Mathematics

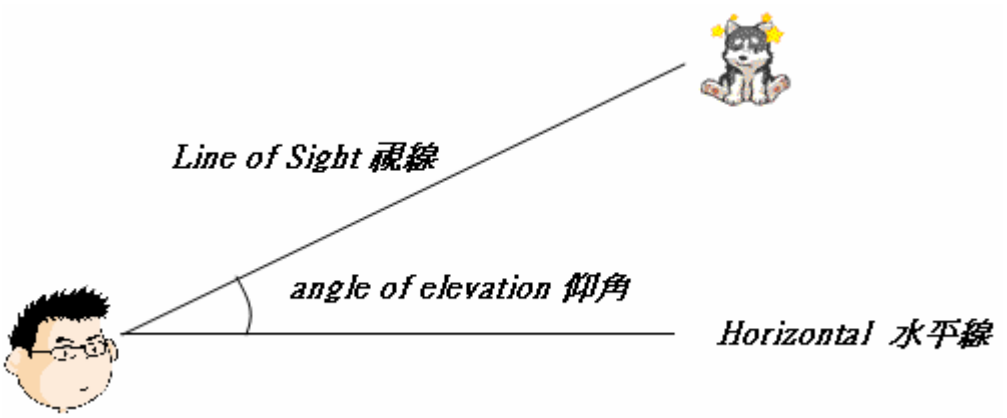
數學科

2-D Trigonometry

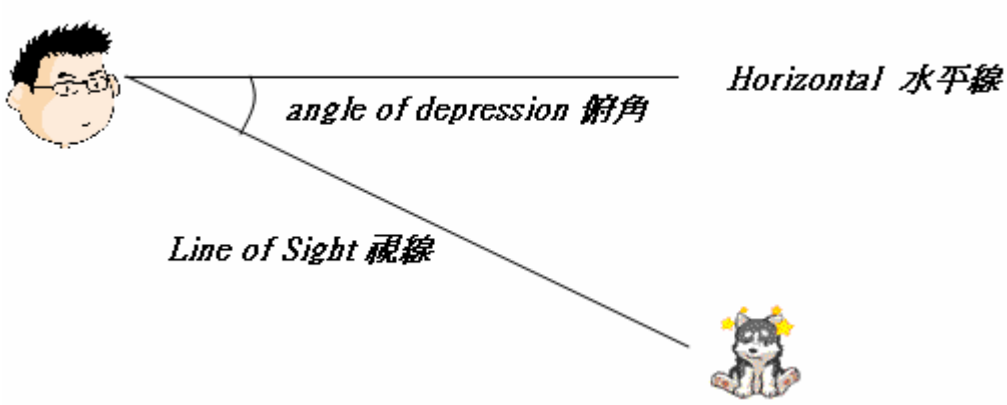
2-D 三角學

Angle of elevation and depression 仰角及俯角

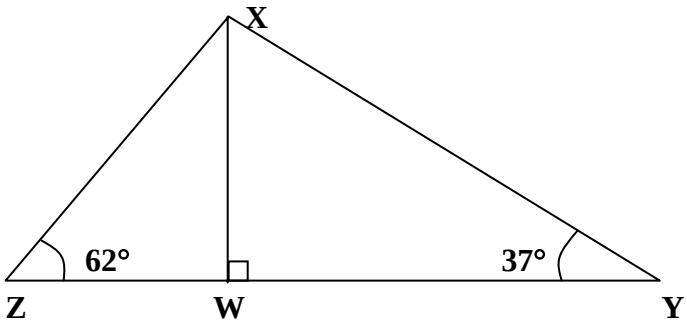
1. Angle of elevation 仰角



2. Angle of depression 俯角



e.g.1 例子一



Referring to the figure, find [參看附圖，求]

(a) angle of elevation of X from Z. [由 Z 測得 X 的仰角.] **(1 Mark 1 分)**

(b) angle of depression of Z from X. [由 X 測得 Z 的俯角.] **(1 Mark 1 分)**

(c) angle of elevation of X from Y. [由 Y 測得 X 的仰角.] **(1 Mark 1 分)**

(d) angle of depression of Y from X. [由 X 測得 Y 的俯角.] **(1 Mark 1 分)**

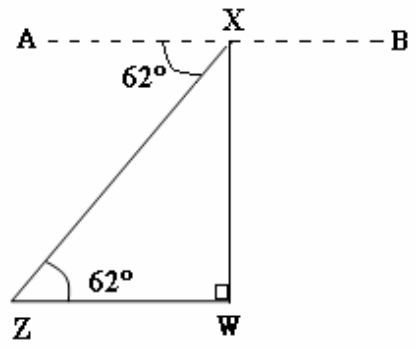
Solutions 題解:

(a) and (b)

Consider 考慮 ΔXWZ **,**

$\angle XZW = 62^\circ$

So Angle of elevation of X from Z = 62°. ---------- (1A)



Adding a dotted line AB such that AB // WZ

加一條虛線 AB 而 AB // WZ.

