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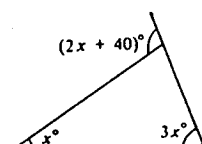
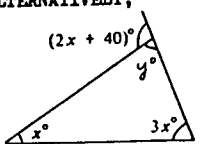
一九八〇年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1980

數學
評卷參考
MATHEMATICS
MARKING SCHEME

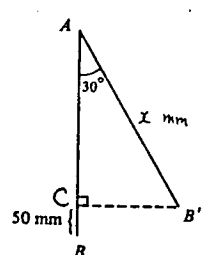
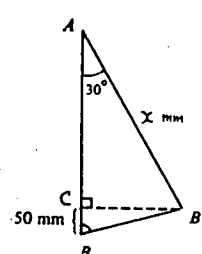
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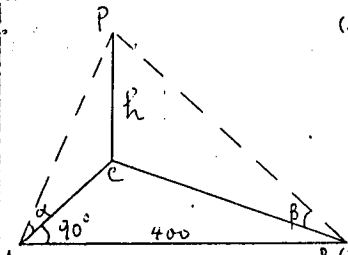
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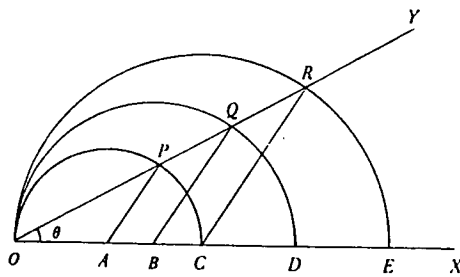
SOLUTION STEPS	MARKS	NOTES
 $2x + 40 = x + 3x$ $2x = 40$ $x = 20$	1M + 1A 2A	Do not penalize cand. for writing $x = 20^\circ$, $x^\circ = 20$ or $x^\circ = 20^\circ$.
<p>ALTERNATIVELY,</p>  $(2x + 40) + y = 180$ $y + x + 3x = 180$ $x = 20$	1M 1M 2A	
<p>ALTERNATIVELY,</p> $x + 3x + 180 - (2x + 40) = 180$ $x = 20$	2M 2A	
<p>2. (a) $a(3b - c) + c - 3b$ $= a(3b - c) - (3b - c)$ or $3b(a - 1) - c(a - 1)$ $= (a - 1)(3b - c)$</p>	1A 1A	
<p>(b) $x^4 - 1$ $= (x^2 + 1)(x^2 - 1)$ $= (x^2 + 1)(x + 1)(x - 1)$</p>	2A 1A	If a cand. writes $x^4 - 1 = 0$ $(x^2 + 1)(x^2 - 1) = 0$ 1A $(x^2 + 1)(x + 1)(x - 1) = 0$ 1A
<p>ALTERNATIVELY,</p> $f(x) = x^4 - 1$ $f(1) = 0,$ $\therefore (x - 1) \text{ is a factor of } f(x).$ <p>By long division,</p> $f(x) = (x - 1)(x^3 + x^2 + x + 1)$ $= (x - 1)(x + 1)(x^2 + 1)$	1A 1A 1A	If a cand. writes $f(1) = 0$, $(x - 1)$ is a factor. 1A $f(-1) = 0$ $(x + 1)$ is a factor. 1A

