1980 Maths I

SECTION A Answer ALL questions in this section.
There is no need to start each question in this section on a fresh page.
Geometry theorems need not be referred to when used.

1. Find the value of x in Figure 1.

(4 marks)

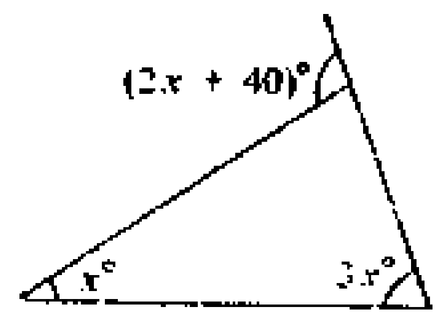

Figure 1

2. Factorize

(a) $a(3b - c) + c - 3b$.

(b) $x^4 - 1$.

(5 marks)

3. What is the product of the roots of the quadratic equation $2x^2 + kx - 5 = 0$? If one of the roots is 5, find the other root and the value of k.

(5 marks)

4. If $0° < \theta < 360°$ and $\sin\theta = \cos 120°$, find θ.

(5 marks)

5. In Figure 2, AB is a vertical thin rod. It is rotated about A to position AB' such that $\angle BAB' = 30°$. If B' is 50 mm higher than B, find the length of the rod, correct to 3 significant figures.

(5 marks)

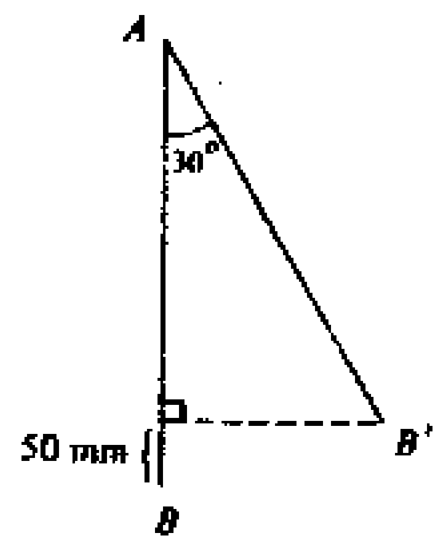

Figure 2

6. At a fun fair, each mother there had brought 2 children along. At the end of the day, it was found that 36 mothers had lost one or both of their children and 62 children had lost their mothers. How many mothers lost only one of their children and how many mothers lost both of their children?

(5 marks)

7. Given that $a(1 + \frac{x}{100}) = b(1 - \frac{x}{100})$, express x in terms of a and b.

(5 marks)

8. A factory employs 10 skilled, 20 semi-skilled, and 30 unskilled workers. The daily wages per worker of the three kinds are in the ratio $4:3:2$. If a skilled worker is paid \$120 a day, find the mean daily wage for the 60 workers.

(5 marks)

SECTION B Answer SIX questions in this section.
Each question carries 10 marks.

9.

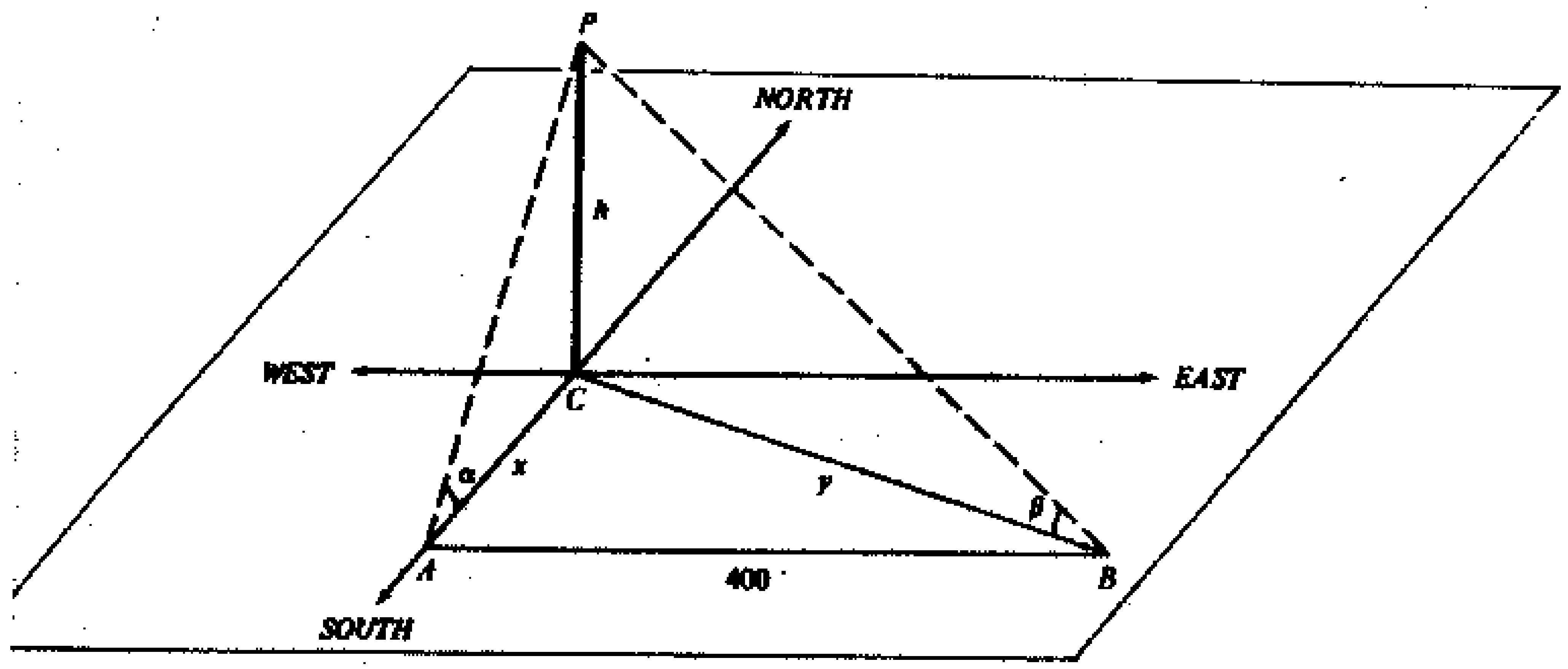

Figure 3

In Figure 3, PC represents a vertical object of height h metres. From a point A, south of C, the angle of elevation of P is α. From a point B, 400 metres east of A, the angle of elevation of P is β. AC and BC are x metres and y metres respectively.