$\angle WZX = \angle AXZ = 62^\circ$ **(alt.∠錯角, AB // WZ)**

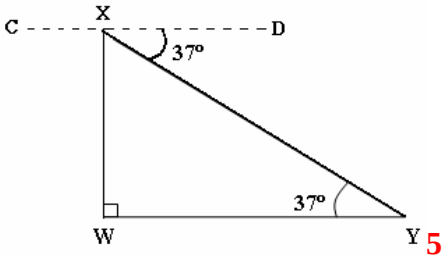
So Angle of depression of Z from X = 62°. -------- (1A)

(c) and (d)

Consider 考慮 ΔXWY **,**

$\angle XYW = 37^\circ$

So Angle of elevation of X from Y = 37°. ---------- (1A)



Adding a dotted line CD such that CD // WY

加一條虛線 CD 而 CD // WY.

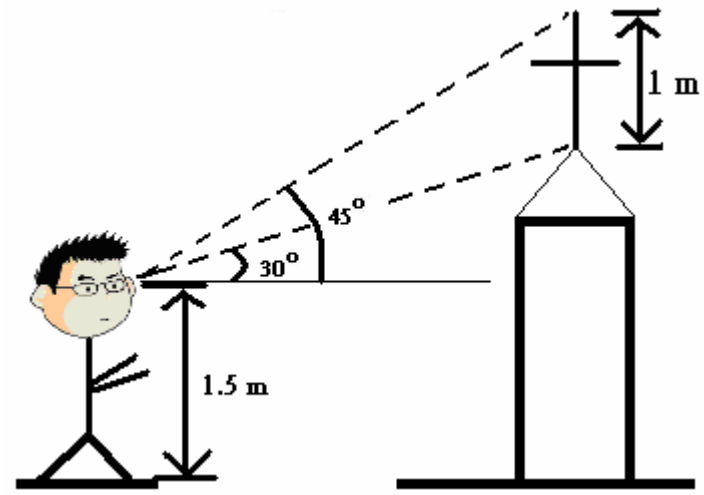
$\angle WYX = \angle DXY = 37^\circ$ **(alt.∠錯角, CD // WY)**

So Angle of depression of Y from X = 37°. -------- (1A)

5

1.

2.



*** **e.g.2 例子二**

In the figure above, A cross with 1 m length is installed on the roof of a church model. Andy records that the angle of elevation of the top and the bottom of the cross is 45° and 30° respectively. Assume Andy's eyes are 1.5 m above his foot.

(a) Find the height of the church model(excluding the length of the cross). (5 marks)

(b) What is the distance between Andy and the church model? (2 marks)

例子二

一個長 1m 的十字架垂直安裝在一教堂模型的頂部. Andy 測得這十字架頂部及底部的仰角分別是 45°及 30°. 假設 Andy 的眼在他的腳之上 1.5 m.

(a) 求這教堂模型的高度(不包括十字架長度). (5 分)

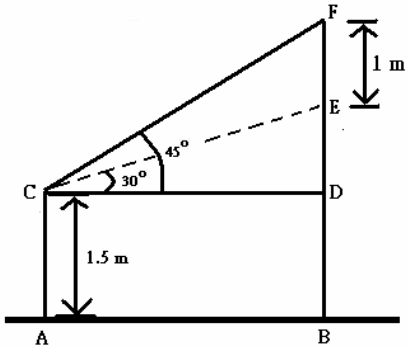
(b) 問 Andy 距離教堂模型有多遠？ (2 分)

Solutions 題解:

(a)

Label the diagram as shown:

標示圖像如下:



The height of the church model 教堂模型的高度

= BE

= BD + DE

= AC + DE

(Rectangular Properties, opposite side equal)

(長方形特性: 對邊相等)

= 1.5m + DE ---------- (1M)

Consider 考慮 ΔCDE **,**

$$\tan 30^\circ = \frac{DE}{CD},$$

$CD = \dfrac{\tan 30^\circ}{DE}$. ---------- **(1)**

Consider 考慮 ΔCDF **,**

$\tan 45^\circ = \dfrac{DF}{CD} = \dfrac{DE + EF}{CD} = \dfrac{DE + 1}{CD}$,

$CD = \dfrac{DE + 1}{\tan 45^\circ}$. ---------- **(2)**

(1) = (2)

$\dfrac{\tan 30^\circ}{DE} = \dfrac{DE + 1}{\tan 45^\circ}$ ---------- **(1M)**

$(\tan 30^\circ)(\tan 45^\circ) = DE^2 + DE$

$DE^2 + DE - (\tan 30^\circ)(\tan 45^\circ) = 0$

$DE^2 + DE - (\dfrac{\sqrt{3}}{3})(1) = 0$

$DE = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-\dfrac{\sqrt{3}}{3})}}{2(1)}$

$DE = -\dfrac{1}{2} \pm \sqrt{(\dfrac{3 + 4\sqrt{3}}{12})}$

$DE = 0.3274$ **or** $DE = -1.3274$ **(rejected 捨棄)**

$DE = 0.33m$ ---------- **(1A)**

The height of the church model 教堂模型的高度

= 1.5m + DE

= 1.5m + 0.33m

= 1.83m ---------- (1A)

(b)

The distance between Andy and the church model

Andy 和教堂模型之間的距離

$= CD$

$= \dfrac{DE}{\tan 30^\circ}$

$= \sqrt{3}(0.3274)$

$= 0.5671$

$= 0.57m$ ---------- **(1A)**