SOLUTION STEPS	MARKS	NOTES
Product of roots = $-\frac{5}{2}$ _____	1A	
Let the other root be α $5\alpha = -\frac{5}{2}$ or $\alpha\beta = -\frac{5}{2}$ _____ $\alpha = -\frac{1}{2}$ _____	1A 1A	1) 已知 $\alpha\beta = -\frac{5}{2}$ (2A)
$-\frac{k}{2} = -\frac{1}{2} + 5$ or $\alpha + \beta = -\frac{k}{2}$ _____ $k = -9$ _____	1M 1A	
ALTERNATIVELY, Product of roots = $-\frac{5}{2}$ _____ Since one of the roots is 5, $2(5)^2 + k(5) - 5 = 0$ _____ $k = -9$ _____	1A 1M 1A	
The equation is $2x^2 - 9x - 5 = 0$ $(x - 5)(2x + 1) = 0$ _____ $x = 5$ or $x = -\frac{1}{2}$	1A	
The other root is $-\frac{1}{2}$ _____	1A	
$\sin \theta = \cos 120^\circ$ $= -\frac{1}{2}$ _____	1A	
ALTERNATIVELY, $\sin \theta = \cos 120^\circ$ $= -\cos 60^\circ$ $= -\sin 30^\circ$ _____	1A	
$\theta = 180^\circ + 30^\circ$ or $360^\circ - 30^\circ$ $= 210^\circ$ or 330° _____	2A + 2A	2) general solution (2A) Accept $\theta = 210^\circ, 330^\circ$ Accept $\theta = 210^\circ$ and 330° 如 330°-360° 內 答 得 亦 要 加 分

SOLUTION STEPS	MARKS	NOTES
Let the length of AB be x mm. $AC = x \cos 30^\circ$ _____ $\therefore x \cos 30^\circ + 50 = x$ _____ $x = \frac{50}{1 - \cos 30^\circ}$ or 373.2 $= 373$ _____ (corr. to 3 sig. fig.) Length of the rod = 373 mm.	2A 1M 1M 1A	any ignition in x. for AC = CB = AB Deleted. 2) 若 $\frac{x-50}{x} = \cos 30^\circ$, 3分
		
ALTERNATIVELY, Let AB = x mm $\angle ABB' = \angle AB'B$ _____ $= 75^\circ$ $\frac{CB'}{CB} = \tan 75^\circ$ $CB' = 50 \tan 75^\circ$ _____ $= 186.6$ $\frac{CB'}{x} = \sin 30^\circ$ $x = \frac{CB'}{\sin 30^\circ}$ $= 373.2$ $= 373$ _____ (corr. to 3 sig. fig.)	1M 1A 1M + 1A 1A	Alternatively, Let AC = R mm. $\frac{R}{R+50} = \cos 30^\circ$... 3M $R = \frac{50 \cos 30^\circ}{1 - \cos 30^\circ}$ or 373.2 1A AB = 373 ... 1A.
		
Suppose x mothers lost only one of their children. Then $(36 - x)$ mothers lost two of their children. _____ $x + 2(36 - x) = 62$ _____ $x = 10$ _____ 10 mothers lost only one of their children. 26 mothers lost both of their children. _____	1A 2M 1A 1A	If one answer given without explanation. 1A If both answers given 3A (i) With checking +1 (ii) With acceptable explanation +2
ALTERNATIVELY, Suppose x mothers lost only one of their children and y mothers lost two of their children. $x + y = 36$ _____ $x + 2y = 62$ _____ Solving, $x = 10$ _____ $y = 26$ _____	1A 2M 1A 1A	

SOLUTION STEPS	MARKS	NOTES
$a(1 + \frac{x}{100}) = b(1 - \frac{x}{100})$ $a + \frac{ax}{100} = b - \frac{bx}{100}$ _____ $\frac{ax}{100} + \frac{bx}{100} = b - a$ _____ $\frac{a+b}{100} x = b - a$ _____ $x = (\frac{b-a}{a+b}) \cdot 100$ _____	1A 1A 1A 2A	
Daily wages for a skilled, semi-skilled and unskilled worker are \$120, \$90, \$60 respectively. _____ Mean daily wage in dollars = $\frac{120 \times 10 + 90 \times 20 + 60 \times 30}{60}$ $= \frac{4800}{60}$ $= 80$ _____	1A 1M 1A 1A 1A	for the ... (for denominator) (for 4800)

