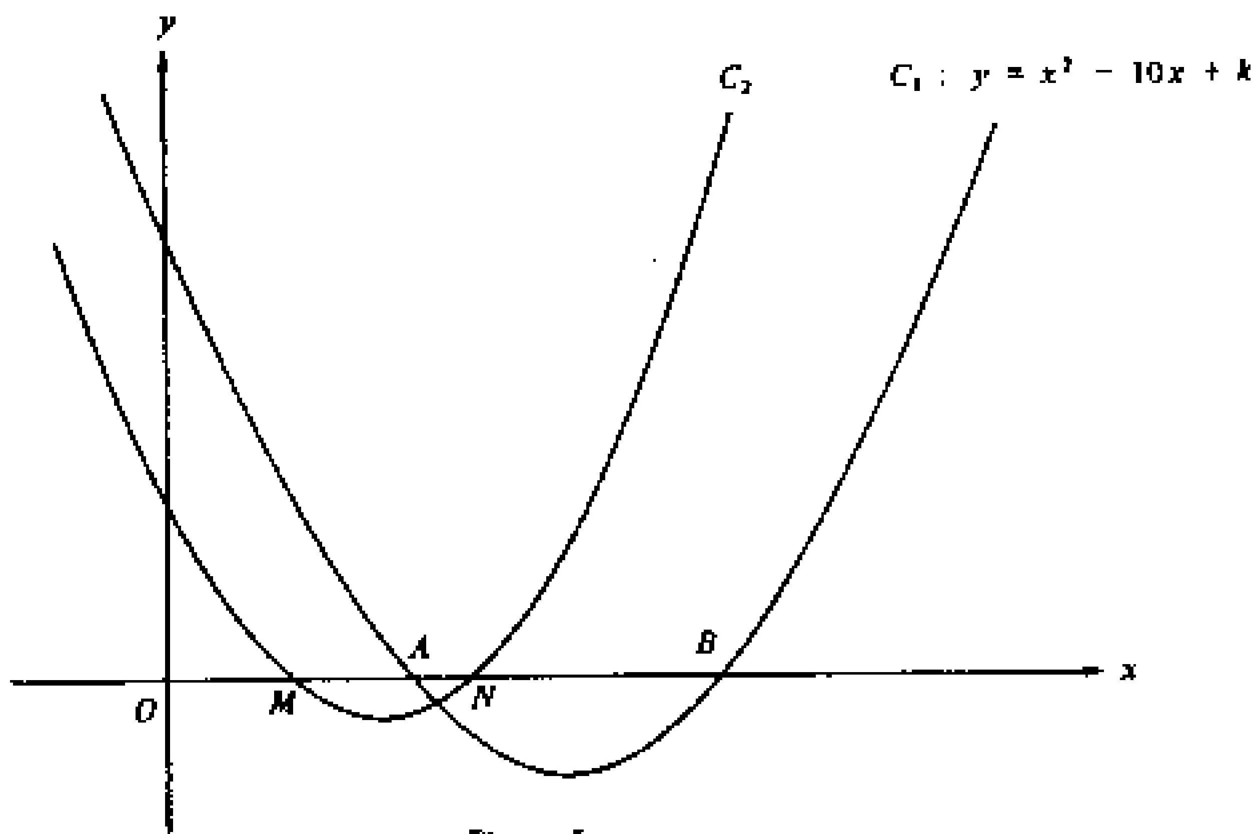


Figure 5

In Figure 5, O is the origin. The curve

C_1 : $y = x^2 - 10x + k$ (where k is a fixed constant)

intersects the x-axis at the points A and B .

(a) By considering the sum and the product of the roots of $x^2 - 10x + k = 0$, or otherwise,

(i) find $OA + OB$,

(ii) find $OA \times OB$ in terms of k .

(4 marks)

(b) M and N are the mid-points of OA and OB respectively (see Figure 5).

(i) Find $OM + ON$.