(a) (i) Express x in terms of h and α.

(ii) Express y in terms of h and β.

(4 marks)

(b) If $\alpha = 60°$ and $\beta = 30°$, find the value of h correct to 3 significant figures.

(6 marks)

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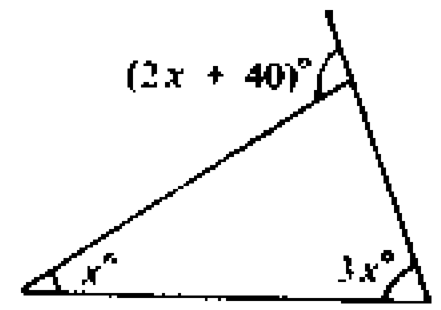

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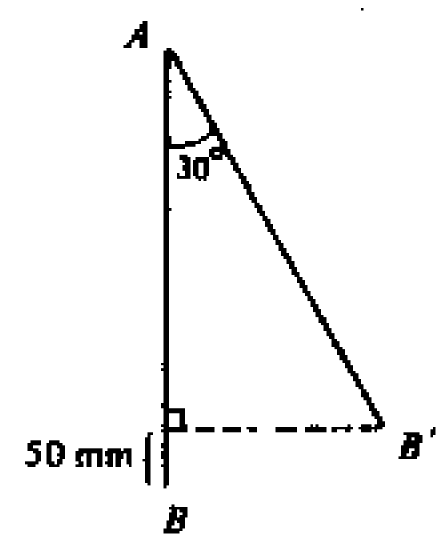

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(5 marks)

7. Find x if $\log_3(x - 3) + \log_3(x + 3) = 3$.

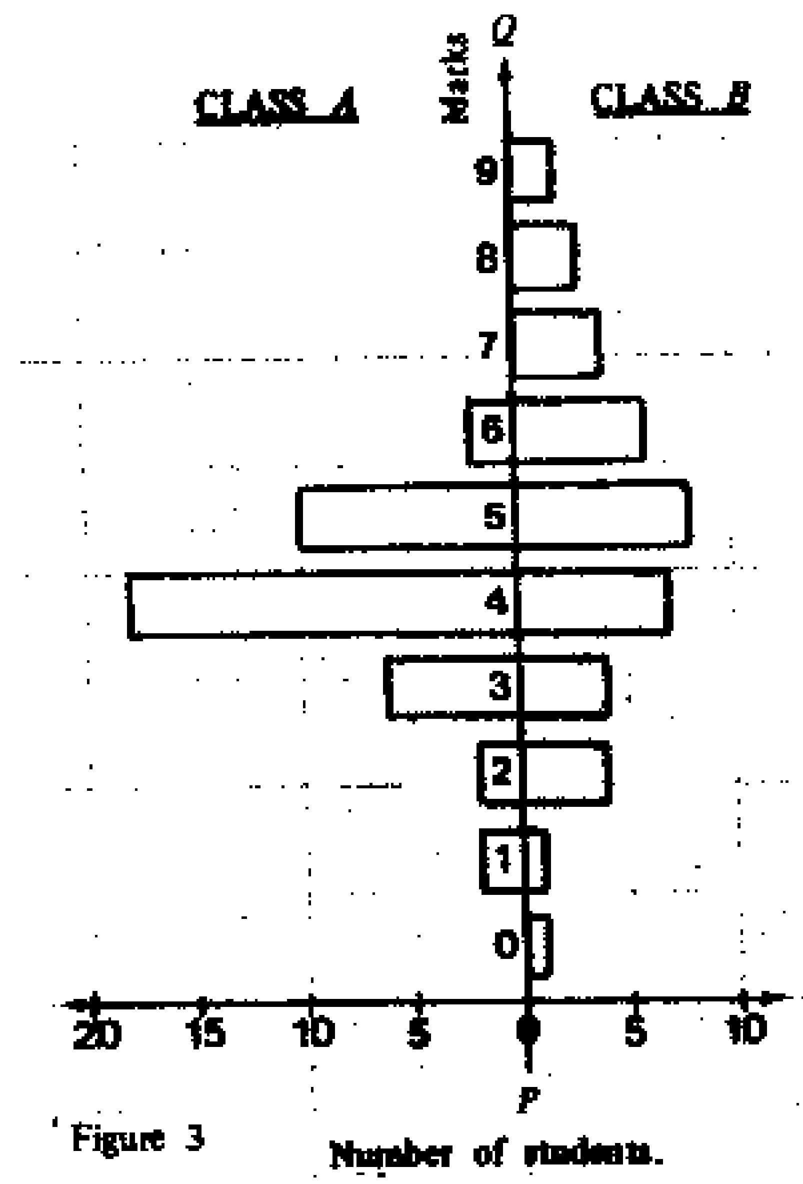
(5 marks)

8. Two classes, A and B, each of 40 students, took a test. In the test, students may score 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9 marks. In Figure 3, the distribution of marks of class A is shown in the bar chart on the left of PQ and that of class B is shown on the right.

(a) Find, by inspection, which class has a greater standard deviation of marks.

(b) If 70 students from the two classes pass the test, what is the minimum mark that a student should get in order to obtain a pass?