SOLUTION STEPS	MARKS	NOTES
 <p>(a) (i) $\tan \alpha = \frac{h}{x}$ _____ $x = \frac{h}{\tan \alpha}$ _____ (ii) $\tan \beta = \frac{h}{y}$ _____ $y = \frac{h}{\tan \beta}$ _____</p> <p>(b) $BC^2 = AC^2 + AB^2$ _____ $y^2 = x^2 + 400^2$ _____</p> <p>$(\frac{h}{\tan \beta})^2 = (\frac{h}{\tan \alpha})^2 + 400^2$ _____ $(\frac{h}{\tan 30^\circ})^2 = (\frac{h}{\tan 60^\circ})^2 + 400^2$ _____ $(\sqrt{3}h)^2 = (\frac{1}{\sqrt{3}}h)^2 + 400^2$ $2\frac{2}{3}h^2 = 400^2$ $h^2 = \frac{3}{8} \times 400^2$ $h = \sqrt{\frac{3}{8}} \times 400$ (or 244.9) _____ ≈ 245 (corr. to 3 sig. fig.) _____</p>	1A 1A 1A 1A 2M 1M 1A 1A 1A 1A	Accept $x = h \cot \alpha$ ✓ Accept $y = h \cot \beta$ For sub. x, y or any figure which rounds off to 245
<p>ALTERNATIVELY,</p> <p>$BC^2 = AC^2 + AB^2$ _____ $y^2 = x^2 + 400^2$ $x = \frac{h}{\tan 60^\circ}$ _____ $\frac{x}{y} = \frac{\tan 30^\circ}{\tan 60^\circ}$ _____ $= \frac{1}{3}$ $3x = y$ $9x^2 = x^2 + 400^2$ _____ $8x^2 = 400^2$ $x = \sqrt{\frac{400^2}{8}} = 141.42$ $h = x \tan 60^\circ$ _____ $= 245$ _____</p>	2M 1A 1M 1A 1A	



(a) $\angle PAX = 2\theta$ _____
 (\because angle at centre is twice as great as angle at circumference)

ALTERNATIVELY,
 $\angle APO = \theta$ or $\angle PAX = \theta + \angle APO$ _____
 $\angle PAX = 2\theta$ _____

Similarly $\left. \begin{array}{l} \angle QBX = 2\theta \\ \angle RCX = 2\theta \end{array} \right\}$ _____

(b) $\angle PAO = \angle QBO = \angle RCO$ _____ 1M
 Sector PAO, sector QBO, sector RCO are similar.
 Area of sector PAO : area of sector QBO :
 $= OA^2 : OB^2 : OC^2$ _____ 2M
 $= 2^2 : 3^2 : 4^2$ or $4 : 9 : 16$ _____ 1A

ALTERNATIVELY,
 $\angle PAO = \angle QBO = \angle RCO = \theta$ _____ 1M
 $\left. \begin{array}{l} \text{Area of sector PAO} = \frac{1}{2} OA^2 \theta \\ \text{Area of sector QBO} = \frac{1}{2} OB^2 \theta \\ \text{Area of sector RCO} = \frac{1}{2} OC^2 \theta \end{array} \right\}$ _____ 1A
 Area of sector PAO : area of sector QBO :
 area of sector RCO = _____
 $= \frac{1}{2} OA^2 \theta : \frac{1}{2} OB^2 \theta : \frac{1}{2} OC^2 \theta$ _____ 1
 $= OA^2 : OB^2 : OC^2$
 $= 2^2 : 3^2 : 4^2$ or $4 : 9 : 16$ _____ 1A

2A

不用证明

1A
1A

1A

Awarded only if both answers are correct

1M

2M
1A

Ratio of areas = ratio of radii²

但 $\angle PAO = \angle QBO = \angle RCO = \theta$
 又给 M 分 θ

1M

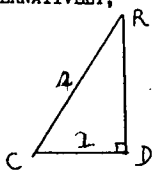
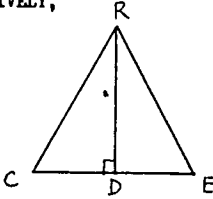
1A

(for any one of the three)