(ii) Find $OM \times ON$ in terms of k .

(4 marks)

(c) Another curve

C_2 : $y = x^2 + px + r$ (where p and r are fixed constants)

passes through the points M and N .

(i) Using the results in (b) or otherwise, find the value of p and express r in terms of k .

(ii) If $OM = 2$, find k .

(4 marks)

12. (a) The pie-chart in Figure 6(a) shows how Mr Wong's income was distributed between his expenses and savings for March.

If his rent is \$2000, find Mr Wong's income for that month.

(3 marks)

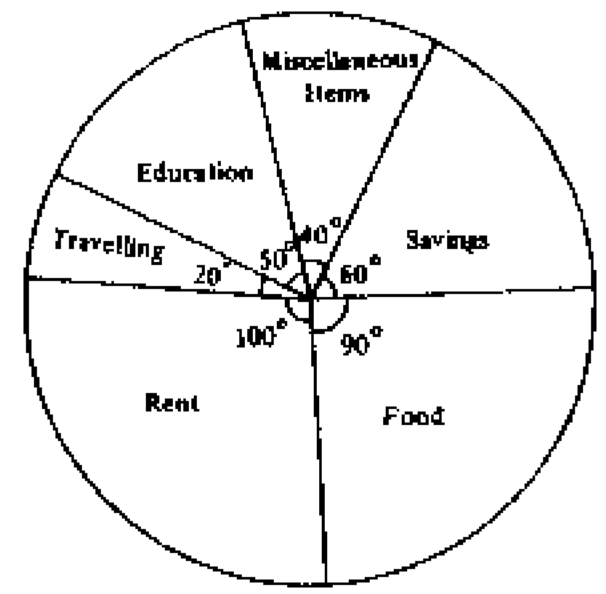


Figure 6(a)

(b) The table below shows the percentage changes when each item of Mr Wong's expenses in April is compared with that in March.

Item	Food	Rent	Travelling	Education	Miscellaneous Items	Savings
Percentage Change	Increased by 10%	Increased by 30%	Increased by 30%	No change	No change	?

The pie-chart in Figure 6(b) shows how Mr Wong's income was distributed between his expenses and savings for April.

(i) Suppose that Mr Wong's income in March and April were the same.

(1) Find x, y and z in Figure 6(b).

(2) Calculate the percentage change in Mr Wong's savings for April when compared with those for March.

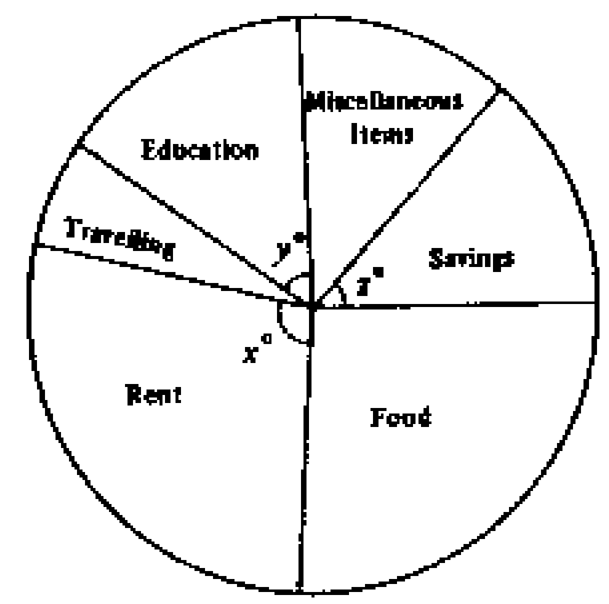


Figure 6(b)

(ii) If Mr Wong's income in April actually increased by 37.5%, what percentage of his income in April was spent on food ?

(9 marks)

13.