(5 marks)



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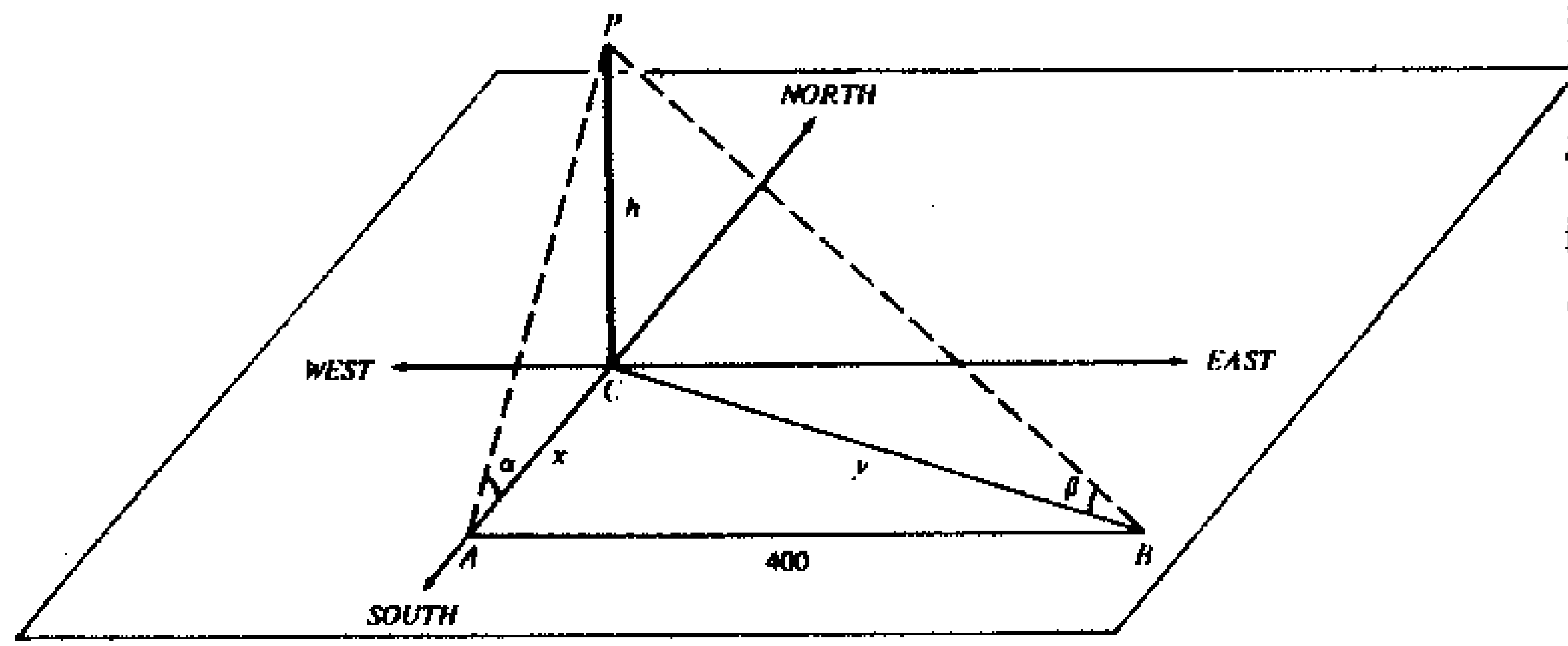

Figure 4

In Figure 4, PC represents a vertical object of height h metres. From a point A, south of C, the angle of elevation of P is α. From a point B, 400 metres east of A, the angle of elevation of P is β. AC and BC are x metres and y metres respectively.

(a) (i) Express x in terms of h and α.

(ii) Express y in terms of h and β.

(4 marks)

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10.

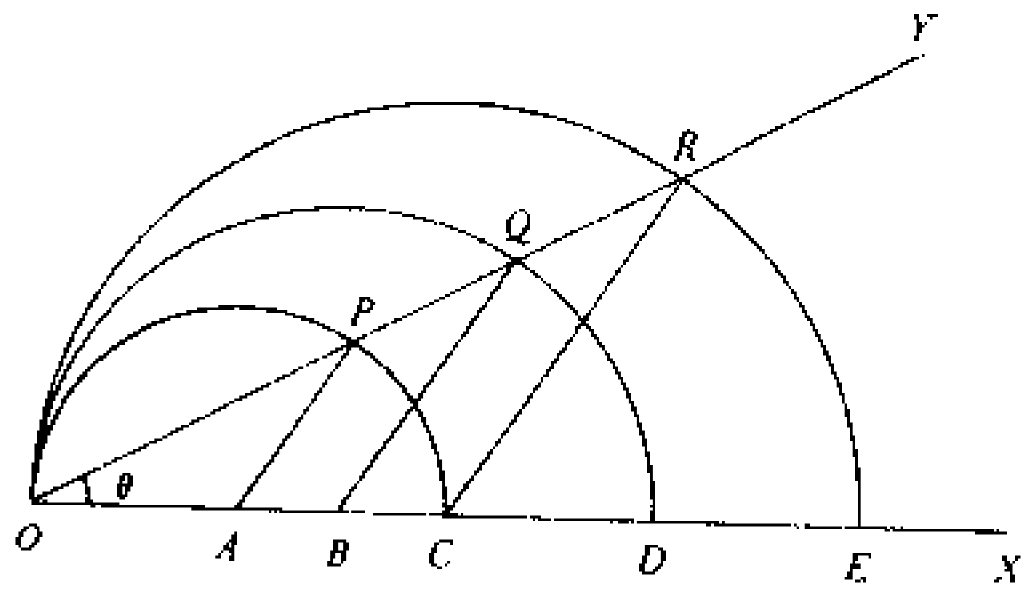


Figure 4

A, B and C are three points on the line OX such that $OA = 2$, $OB = 3$ and $OC = 4$. With A, B, C as centres and OA, OB, OC as radii, three semi-circles are drawn as shown in Figure 4. A line OY cuts the three semi-circles at P, Q, R respectively.

(a) If $\angle YOX = \theta$, express $\angle PAX$, $\angle QBX$ and $\angle RCX$ in terms of θ.

(3 marks)

(b) Find the following ratios:-

area of sector OAP : area of sector OBQ : area of sector OCR.

(4 marks)

(c) If $RD \perp OX$, calculate the angle θ.

(3 marks)

11. Let $k > 0$.

(a) (i) Find the common ratio of the geometric progression

$$k,\ 10k,\ 100k.$$

(ii) Find the sum of the first n terms of the geometric progression

$$k,\ 10k,\ 100k,\ \ldots.$$

(3 marks)

(b) (i) Show that

$$\log_{10} k,\ \log_{10} 10k,\ \log_{10} 100k$$

is an arithmetic progression.