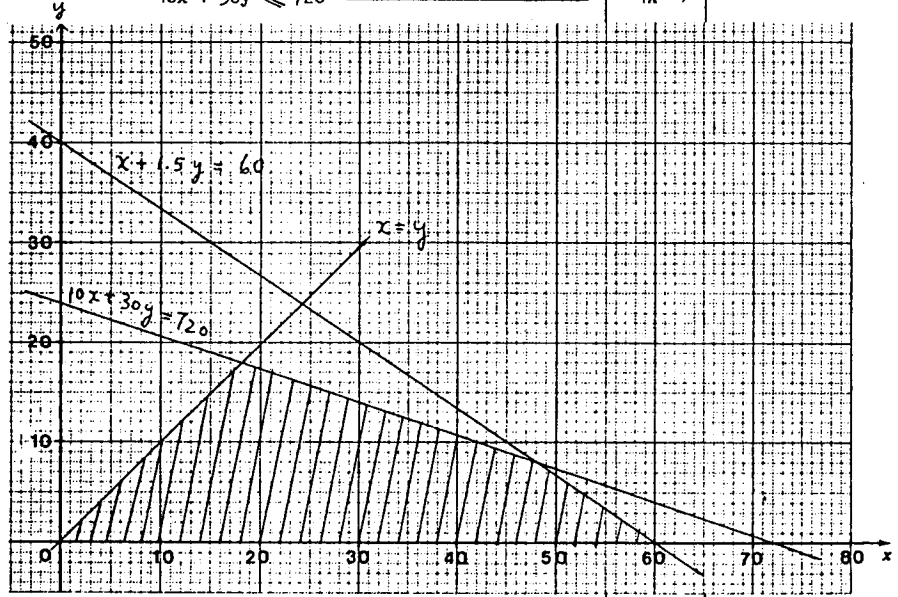
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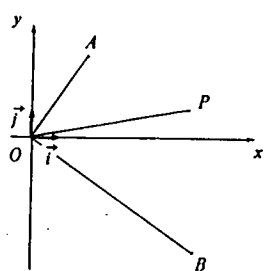
(provided all three expressions are correct)

1A

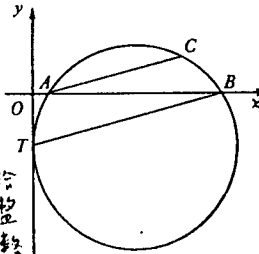
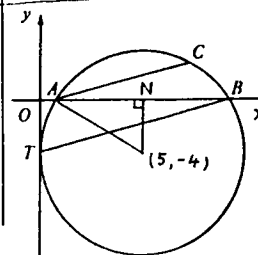
SOLUTION STEPS	MARKS	NOTES
(c) If $RD \perp OX$, $RD^2 = CR^2 - CD^2$ _____ $= OC^2 - CD^2$ $= 4^2 - 2^2$ $= 12$	1M	
$\tan \theta = \frac{RD}{OD}$ _____ $= \frac{\sqrt{12}}{6} = \frac{\sqrt{3}}{3}$ $\theta = 30^\circ$ _____	1M 1A	
ALTERNATIVELY,  $\angle RCD = 60^\circ$ _____ $\theta = 30^\circ$ _____	1M 1A 1A	
ALTERNATIVELY,  Mentioning RD is the perpendicular bisector of CE 1M $\therefore RC = RE$ $RC = CE$ (radii) Hence $\triangle RCE$ is an equilateral \triangle 1M $\therefore \angle RCE = 60^\circ$ $\therefore \theta = 30^\circ$ _____ 1A 1A		