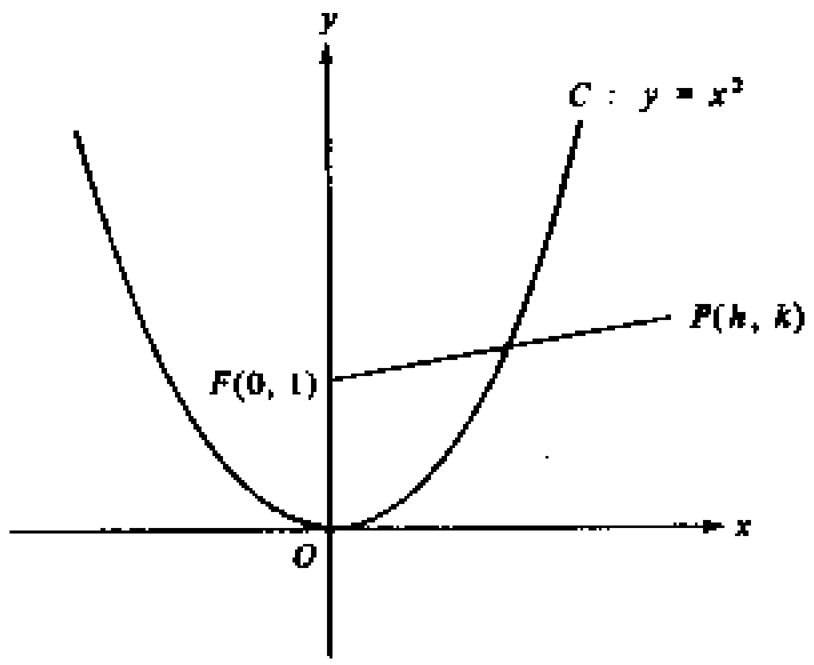


Figure 7

Figure 7 shows the sketch of the curve $C: \ y = x^2$.

(a) Find the coordinates of the point R on the curve C such that the slope of the tangent to the curve C at R is 1 .

(4 marks)

(b) $F(0, 1)$ is a fixed point and $P(h, k)$ is a variable point. $M(a, b)$ is the mid-point of FP.

(i) Express a in terms of h and b in terms of k .

(ii) If P moves such that M always lies on the curve C, show that the equation of the locus of P is $x^2 = 2(y + 1)$.

(8 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

14. **If you attempt this question, fill in the details in the first three boxes above and tie this sheet into your answer book.**

Three gold cubes have sides of length $(x + 1)$ cm, x cm and $(x - 1)$ cm respectively (see Figure 8(a)).

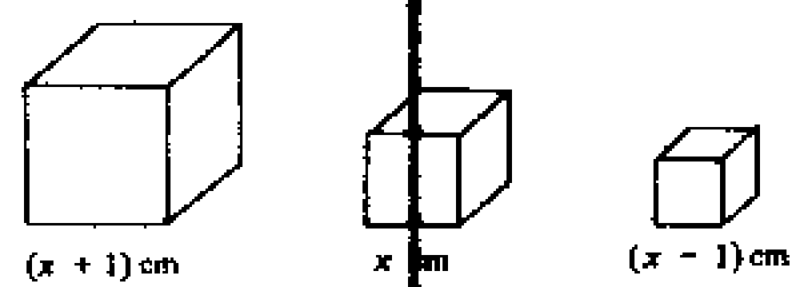


Figure 8(a)

(a) (i) Find, in terms of x, the total volume of these three cubes.

(ii) If the total volume of these three cubes is 12 cm³, show that $x^3 + 2x - 4 = 0$.

(5 marks)

(b) Figure 8(b) shows the graph of $y = x^3$ for $0 \leq x \leq 2$.