(ii) Find the sum of the first n terms of the arithmetic progression

$$\log_{10} k,\ \log_{10} 10k,\ \log_{10} 100k,\ \ldots.$$

Also, if $n = 10$, what is the sum?

(7 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page

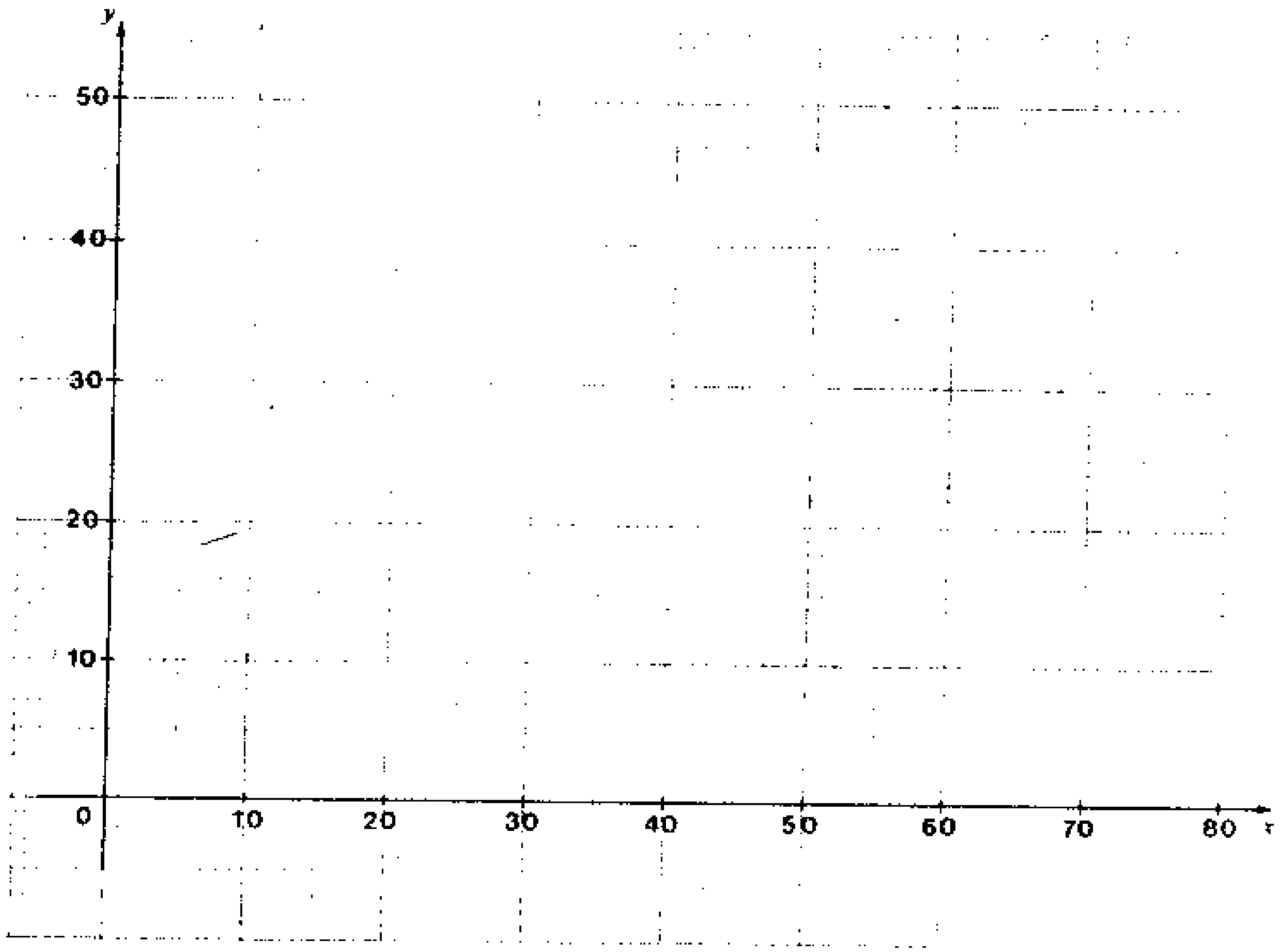
12. **If you attempt this question, fill in the details in the first three boxes above and tie this sheet into your answer book.**

An airline company has a small passenger plane with a luggage capacity of 720 kg, and a floor area of 60 m^2 for installing passenger seats. An economy-class seat takes up 1 m^2 of floor area while a first-class seat takes up 1.5 m^2. The company requires that the number of first-class seats should not exceed the number of economy-class seats. An economy-class passenger cannot carry more than 10 kg of luggage while a first-class passenger cannot carry more than 30 kg of luggage.

The profit from selling a first-class ticket is double that from selling an economy-class ticket. If all tickets are sold out in every flight, find graphically how many economy-class seats and how many first-class seats should be installed to give the company the maximum profit.

(10 marks)

(Let x be the number of economy-class seats installed,
y be the number of first-class seats installed.)



13. A, B, P are three points in the xy-plane whose position vectors are respectively :

$$\vec{OA} = 3\vec{i} + 4\vec{j},$$
$$\vec{OB} = 8\vec{i} - 6\vec{j},$$
$$\vec{OP} = x\vec{i} + y\vec{j}.$$

O is the origin, $\vec{i}$ and $\vec{j}$ are perpendicular unit vectors as shown in Figure 5.