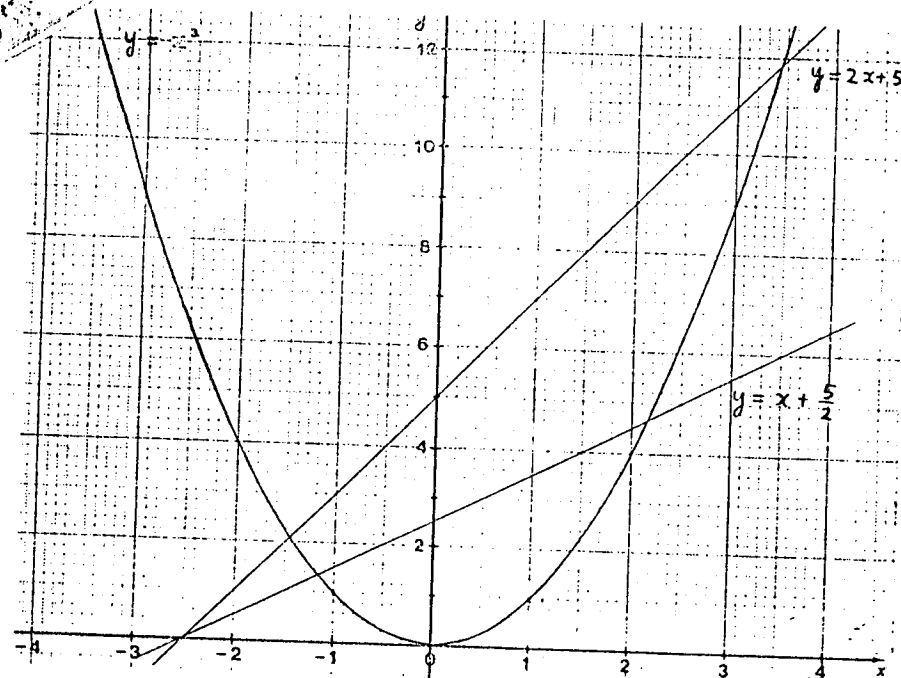
SOLUTION STEPS	MARKS	NOTES
1. (a) (i) Common ratio = 10 _____	1A	
(ii) Sum of n terms = $\frac{a(r^n - 1)}{r - 1}$ _____	1M	
$= \frac{k(10^n - 1)}{10 - 1}$		
$= \frac{k}{9} (10^n - 1)$ _____	1A	
(b) (i) One mark would be awarded if a cand. shows the correct <u>idea</u> of proving		
either 3rd term - 2nd term = 2nd term - 1st term	1M	
or 1st term + 3rd term = 2 x 2nd term		
$\log 10k - \log k = \log \frac{10k}{k}$		
$= \log 10$ or 1 _____	1A	
$\log 100k - \log 10k = \log \frac{100k}{10k}$		
$= \log 10$ or 1 _____	1A	
\therefore It is an A.P.		
ALTERNATIVELY,		
$\frac{\log k + \log 100k}{2} = \frac{1}{2} \log 100k^2$		
$= \log 10k$ _____	2A	
It is an A.P.		
(ii) Quoting correct formula for the sum of A.P.		
$\frac{n}{2}[2a + (n - 1)d]$ or $\frac{n}{2}[T(1) + T(n)]$ _____	1M	This may be omitted
Sum of the first n terms		
$= \frac{n}{2}[2 \log k + (n - 1) \log 10]$ _____	1A	(ii) $S_n = \frac{n}{2} [2 \log k + (n-1) \log 10]$ $= \frac{n}{2} [2 \log k + (n-1) \log 10]$ 3分
$= \frac{n}{2}[2 \log k + (n - 1)]$		
Sum of the first ten terms		
$= \frac{10}{2}[2 \log k + 9 \log 10]$ _____	1A	
$= 10 \log k + 45$ _____	1A	

SOLUTION STEPS	MARKS	NOTES
2. Let x be the no. of economy class seats and y be the no. of first class seats.		
Constraints: $x, y \in \mathbb{N}$, may be omitted		
$x + 1.5y \leq 60$ _____	1A	滿分皆得 10-6分 may be omitted
$x \geq y$ _____	1A	
$10x + 30y \leq 720$ _____	1A	
		
Graphs of the lines: $x = y$ _____	1A	Labelling of graphs not necessary.
$x + 1.5y = 60$ _____	1A	
$10x + 30y = 720$ _____	1A	
Correct region _____	1A	
Testing optimization _____	1M	
No. of first class seats = 8 _____	1A	Awarded only if region correct
No. of economy class seats = 48 _____	1A	
		空位一空, 最多6分 ... = ... 5分 ... = ... 4分 滿分皆得 10-6分