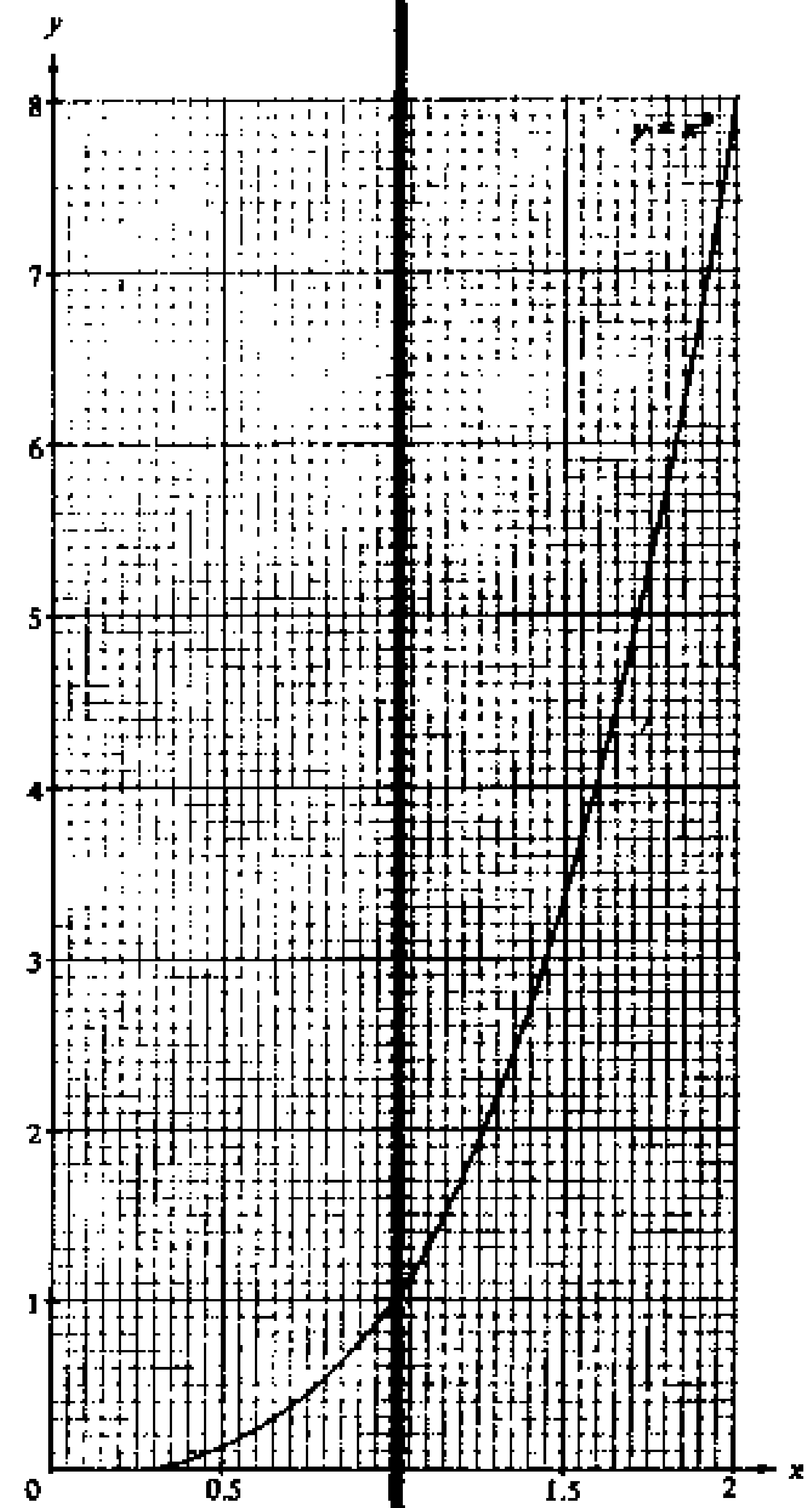
(i) Draw a suitable straight line in Figure 8(b) to solve the equation

$$x^3 + 2x - 4 = 0$$

for $0 \leq x \leq 2$. Give the root of the equation correct to 2 significant figures.

(ii) Use the method of magnification to find the root in (b)(i) correct to 3 significant figures.

(7 marks)



END OF PAPER

Figure 8(b)