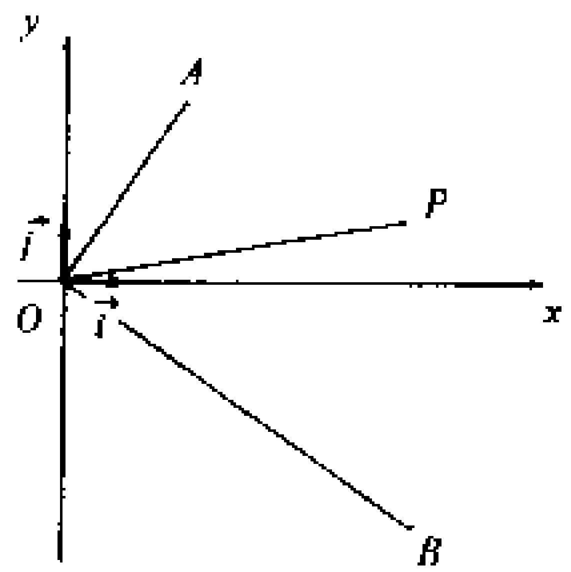


Figure 5

(a) (i) Evaluate $(3\vec{i} + 4\vec{j}) \cdot (x\vec{i} + y\vec{j})$.

(ii) Find $|\vec{OA}|$ and $|\vec{OP}|$.

(iii) Hence, express $\cos \angle AOP$ in terms of x and y.

(4 marks)

(b) Express $\cos \angle BOP$ in terms of x and y.

(3 marks)

(c) Using the results of (a) and (b), find the equation of the internal bisector of $\angle AOB$.

(3 marks)

14. The examination for a professional qualification consists of a theory paper and a practical paper. To obtain the qualification, a candidate has to pass both papers. If a candidate fails in either paper, he need only sit that paper again.

The probabilities of passing the theory paper for two candidates A and B are both $\frac{9}{10}$ and their probabilities of passing the practical paper are both $\frac{2}{3}$. Find the probabilities of the following events :

(a) Candidate A obtaining the qualification by sitting each paper only once.

(3 marks)

(b) Candidate A failing in one of the two papers but obtaining the qualification with one re-examination.

(4 marks)

(c) At least one of the candidates A and B obtaining the qualification without any re-examination.

(3 marks)

15. The circle

$$x^2 + y^2 - 10x + 8y + 16 = 0$$

cuts the x-axis at A and B and touches the y-axis at T as shown in Figure 6.

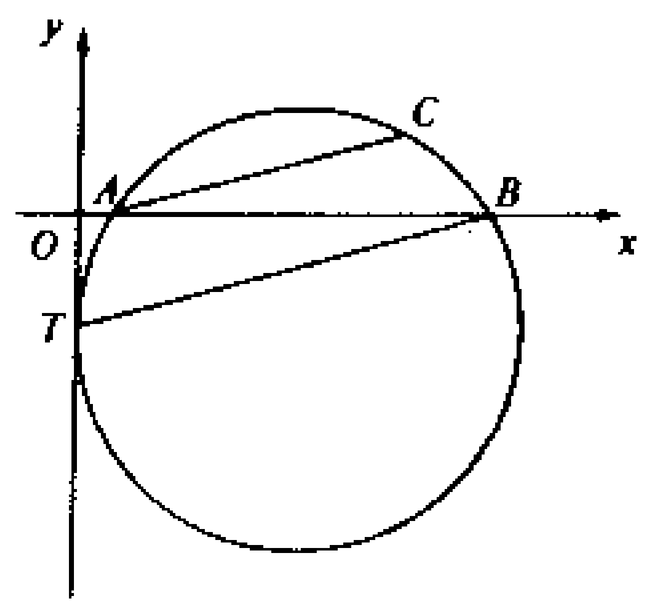


Figure 6

(a) Find the coordinates of A, B and T.

(5 marks)

(b) C is a point on the circle such that $AC \parallel TB$.

(i) Find the equation of AC.

(ii) Find the coordinates of C by solving simultaneously the equation of AC and the equation of the given circle.

(5 marks)

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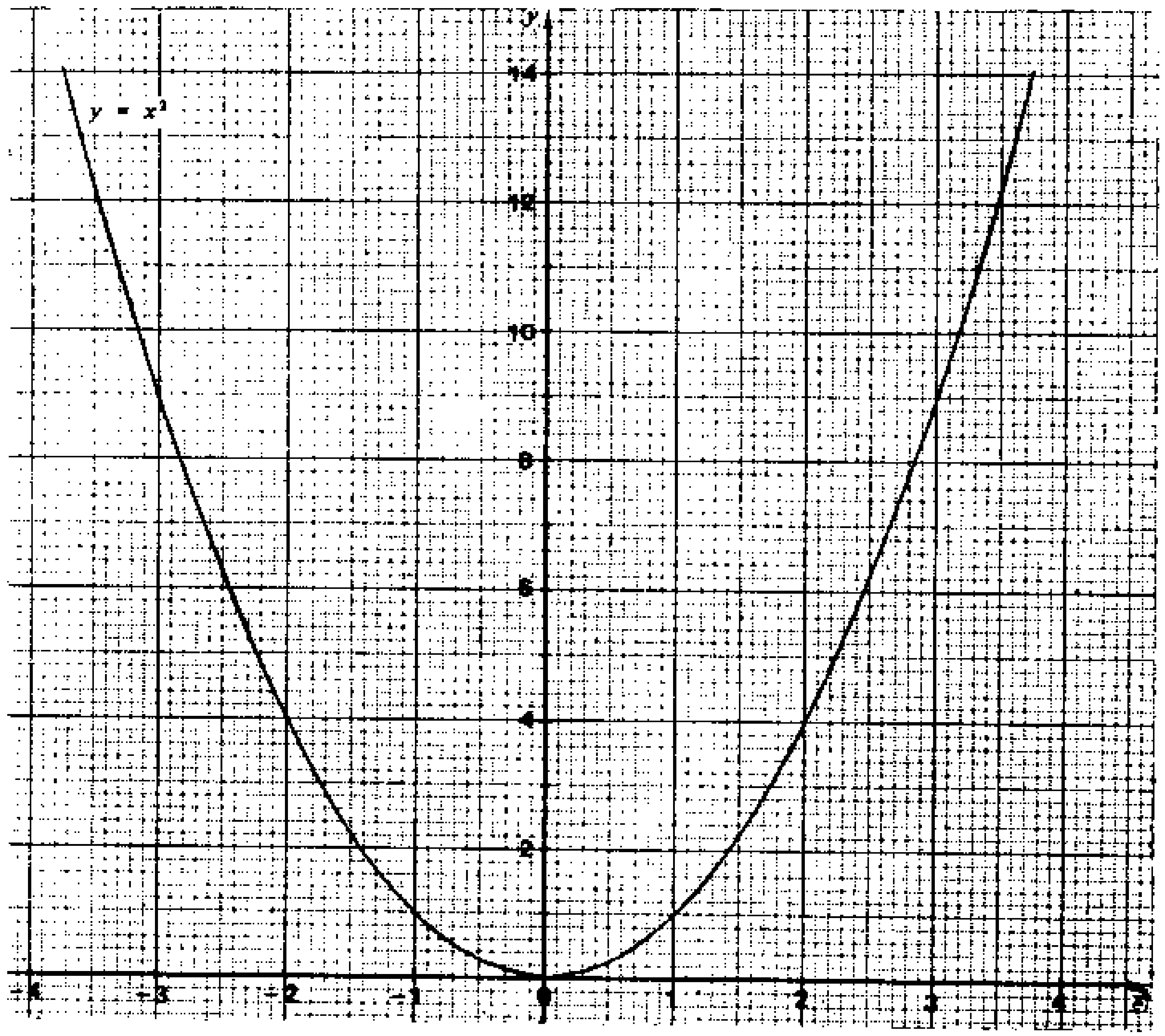