SOLUTION STEPS	MARKS	NOTES
(a) (i) $(3\vec{i} + 4\vec{j}) \cdot (x\vec{i} + y\vec{j})$ $= 3x + 4y$	1A	If "." omitted, do not deduct mark.
(ii) $ \vec{OA} = \sqrt{3^2 + 4^2}$ $= 5$	1A	$ \vec{OA} = OA = OA$ Accept
$ \vec{OP} = \sqrt{x^2 + y^2}$	1A	$ \vec{OA} = \sqrt{(3\vec{i})^2 + (4\vec{j})^2}$
(ii) $\cos \angle AOP = \frac{\vec{OA} \cdot \vec{OP}}{ \vec{OA} \vec{OP} }$ $= \frac{3x + 4y}{5\sqrt{x^2 + y^2}}$	1M	Accept $\begin{pmatrix} 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 3x + 4y$, $(3, 4) \cdot (x, y) = 3x + 4y$
(b) $\vec{OB} \cdot \vec{OP} = 8x - 6y$ $ \vec{OB} = 10$ $\cos \angle BOP = \frac{\vec{OB} \cdot \vec{OP}}{ \vec{OB} \vec{OP} }$ $= \frac{8x - 6y}{10\sqrt{x^2 + y^2}}$	1A 1M + 1A	
(c) Equation of internal bisector of $\angle AOB$: $\frac{3x + 4y}{5\sqrt{x^2 + y^2}} = \frac{8x - 6y}{10\sqrt{x^2 + y^2}}$ $3x + 4y = 4x - 3y$ $x - 7y = 0$	2M 1A	
		If "→" is omitted three times or more in the solution, deduct 1 mark for poor presentation.

SOLUTION STEPS	MARKS	NOTES
(a) Probability = $\frac{9}{10} \times \frac{2}{3}$ $= \frac{3}{5}$ or $\frac{6}{10}$ or $\frac{60}{100}$ or 60% or 0.6	2A 1A $\leftarrow \frac{11}{30}$	Award 2 or 0
(b) Probability of obtaining the qualification with one re-examination of the theory paper $= \frac{1}{10} \times \frac{9}{10} \times \frac{2}{3}$ $= \frac{3}{50}$	1A	
Probability of obtaining the qualification with one re-examination of the practical paper $= \frac{9}{10} \times \frac{1}{3} \times \frac{2}{3}$ $= \frac{1}{5}$	1A	
Required probability $= \frac{3}{50} + \frac{1}{5}$ $= \frac{13}{50}$	1M 1A	Award this mark for the + sign. Even when $\frac{3}{50}$ and $\frac{1}{5}$ are both incorrect, still give this mark.
(c) Probability that A (or B) does not obtain the qualification by sitting each paper once. $= 1 - \frac{3}{5}$ $= \frac{2}{5}$	1A	
Probability that A and B do not obtain the qualification by sitting each paper once. $= \frac{2}{5} \times \frac{2}{5}$	1M 1A	<i>cancelled</i> $\frac{3}{5} + \frac{3}{5} - \frac{3}{5} \times \frac{3}{5}$ This method mark should be given when the expression is of the form $1 - p^2$, where $0 < p < 1$.
Required probability $= 1 - \frac{2}{5} \times \frac{2}{5}$ $= \frac{21}{25}$	1M 1A	This method mark should be given when the expression is of the form $p_1 + p_2 p_3$
ALTERNATIVELY, Required probability $= \frac{3}{5} + \frac{2}{5} \times \frac{3}{5}$ $= \frac{21}{25}$	1M + 1A 1A	This method mark should be given when the expression is of the form $p_1 p_2 + p_3 p_4 + p_5 p_6$
ALTERNATIVELY, Required probability $= \frac{3}{5} \times \frac{3}{5} + \frac{3}{5} \times \frac{2}{5} + \frac{2}{5} \times \frac{3}{5}$ $= \frac{21}{25}$	1M + 1A 1A	This method mark should be given when the expression is of the form $p_1 p_2 + p_3 p_4 + p_5 p_6$