Figure 7

Figure 7 shows the graph of $y = x^2$. By drawing suitable lines in the same figure, solve the following :

(a) $x^2 - 2x - 5 = 0$,

(4 marks)

(b) $x^2 - 2x - 5 > 0$,

(2 marks)

(c) $2x^2 - 2x - 5 = 0$.

(4 marks)

(Answers should be correct to 1 decimal place. All straight lines should be labelled.)

END OF PAPER

(ALT)

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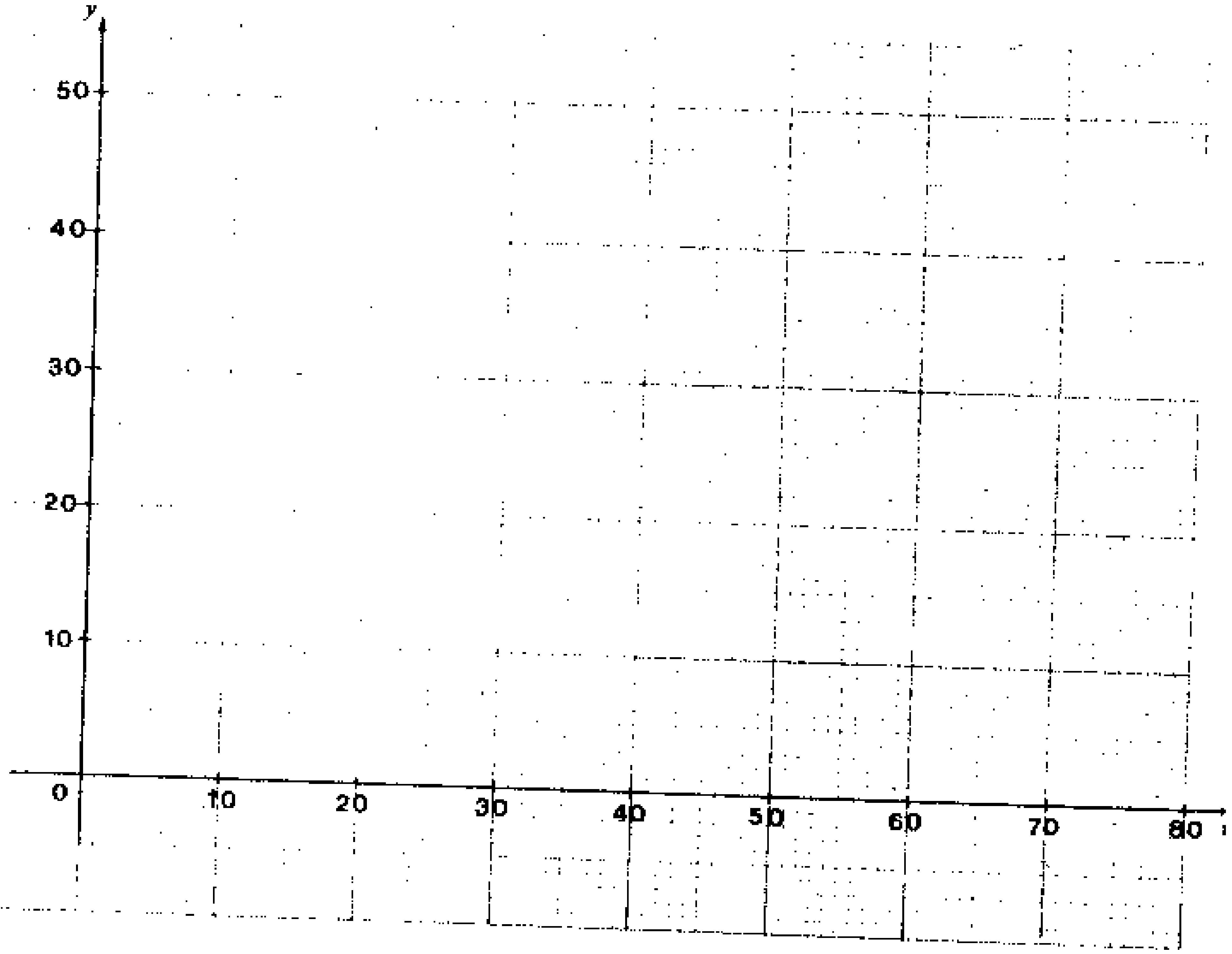


13. (a) It is given that $f(x) = 2x^2 + ax + b$.

(i) If $f(x)$ is divided by $(x - 1)$, the remainder is -5.
If $f(x)$ is divided by $(x + 2)$, the remainder is 4.
Find the values of a and b.

(ii) If $f(x) = 0$, find the value of x.

(6 marks)

(b) Solve the equation

$$1 - 2x = \sqrt{2 - x}.$$

Check to see whether the solutions satisfy the equation.

(4 marks)

14.