SOLUTION STEPS	MARKS	NOTES
$x^2 + y^2 - 10x + 8y + 16 = 0$ (*) (a) $\begin{cases} y = 0 \\ x^2 + y^2 - 10x + 8y + 16 = 0 \\ x^2 - 10x + 16 = 0 \\ (x-2)(x-8) = 0 \\ x = 2 \text{ or } 8 \\ A = (2, 0), B = (8, 0) \end{cases}$ $\begin{cases} x = 0 \\ x^2 + y^2 - 10x + 8y + 16 = 0 \\ y^2 + 8y + 16 = 0 \\ (y+4)^2 = 0 \\ y = -4 \\ T = (0, -4) \end{cases}$	1M $\frac{1}{2}A + \frac{1}{2}A$ $\frac{1}{2}A + \frac{1}{2}A$ 1M 1A	 <p>19. 1/6分 1分.</p>
(b) (i) Slope of BT = $\frac{0 - (-4)}{8 - (0)} = \frac{1}{2}$ Equation of AC : $\frac{y - 0}{x - 2} = \text{slope of BT}$ $\frac{y - 0}{x - 2} = \frac{1}{2}$ $x = 2y + 2$ or $y = \frac{1}{2}x - 1$	1M 1A	
(ii) Substitute $x = 2y + 2$ in (*), $(2y + 2)^2 + y^2 - 10(2y + 2) + 8y + 16 = 0$ $5y^2 - 4y = 0$ $y(5y - 4) = 0$ $y = 0$ or $y = \frac{4}{5}$ $y = 0$ is rejected, $y = \frac{4}{5}$ $x = \frac{18}{5}$ $C = (\frac{18}{5}, \frac{4}{5})$	1M 1A 1A	OR Sub. $y = \frac{x}{2} - 1$ in (*) 1M $5x^2 - 28x + 36 = 0$ $(x-2)(5x-18) = 0$ $x = 2$ or $\frac{18}{5}$ 1A $x = 2$ is rejected, $x = \frac{18}{5}$ $y = \frac{4}{5}$ $C = (\frac{18}{5}, \frac{4}{5})$ 1A
(a) ALTERNATIVELY, $(x-5)^2 + (y+4)^2 = 25$ Centre = (5, -4) Radius = 5 $T = (0, -4)$ $AN = BN = \sqrt{5^2 - 4^2} = 3$ $OA = 5 - 3 = 2$ $OB = 5 + 3 = 8$ $A = (2, 0), B = (8, 0)$	1A 1A 1A $\frac{1}{2}A$ $\frac{1}{2}A$ $\frac{1}{2}A + \frac{1}{2}A$	



(a) $x^2 - 2x - 5 = 0$ $x^2 = 2x + 5$ Equation of straight line: $y = 2x + 5$ Graph of the straight line: $y = 2x + 5$	1A 1A	May be written on the graph
$x_1 = -1.4$ (-1.5 to -1.4) $x_2 = 3.4$ (3.4 to 3.5)	1A 1A	Awarded only if the line is correctly drawn
(b) $x^2 - 2x > 5$ $x < -1.4$ or $x > 3.4$	2M	(i) If the word "or" is omitted award 1 mark. (ii) If a candidate writes $-1.4 > x > 3.4$ or $x < -1.4$ and $x > 3.4$, award 1 mark.
(c) $2x^2 - 2x - 5 = 0$ $x^2 - x - \frac{5}{2} = 0$ Equation of straight line: $y = x + \frac{5}{2}$ Graph of the straight line: $y = x + \frac{5}{2}$	1A 1A	May be written on the graph
$x_1 = -1.2$ (-1.2 to -1.1) $x_2 = 2.2$ (2.1 to 2.2)	1A 1A	Awarded only if the line is correctly drawn
Do not deduct mark if st. line not labelled or labelled as $x^2 - 2x - 5 = 0$. If answers in ordered pairs, deduct 1 mark as poor presentation.		