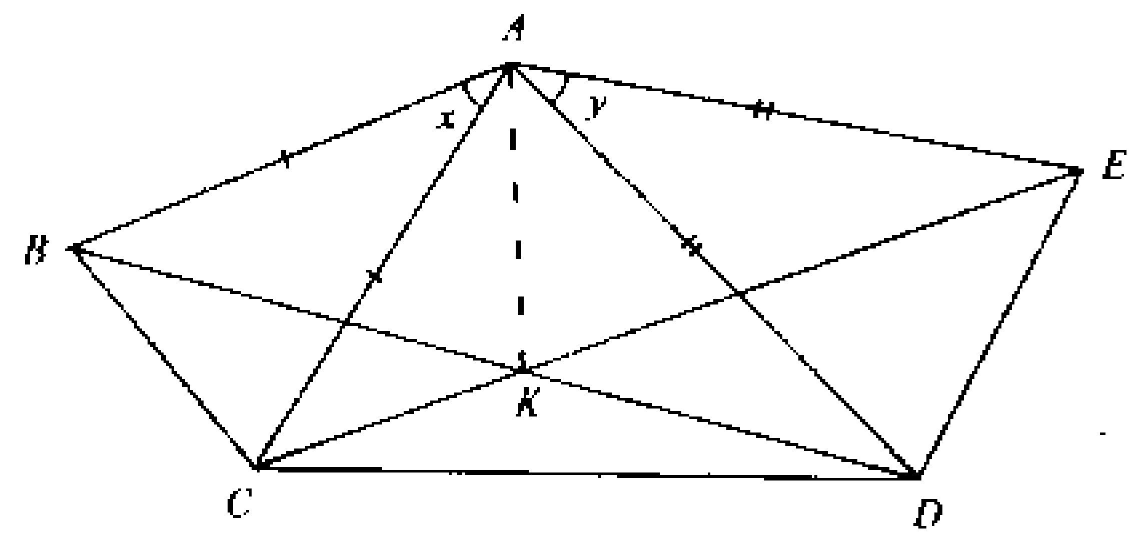


Figure 6

In Figure 6, $AB = AC$, $AD = AE$, $\angle x = \angle y$. Straight lines BD and CE intersect at K.

(a) Prove that ΔABD and ΔACE are congruent.

(5 marks)

(b) Prove that $ABCK$ is a cyclic quadrilateral.

(3 marks)

(c) Besides the quadrilateral $ABCK$, there is another cyclic quadrilateral in the figure. Write it down (proof is not required).

(2 marks)

15. In ΔABC (see Figure 7), $BD = \frac{1}{4} AB$, $CE = \frac{1}{3} AC$, BE intersects CD at P. $\angle x = \angle y$.

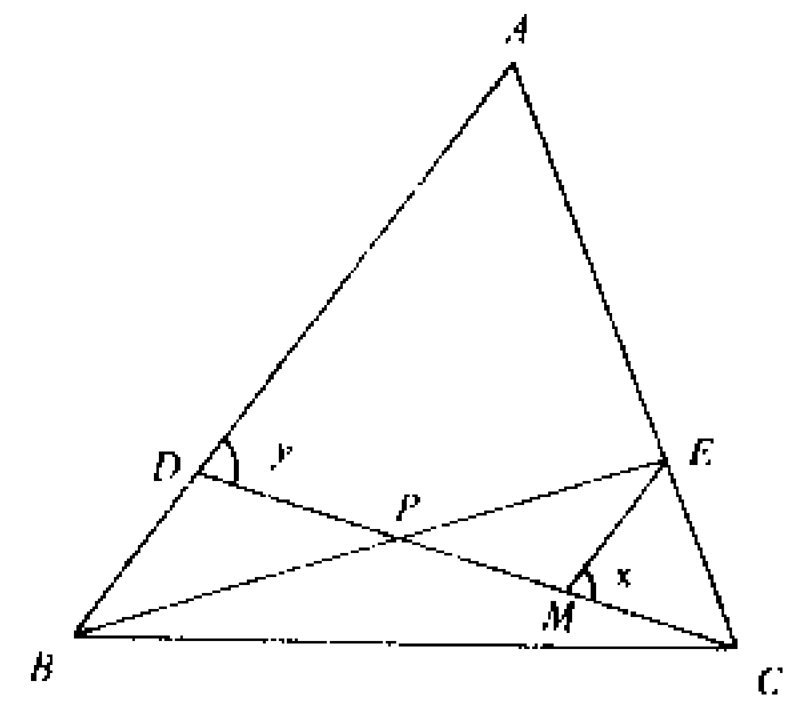
Prove that

(a) ΔEMC and ΔADC are similar and $EM = \frac{1}{4} AB$. (4 marks)

(b) ΔBDP and ΔEMP are congruent. (2 marks)

(c) $PM = CM$. (2 marks)

(d) area of triangle BDP is half the area of triangle PEC. (2 marks)



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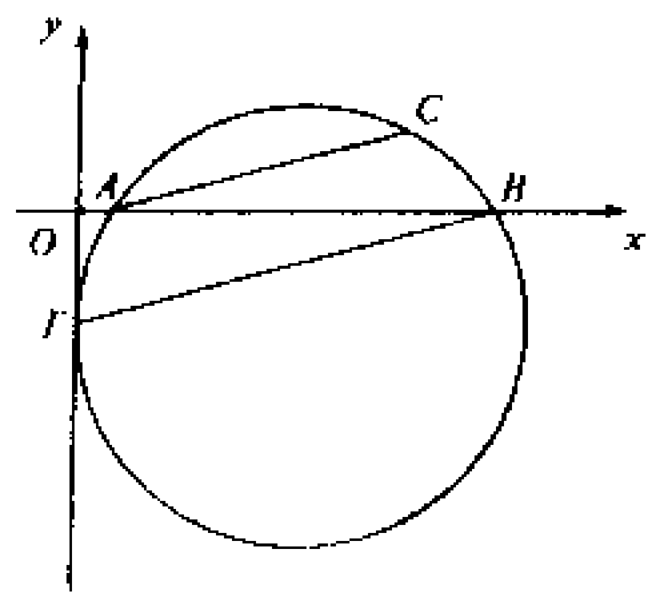

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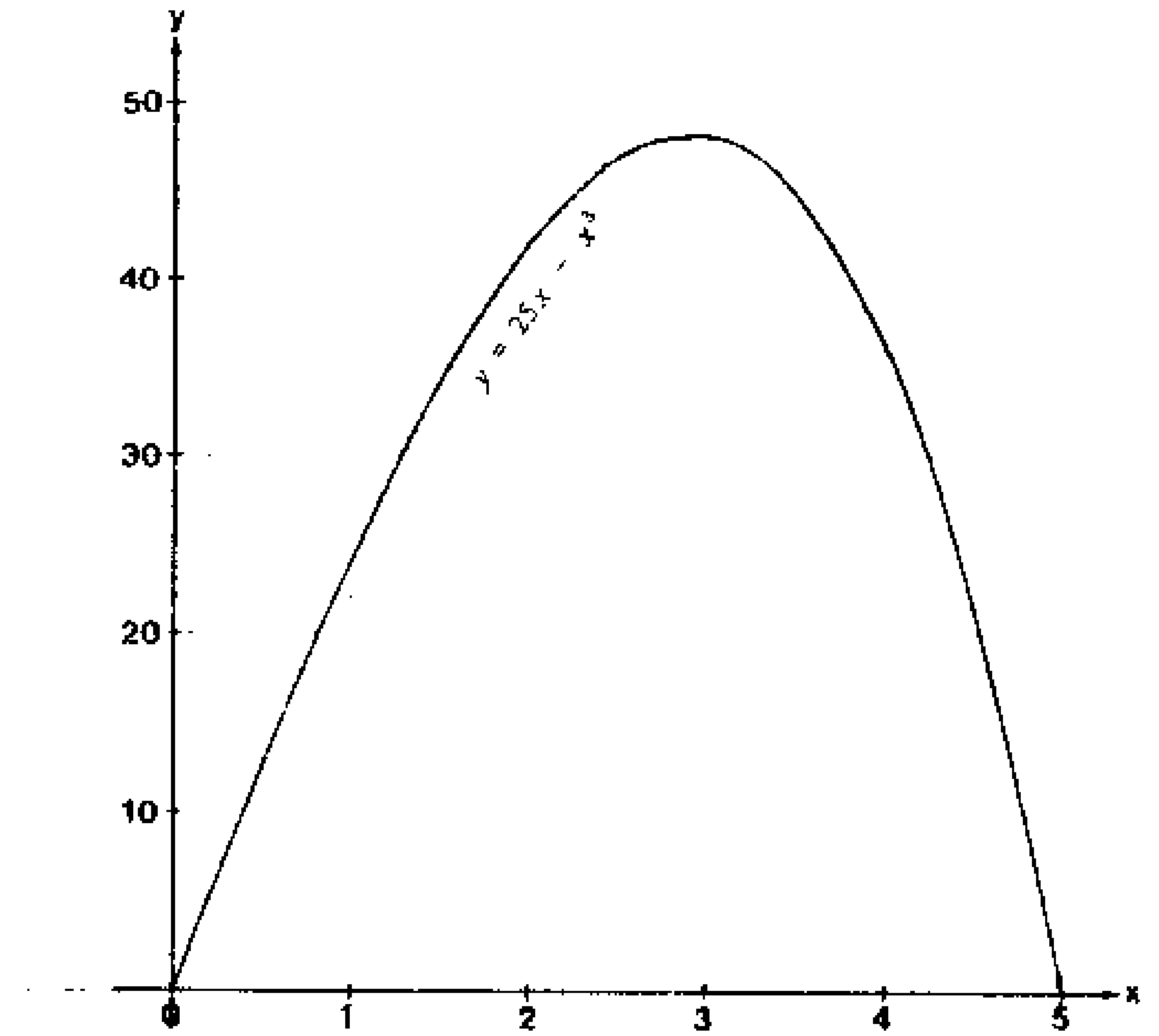

Figure 7

(a) Figure 7 shows the graph of $y = 25x - x^3$ for $0 \leq x \leq 5$. By adding a suitable straight line to the graph, solve the equation

$$30 = 25x - x^3,$$

where $0 \leq x \leq 5$. Give your answers correct to 2 significant figures.

(2 marks)

(b) Figure 8 shows a right pyramid with a square base $ABCD$. $AB = b$ units and $AE = 5$ units. The height of the pyramid is h units and its volume is V cubic units.

(i) Express b in terms of h.
Hence show that $V = \frac{2}{3}(25h - h^3)$.

(3 marks)

(ii) Using (a), find the two values of h such that $V = 20$.
(Your answers should be correct to 2 significant figures.)

(2 marks)

(iii) Use the "method of magnification" to find the smaller value of h in (b) (ii) correct to 3 significant figures.

(3 marks)

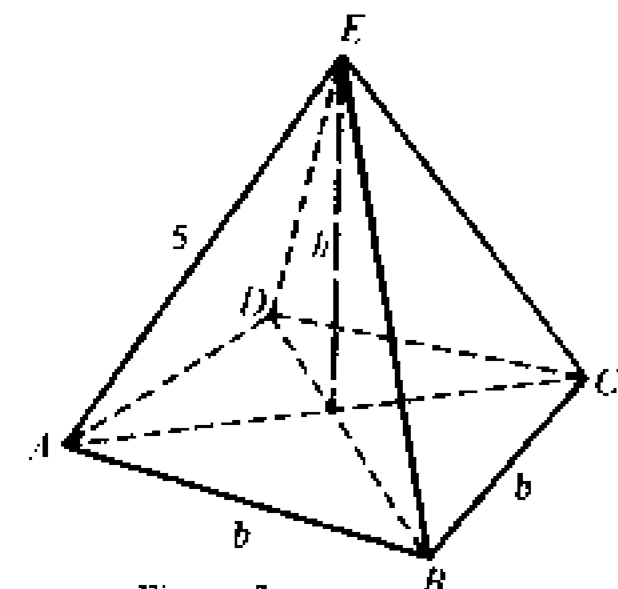

Figure 8

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